Collusion in Dynamic Buyer-Determined Reverse Auctions

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While binding reverse auctions have attracted a good deal of interest in the academic literature, in practice dynamic non-binding auctions, sometimes referred to as bidding contests, are the norm in procurement. In those bidding contests, suppliers submit price quotes and can respond to quotes of their competitors during a live auction event. However, the lowest quote does not necessarily determine the winner. The buyer decides after the contest, taking further supplier information into account, on who will be awarded the contract. We show, both theoretically and empirically, that this bidding format enables suppliers to collude, thus leading to non-competitive prices.

Key words: Bidding, Procurement, Reverse Auctions, Multi-Attribute Auctions, Behavioral Game Theory, Experimental Economics

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1. Introduction

In non-binding reverse auctions, bidders compete against each other like in a standard reverse auction, but the winner is not necessarily the supplier with the lowest bid. Rather, buyers decide, based on the final quotes and further information about the suppliers, who will be awarded the contract. These buyer-determined reverse auctions are virtually the norm in competitive procurement today. Ariba, a major commercial provider of on-line reverse auctions and other sourcing solutions, uses non-binding auctions almost exclusively. In a recent survey, Elmaghraby
(2007) notes that “The exact manner in which the buyer makes her final selection still remains unclear. With either an online auction or a RFP, the buyer may still leave some terms of trade unspecified.” (p. 411).1

In the context of multi-attribute auction events, the advantage of a non-binding format from the buyers' perspective seems evident. The winner should not be the supplier with the lowest quote, but further attributes, such as quality, reliability, capacity, reputation, incumbent status and other suppliers capabilities, etc. should be taken into account. However, we show in this paper that there is a serious disadvantage of such a dynamic non-binding auction: If bidders are uncertain about the exact way different criteria affect the final decision by the buyer, then a non-binding auction enables them to implicitly coordinate on high prices.

The collusive arrangement in the non-binding auction works as follows: The suppliers begin the contest with a relatively high quote. These offers are such that if the process were to stop at this point, all have a positive probability of winning, given the uncertain criteria of the buyer's award decision. In equilibrium no supplier makes an improvement on his offer, so the bidding stops at a high price. If one supplier were to lower the offer, this would trigger a response by the other suppliers, and they would also lower their quotes. Thus the deviating supplier has to reduce his price even further, which makes it unattractive to lower the price in the first place. Note that the stabilizing element in this collusion is that suppliers do not know how the buyer will ultimately determine the winner. Thus, with their initial offers, all suppliers have a chance of winning.

Binding auctions, where the final decision rule is known in advance, do not allow for this form of collusion. In a (reverse) English auction, for example, at any moment during the auction

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1 SAP (2006) notes in a document on best practice in reverse auctions: “Often, you may find that the lowest bidder is not meeting quality and service grades and thus may select the second-lowest bidder.”
firms do not have any uncertainty about whether they would receive the contract or not if the auction were to stop at this point. Thus, suppliers who know that they will not be awarded the contract at the current price, have to improve their offer, which in turn puts pressure on their competitors. Therefore collusion cannot be sustainable in binding auctions.

Buyer-determined auction mechanisms have not been widely studied and are not well-understood, especially theoretically. Jap (2002) was the first to point out that most reverse auctions that are conducted in industry do not determine winners – thus they are non-binding. Jap (2003) and Jap (2007) show that dynamic non-binding auctions often have a more detrimental effect on buyer-supplier relationships than do sealed-bid auctions. Stoll and Zöttl (2012) analyze field data and estimate the consequences of different information structures regarding the buyer’s preferences and bidders’ qualities. In another study, Engelbrecht-Wiggans et al. (2007) examine sealed-bid first price auctions. They compare price-based and buyer-determined mechanisms, both theoretically and using laboratory experiments, and find that buyer-determined mechanisms generate higher buyer surplus only as long as there are enough suppliers competing for the contract. Haruvy and Katok (2012) investigate the effect of information transparency on sealed-bid and dynamic buyer-determined mechanisms, and find that sealed-bid formats are generally better for buyers, especially when suppliers are aware of their competitors' non-price attributes. In both of those studies, suppliers know the value the buyer attaches to their own non-price attributes.\(^2\) In contrast, in the present paper we investigate the effect of having this information on the performance of dynamic non-binding auctions. We show that it is precisely the combination of the dynamic nature of the bidding process, that allows bidders to react to their competitors' bids, and the lack of knowledge about the valuation of the non-price attributes by the buyer, that ensures

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\(^2\) Thomas and Wilson (2005) compare experimentally multilateral negotiations and auctions. They explicitly assume that during the negotiations offers are observable, so that this case resembles our buyer determined bidding mechanism. However, everyone knew preferences of the parties in advance. So the effect we analyze here could not occur.
that each bidder has some probability of winning even at a high price, that enables bidders to collude.

The way collusion works in our model has some similarity to the collusive behavior in the context of strategic demand reduction (Brusco and Lopomo, 2002; Ausubel and Cramton, 2002) and to the industrial organization literature on price clauses (see e.g. Salop, 1986 or Schnitzer, 1994 and references therein). Strategic demand reduction describes the phenomenon that, in a procurement multi unit auction, bidders might prefer to win a smaller number of units at a higher price than a larger number of units at a lower price. Sherstyuk (1999) and Sherstyuk (2002) show in an experimental study that the improvement rule has an influence on the bidders’ ability to collude in repeated auctions. Our paper analyzes a single unit situation in which bidders are content with a small probability of winning at a higher price.

Price clauses like "Meet-the-Competition-Clause" or a "Price-Matching-Clause" might be used to sustain collusion in a market similarly to the present analysis, where suppliers refrain from lowering their quotes, as this will trigger a lower price by their competitors. The literature on price clauses differs from this paper in two respects, however. First, in the pricing literature, it is either assumed that trade takes place in several periods (e.g. Schnitzer, 1994) or that contingent contracts can be written in which the price depends on the prices of the competitors (e.g. Doyle, 1988; Logan and Lutter, 1989). In the present case, trade only takes place once and contingent bidding is not possible. Second, the main argument why collusion is feasible, namely the remaining uncertainty about the final decision the buyer will take, has to our knowledge not been investigated so far.

Several authors have analyzed collusion in the context of auctions (see e.g. Robinson, 1985, Graham and Marshall, 1987, for an overview see Klemperer, 1999, and Kwasnica and
Sherstyuk, 2012). Usually this literature assumes that before the auction takes place, a designated winner is selected. In addition, there must be some means to divide the gains of collusion between the participating bidders. This is different to the form of collusion described here. First, all participating firms have a chance of winning the contract, thus there is no predetermined winner and no pre-play communication required. Second, during the contest, all firms have a positive expected profit, even if after the decision by the buyer only one firm receives the contract. This makes it unnecessary to divide the gains of collusion after the contest.

The paper is structured as follows: In the next section we develop the model and analyze the collusive behavior in a buyer-determined auction. In Section 3 we describe our experimental setting and present the results. In section 4 we conclude the paper with a discussion of ways for overcoming the problem of collusion.

2. Analytical Results

2.1 Model set-up

The auction format we consider is one in which suppliers bid on price, but different suppliers may provide different value to the buyer. This value can be viewed as exogenous attributes of suppliers themselves, rather than a part of their bids, and we will refer to it as quality. Our modeling approach is similar to Engelbrecht-Wiggans et al. (2007) and Haruvy and Katok (2012). There are \( n \) potential suppliