Efficiency and Redistribution Effects of Environmental Taxes under Capital Mobility

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Abstract

When capital taxes are used to finance a public good, capital mobility reduces efficiency since it narrows the tax base. This motivates the concern that capital mobility might also interfere with unilateral efforts to provide the public good of environmental quality through pollution taxes. I show in this paper that capital mobility does not affect efficiency in a small open economy, even though the pollution taxes reduce capital productivity. Capital mobility does, however, affect the distribution of the policy effects within the domestic economy. Compared to a closed economy setting capital income is higher, while labor income bears the full burden of the policy.

JEL classification:

Keywords: capital mobility, climate policy, competitiveness
Indeed, we have granted large reductions in energy prices for energy intensive firms... because we have a strong interest to maintain jobs in the energy sector in Germany.

Peter Altmaier, German minister of the environment, in BMU (2013)

1 Introduction

Politicians frequently claim that climate policy would cost jobs because firms might relocate abroad. This concern can be understood as a hypothesis that capital mobility introduces a reduction in aggregate household income in excess of the reduction that would occur if capital were prevented from crossing borders. Using a static short-term analysis (without technological adaptation) I find that capital does indeed leave the country following the introduction of a tax on carbon dioxide pollution. However, the exact same combinations of pollution reduction and income loss can be achieved under autarky and with capital mobility. Capital mobility thus does not limit the solution space for politicians in implementing climate policy.

When carbon tax revenues are returned to households lump-sum then income must decrease, whether capital is mobile or not. The reason is that the tax increases the effective pollution price and thus reduces its supply. The difference thus lies in how income is reduced. In the closed economy, capital simply becomes less productive. When capital is mobile, less capital is used for domestic production, but the marginal productivity remains constant (at the level of the international interest rate). These two effects have exactly the same magnitude.

The importance to distinguish between the “baseline” effect of carbon taxes and the additional effect of capital mobility can also be seen in the distributional effect. Carbon taxes introduce a comparatively large loss in labor income for workers in energy intensive industries, but this is true independent of capital mobility. The additional effect of capital mobility consists of shifting losses from capital owners towards workers, in particular in capital intensive industries.

Rauscher (1991) and Kim and Wilson (1997) consider Nash equilibria of environmental policy under capital mobility. They focus on the effect of capital mobility on the international interest rate and the efficiency of foreign taxation, both identifying a potential of cooperation. By contrast, this paper focuses on the effect of environmental taxes on domestic income, both at the aggregate level as well as for sectors with different production functions.
The effect of carbon taxes in an open economy has been investigated intensively in international trade. A comprehensive review is provided by Copeland and Taylor (2004). The key question here is the net environmental effect of a unilateral climate policy when trade patterns can adjust and shift the polluting industry abroad. As Eichner and Pethig (2011) point out a further international link is provided by the world demand for fossil resources. Contrary to this literature, I consider the non-environmental domestic effect and capital mobility instead of trade. While shifts in trade patterns affects the type of production in a given country, the volume of production is at stake when production factors are mobile.

Since the amount of carbon dioxide pollution used in production affects the marginal productivity of capital, this paper is also related to the literature on capital taxes under capital mobility. Early treatments like Gordon (1986) and Razin and Sadka (1991) find that capital mobility introduces an additional distortion, the race to the bottom, which makes capital taxes unattractive for revenue generation. More recent treatments like Baldwin and Krugman (2004) weaken this result, in this case because of agglomeration economies. Nevertheless, this literature inspires the concern that capital mobility might interact negatively with carbon taxes.

Böhringer and Rutherford (1997) and Babiker et al. (2003) also investigate the introduction of climate policy under capital mobility. However, they provide a quantitative analysis, using a calibrated numerical model. In contrast to these papers, I identify the theoretical mechanisms at work. The paper most closely related might be De Mooij and Bovenberg (1998). It also investigates environmental taxes both with and without capital mobility. However, it focuses on the most efficient way of introducing environmental taxes, while this paper investigates the effect of capital mobility on the effect of pollution taxes.

Section 2 derives the effect of capital mobility from the point of view of aggregate efficiency. Section 3 considers an extension of the model to two sectors, which allows analyzing distributional effects between the two sectors. Section 4 briefly discusses further extensions of the model, which will be addressed in future drafts. Section 5 concludes.

2 Aggregate Efficiency

In this section we consider a representative household, which earns labor income and receives capital revenue. This allows comparing how capital mobility affects aggregate efficiency in the case of public good provision through capital taxes and and in the case of environmental
regulation through pollution taxes.

2.1 The Model

We consider a small open economy. It faces an international interest rate $\rho$, which it cannot influence.

*Firms.* Production requires capital $k$, (carbon dioxide) pollution $p$ and labor. Since labor is supplied in a fixed amount we do not note it explicitly,

$$ y = f(k,p) .$$

(1)

Capital is borrowed from either foreign or domestic investors at the international rate. We assume the production function to be concave and $f_{kp} > 0$. There is a pollution tax $t_p$ and a capital tax $t_k$. Profits are given by

$$ \Pi = f(k,p) - (\rho + t_k)k - t_pp .$$

(2)

The first order conditions of the firm are

$$ f_k = \rho + t_k ,$$

(3)

$$ f_p = t_p .$$

(4)

*Household.* The representative household owns an amount of $a$ in capital (“assets”), for which it earns capital income of $\rho a$ on the international market.

It supplies a fixed amount of labor to the firm. As the owner of the firm it receives the entire firm revenue less expenses for capital and pollution as wage payment,

$$ w = \Pi .$$

(5)

As we do not make the assumption that the production function is homogeneous of degree one, this wage payment might exceed the marginal productivity of labor, so that it effectively includes profits. I use this wage as a proxy for household welfare. Instead of explicitly considering unemployment, a reduction in household welfare will take the form of reducing wages.
Government. Government revenue stems from taxes,

\[ G = t_p p + t_k k . \]  

(6)

It may be used for either public good provision or returned to households as transfers.

Autarky. In order to identify which of the reactions to the policy experiments considered below stem from capital mobility, we consider an autarkic version of the described economy for comparison. For this we replace the assumption of the international capital market with the assumption that the domestic economy can make use only of domestically owned capital, \( k = a \). Firms thus face fixed amounts of capital and labor and can decide only on the amount of pollution to be used. The optimal decision in this case is again

\[ f_p = t_p . \]  

(7)

2.2 Public Good Provision through Capital Taxes

In this part we briefly present the “race to the bottom” effect described in Wilson (1986), Zodrow and Mieszkowski (1986) and others for comparison. We assume that the government uses tax revenues to provide a public good.

Total household income is given as the sum of wages and capital income,

\[ x = f(k, p) - (\rho + t_k)k - t_p p + \rho a . \]  

(8)

An increase in capital taxes has the following effect on income and public good provision,

\[ \frac{dx}{dt_k} \bigg|_{Mobility} = -k , \]  

(9)

\[ \frac{dG}{dt_k} \bigg|_{Mobility} = k + t_k \frac{dk}{dt_k} + t_p \frac{dp}{dt_k} . \]  

(10)

The terms \( \frac{dk}{tk} = \frac{f_{pp}}{f_{kk}f_{pp} - f_{kp}} < 0 \) and \( \frac{dp}{tk} = \frac{-f_{kp}}{f_{kk}f_{pp} - f_{kp}} < 0 \) are specific to the case of capital mobility. The increase in the capital tax drives capital out of the country. This does not affect income since this capital is paid its marginal productivity. However it does affect public good provision since it narrows the tax base. The pollution tax base is reduced since the firm employs less pollution when it reduces capital.

In the autarkic economy, all of the domestic capital remains in domestic use, \( k = a \). The
firm does not adjust the amount of pollution employed. Therefore additional capital taxation moves private consumption towards public goods without any loss,

\[
\frac{dx}{dt} \bigg|_{\text{Autarky}} = -k, \tag{11}
\]

\[
\frac{dG}{dt} \bigg|_{\text{Autarky}} = k. \tag{12}
\]

We can summarize this result in the following way:

**Proposition 1** *Capital mobility makes the transformation of private goods into public goods through capital taxation less efficient.*

When a utility function for the household with private consumption and public good provision is assumed than capital mobility makes a difference. When the transformation is less efficient, the optimal combination is a bundle with more private consumption and less public goods. The optimal capital tax under capital mobility is thus lower than in autarky. This explains the expression “race to the bottom”.

### 2.3 Environmental Regulation through Pollution Taxes

We now assume that the government aims to improve environmental quality by taxing pollution. It returns tax revenues to the households lump-sum as transfers, \( T = G \). Total household income, the amount available for consumption, is thus

\[
x = w + \rho a + T = f(k, p) - \rho k + \rho a. \tag{13}
\]

Making use of the first order conditions in (3) and (4) the effect of carbon taxes on income and pollution can be written as

\[
\frac{dx}{dt} \bigg|_{\text{Mobility}} = t_p \frac{dp}{dt}, \tag{14}
\]

\[
\frac{dp}{dt} \bigg|_{\text{Mobility}} = \frac{f_{kk} f_{kp} - f_{kp}^2}{f_{kk} f_{pp} - f_{kp}^2} < 0. \tag{15}
\]

Under autarky, the domestic economy can use only its own capital \( k = a \). Income is thus

\[
x = f(k, p) \tag{16}
\]
Since capital is fixed, the firm determines pollution using the first order condition on pollution (7) alone.

The effect of carbon taxes on income and pollution is thus given by

\[
\frac{dx}{dt} \bigg|_{\text{Autarky}} = t_p \frac{dp}{dt}, \\
\frac{dp}{dt} \bigg|_{\text{Autarky}} = \frac{1}{f_{pp}} < 0.
\]

(17) (18)

Comparing equations (14) and (15) with (17) and (18) we can make two observations. One is that the transformation from private consumption to the policy objective is possible at the same rate in both cases. The ratio is always \( t_p \). The other observation is that in order to achieve a certain income/pollution combination an economy with capital mobility has to set a different pollution tax than the autarkic economy, since pollution reacts with a different sensitivity to changes in the carbon tax. Depending on the functional form of the production function, pollution may react more or less sensitively under capital mobility.

**Proposition 2** Capital mobility does not affect the efficiency of transformation of income into environmental quality. It does, however, call for a different tax rate to achieve a certain income/pollution combination.

Both under capital mobility and under autarky, the input of pollution reduces. Under capital mobility capital input also reduces:

\[
\frac{dk}{dt_p} = \frac{-f_{kp}}{f_{kk}f_{pp} - f_{kp}^2} < 0.
\]

(19)

This, however, does no harm to the economy as we have seen above. Under autarky, the domestic capital becomes less productive, while under capital mobility less capital remains in the domestic economy, but the marginal productivity remains high.

Coming to the application of this result, it means that a government wanting to achieve a given environmental objective can do so at the same cost under capital mobility and autarky. All the government needs to consider is that under capital mobility the tax rate might need to be set a bit different than in autarky since the economy reacts a bit more or less sensitively. The distortion from capital mobility works through the marginal productivity of pollution and can be positive or negative.
3 Distributional Aspects

In section 2, we used a single production function for the entire economy, implicitly making the assumption that production factors are perfectly mobile across sectors. When politicians motivate carbon tax exemptions for the energy intensive sector with a concern for employment in that sector, they might have mainly distributional concerns in mind. In this section, we thus relax the assumption of a representative household and extend the previous model to two sectors, one energy-intensive, the other not.

In order to compare the effect of climate policy on the employees in the two sectors, we assume that they cannot move from one sector to the other in this section. While capital is assumed to be mobile between the domestic and the foreign economy, labor is restricted to the sector in which it was employed originally. Pollution and capital are thus control variables of the firm, while labor is effectively exogenous.

3.1 The Model

Again, we consider a small open economy facing an international interest rate $\rho$.

**Firms.** Firms are modeled as in Section 2.1. They belong either to the energy intensive sector $E$ or the not energy intensive sector $N$. The production function and factor inputs are labeled $f^i, k^i, p^i$ with $i \in \{E, N\}$ accordingly. $f^E$ and $f^N$ are assumed concave and $f^E_{kp} > 0$ and $f^N_{kp} > 0$.

**Household.** As in section 2.3, household income has three components: wage income, capital income and transfers. A household works in either sector $E$ or sector $N$. It can not move from one sector to the other even when taxes are changed. The wage income of a household working in sector $i$ is thus given by

$$w^i = \Pi^i = f^i(k^i, p^i) - \rho k^i - t_{pp}^i, \quad \forall i \in \{E, N\}.$$  \hspace{1cm} (20)

The aggregate capital income is given $\rho a$.

I do not make any explicit assumption on which of the household owns how much capital and receives how many transfers. Instead, we consider these income components separately.

**Government.** The government returns pollution tax revenue lump-sum,

$$t_p(p^E + p^N) = T.$$  \hspace{1cm} (21)
Autarky. When no capital can leave or enter the domestic economy, firms must use the local capital, \( k^E + k^N = a \). Instead of the fixed international interest rate, there will be a national interest rate \( r \) and the marginal interest rate of the two sectors will be equal, \( f_k^E = r = f_k^N \). While the international interest rate \( \rho \) cannot be manipulated, the domestic interest rate \( r \) is subject to domestic manipulation. The marginal productivity of pollution will again equal to the pollution tax, \( f_p^E = t_p = f_p^N \).

3.2 The income components

We first consider the three income components capital income, transfers and wage income in the case of capital mobility. Since the interest rate is set internationally, the government sets the pollution tax and households are not mobile, there is actually no interaction between the sectors, so that the sectors resemble individual countries.

When capital is mobile, the income on domestic assets is unaffected at \( \rho a \), since both the international interest rate and the domestic asset level are exogenous.

The total amount of transfers may increase or decrease, depending on the initial level of the taxes,

\[
\left. \frac{dT}{dt} \right|_{\text{Mobility}} = p^E + p^N + t_p \left( \frac{dp^E}{dt_p} + \frac{dp^N}{dt_p} \right).
\]

(22)

Under capital mobility, the firms optimize in the same way as above so that \( \frac{\partial p^E}{dt_p} \) is given by (15) and is thus less than zero. The transfers show a Laffer curve behavior. As the tax rate increases tax revenue increases at first and then decreases.

The change in wages, given in (20), is given by

\[
\left. \frac{dw^i}{dt} \right|_{\text{Mobility}} = f_k^i \frac{dk^i}{dt_p} + f_p^i \frac{dp^i}{dt_p} - \rho \frac{dk^i}{dt_p} - p^i - t_p \frac{dp^i}{dt_p} = -p^i < 0.
\]

(23)

An increase in pollution taxes thus lowers income in both sectors.

In the closed economy the two sectors compete for scarce capital. The equilibrium in the endogenous variables \( k^N, k^E, p^N, p^E \) is thus given by capital constraint and the first order
conditions of the two sectors,

\[ k^E + k^N = a \]  \hspace{1cm} (24)

\[ f_k^E(k^E, p^E) = f_k^N(k^N, p^N) \]  \hspace{1cm} (25)

\[ f_p^E(k^E, p^E) = t_p \]  \hspace{1cm} (26)

\[ f_p^N(k^N, p^N) = t_p \]  \hspace{1cm} (27)

It will turn out that the effect of the pollution tax on the domestic interest rate \( r \) plays a central role. Using the first order condition \( r = f_k^N \) we obtain

\[
\frac{dr}{dt_p} = \frac{df_k^N}{dt_p} = f_{kk}^N \frac{dk^N}{dt_p} + f_{kp}^N \frac{dp^N}{dt_p} . \]  \hspace{1cm} (28)

We chose here to work with \( f^N \), but of course the same result will be obtained from \( f^E \). The effects of the pollution tax on capital and pollution, \( \frac{dk^N}{dt_p} \) and \( \frac{dp^N}{dt_p} \) can be obtained from the general equilibrium as defined in equations (24) to (27). This will be done in the proof of Proposition 3.

The capital return is now written as \( ra \). The effect of pollution taxes is given as

\[
\frac{d(ra)}{dt_p} = \frac{dr}{dt_p} a = \frac{dr}{dt_p} (k^E + k^N) . \]  \hspace{1cm} (29)

The effect of pollution taxes on transfers is given by

\[
\left. \frac{dT}{dt_p} \right|_{Autarky} = p^E + p^N + t_p \left( \frac{dp^E}{dt_p} + \frac{dp^N}{dt_p} \right) . \]  \hspace{1cm} (30)

The effect of the tax on pollution in the two sectors is, again, to be obtained from the general equilibrium. As for the case of autarky, there is a Laffer curve, so that the net effect of pollution taxes on transfers depends on the level of the pollution tax.

The wage in sector \( i \) can be written as

\[ w^i = \Pi^i = f^i(k^i, p^i) - rk^i - t_pp^i , \quad \forall i \in \{E, N\} . \]  \hspace{1cm} (31)
The reaction of the wage in sector $i$ to the pollution tax rate is thus given by

$$\frac{dw^i}{dt_p} \bigg|_{\text{Autarky}} = f_k^i \frac{dk^i}{dt_p} + f_p^i \frac{dp^i}{dt_p} - \frac{dr}{dt_p} k^i - \frac{dp^i}{dt_p} - p^i - t_p \frac{dp^i}{dt_p}$$

$$= - \frac{dr}{dt_p} k^i - p^i \quad . \quad (32)$$

We can now analyze the differential effect of capital mobility on the three income components.

**Proposition 3** When pollution taxes are increased,

(i) capital income is less affected under capital mobility

$$\frac{d(pa)}{dt_p} = 0 > \frac{dr}{dt_p} a = \frac{d(ra)}{dt_p} ,$$

(ii) wage income is more affected under capital mobility

$$\frac{dw^i}{dt_p} \bigg|_{\text{Mobility}} = -p^i < - \frac{dr}{dt_p} k^i - p^i \quad = \quad \frac{dw^i}{dt_p} \bigg|_{\text{Autarky}} ,$$

(iii) transfers may be more or less affected.

The income components are thus affected differently under capital mobility and autarky. Capital owners benefit from capital mobility since the international market provides a certain source of income. This is not the case in autarky since distortionary taxes reduce the productivity of capital. As we have seen in Section 2.3, the economic loss per unit of pollution reduced is unaffected by capital mobility. However, under autarky, this loss is realized by the wage income and the capital income together. In the case of capital mobility it is shouldered by the wage income alone.

The ambiguous effect in the case of transfers can be understood when combining (31) with (21),

$$\left. \frac{dT}{dt_p} \right|_{\text{Autarky}} = \left. \frac{d(f^N + f^E)}{dt_p} \right|_{\text{Autarky}} - \left. \frac{d(ra)}{dt_p} \right|_{\text{Autarky}} - \left. \frac{d(w^N + w^E)}{dt_p} \right|_{\text{Autarky}} . \quad (33)$$

While the pollution tax reduces both capital and wage income, it also reduces total output (at least in most cases). The net effect could be either positive or negative. The same holds in the case of capital mobility, but whether transfers increase more or less under capital mobility
depends on the exact forms of the production functions.

### 3.3 The distributional effect of capital mobility

Proposition 3 allows us to determine the differential effect of capital mobility on the wages in the two sectors. Let \( l^i \) be the amount of workers employed in sector \( i \). Then, using (20), the wage per worker changes with pollution taxes according to

\[
\frac{d w^i_l}{dt_p} \bigg|_{\text{Autarky}} = \frac{p^i}{l^i}.
\]  

(34)

When energy intensity is defined as pollution per worker, then the loss in wages is indeed greater in the energy intensive industry. This effect, however, occurs under autarky in a very similar way.

Using (31) we can identify the differential effect of capital mobility as

\[
\frac{d w^i_l}{dt_p} \bigg|_{\text{Mobility}} - \frac{d w^i_l}{dt_p} \bigg|_{\text{Autarky}} = \frac{dr}{dt_p} k^i.
\]

(35)

When workers are not mobile, it will be workers in a capital intensive industry which will be harder hit by climate policy under capital mobility than they would be under autarky.

We can take from this that workers in the energy intensive industry are harder hit by climate policy, whether capital is mobile or not. The migration of firms abroad is just a manifestation of the economic loss through climate policy which would occur in a different form under autarky as well.

### 4 Model extensions

#### 4.1 Redistribution policy

When the government exempts some industries from pollution taxes, it engages in redistribution policy between the sectors. Hoel (1996), Böhringer and Rutherford (1997) and De Mooij and Bovenberg (1998) suggest that there are more efficient ways to achieve redistribution than exemptions. The framework in Section 3 could be used to suggest more efficient policy instruments than differential carbon prices \( t^N_p \) and \( t^E_p \). This would, however, require the definition of a social welfare function, so that the welfare effect of policy instruments can be
4.2 Unemployment

Unemployment is treated only indirectly as a loss in labor income in this model. A more detailed analysis of unemployment, however, would require the introduction of labor market imperfections as in Babiker and Eckaus (2007) and Guivarch et al. (2011). This would also give a greater role for wage subsidies, an instrument suggested by Böhringer and Rutherford (1997) as a means of redistribution towards particularly hard-hit industries.

4.3 Dynamics

Ogawa and Wildasin (2009) provide a static model with mobile capital in which decentralized decision-making on capital taxation leads to an efficient outcome. Eichner and Runkel (2012) counter this by showing that the outcome becomes inefficient when the distortion of savings decisions are considered. Similarly, it would be interesting to analyze how savings decisions affect the results presented in this paper. Possibly, savings decisions introduce an additional effect of capital mobility which needs to be taken into account.

5 Conclusion

I find that capital mobility does not systematically alter the solution space of the government for the worse. The same combinations of pollution and income can be achieved with and without it. In particular, there is no reason for a harmful race to the bottom, where climate policy is biased by a concern of bleeding out in terms of capital.

In other words, jobs will indeed be lost to foreign firms when climate policy is introduced and capital is mobile. But the same amount of jobs would be lost if capital were not mobile. The loss might simply be more visible when capital is mobile.

On the contrary, if tax revenues are used in a productive way, the solution space for the government might even enlarge through climate policy. These productive uses might either be an improvement of a previously inefficient tax system, see Goulder (2013). Or it might be a capital subsidy to attract more capital from abroad. In any case these effects are short-term. The long-term effect of climate policy on income will be dominated by its steering effect on technology, see for example the Porter hypothesis (Porter and Linde, 1995).
A Appendix

A.1 Proofs

Derivations for Section 2

First order conditions (3) and (4) can be used to define a function $F$ on the parameter $t_p$ and the endogenous variables $k$ and $p$:

$$ F : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2, (t_k, t_p, k, p) \mapsto \begin{pmatrix} f_k - \rho - t_k \\ f_p - t_p \end{pmatrix}. $$

When the assumptions of the implicit function theorem are given we can write

$$ \frac{d}{dt_p} \begin{pmatrix} k \\ p \end{pmatrix} = - \left( \begin{pmatrix} \frac{\partial F_1}{\partial k} & \frac{\partial F_1}{\partial p} \\ \frac{\partial F_2}{\partial k} & \frac{\partial F_2}{\partial p} \end{pmatrix} \right)^{-1} \begin{pmatrix} \frac{\partial F_1}{\partial t_k} & \frac{\partial F_1}{\partial t_p} \\ \frac{\partial F_2}{\partial t_k} & \frac{\partial F_2}{\partial t_p} \end{pmatrix} \begin{pmatrix} f_k - \rho - t_k \\ f_p - t_p \end{pmatrix}. $$

By the concavity of $f$ we have $f_{kk}f_{pp} - f_{kp}^2 \geq 0$, see Mas-Colell et al. (1995), Theorem M.C.2 and Example M.D.1. Using $f_{kp} > 0$ and $f_{kk} < 0$ we have $\frac{\partial k}{\partial t_p} < 0$ and $\frac{\partial p}{\partial t_p} < 0$.

Proof of Proposition 3

Using the equation system (24) to (27) I define an auxiliary function $G$ as

$$ G : \mathbb{R} \times \mathbb{R}^3 \to \mathbb{R}^3, (t_k, k^N, p^N, p^E) \mapsto \begin{pmatrix} f_k^N (k^N, p^N) - f_k^N (a - k^N, p^E) \\ f_p^N (k^N, p^N) - t_p \\ f_p^E (a - k^N, p^E) - t_p \end{pmatrix}. $$
The effect of a change in pollution tax can now be written as

\[
\frac{d}{dt_p} \begin{pmatrix} k^N \\ p^N \\ p^E \end{pmatrix} = - \begin{pmatrix}
\frac{\partial G_1}{\partial k} & \frac{\partial G_1}{\partial p^N} & \frac{\partial G_1}{\partial p^E} \\
\frac{\partial G_2}{\partial k} & \frac{\partial G_2}{\partial p^N} & \frac{\partial G_2}{\partial p^E} \\
\frac{\partial G_3}{\partial k} & \frac{\partial G_3}{\partial p^N} & \frac{\partial G_3}{\partial p^E}
\end{pmatrix}^{-1} \begin{pmatrix}
\frac{\partial G_1}{\partial \sigma_p} \\
\frac{\partial G_2}{\partial \sigma_p} \\
\frac{\partial G_3}{\partial \sigma_p}
\end{pmatrix}
\begin{pmatrix}
f_{kp}^N p^E - f_{kp}^E p^N \\
-f_{kk} f_{pp}^E - f_{kk} f_{pp}^N + (f_{kp}^E)^2 + f_{kp} f_{kp}^N \\
-f_{kk} f_{pp}^N - f_{kk} f_{pp}^N + (f_{kp}^N)^2 + f_{kp} f_{kp}^N
\end{pmatrix}
\]

\[
= - \frac{1}{\det(DG)} \begin{pmatrix}
f_{kp} f_{pp}^E + f_{kp} f_{pp}^N \\
-f_{kk} f_{pp}^E - f_{kk} f_{pp}^N + (f_{kp}^E)^2 + f_{kp} f_{kp}^N \\
-f_{kk} f_{pp}^N - f_{kk} f_{pp}^N + (f_{kp}^N)^2 + f_{kp} f_{kp}^N
\end{pmatrix}
\]

The Jacobian matrix \( DG \) is given by

\[
\det(DG) = f_{kk}^E (f_{kp}^N f_{pp}^N - (f_{kp}^N)^2) + f_{pp}^N (f_{kk} f_{pp}^E - (f_{kp}^E)^2) .
\]

By the concavity of both \( f^N \) and \( f^E \) we have \( \det(DG) < 0 \).

Using (28) we have

\[
\frac{dr}{dt_p} = \frac{df^N}{dt_p} (k^N, p^N) + \frac{dp^N}{dt_p} \\
= f_{kk}^N \frac{dk^N}{dt_p} + \frac{dp^N}{dt_p} \\
= f_{kk}^N \left( - \frac{f_{kp} f_{pp}^E - f_{kp} f_{pp}^N}{\det(DG)} \right) + \frac{dp^N}{dt_p} \left( - \frac{f_{kk} f_{pp}^E - f_{kk} f_{pp}^N + (f_{kp}^E)^2 + f_{kp} f_{kp}^N}{\det(DG)} \right) \\
= - \frac{1}{\det(DG)} (-f_{kp} f_{kk} f_{pp}^N - (f_{kp}^N)^2) - f_{kp} (f_{kk} f_{pp}^E - (f_{kp}^E)^2)
\]

\[
< 0 .
\]

From this the results follow directly.

Note that \( \frac{dp^E}{dt_p} + \frac{dp^N}{dt_p} = \frac{(f_{kk}^E + f_{kp}^N)(f_{kp}^E + f_{pp}^N) - (f_{kp}^E + f_{pp}^N)^2}{\det(DG)} \). Therefore, if \( f^N + f^E \) is concave then

\[
\frac{dp^E}{dt_p} + \frac{dp^N}{dt_p} < 0. \text{ By assumption } \frac{dk^E}{dt_p} + \frac{dk^N}{dt_p} = 0.
\]
References


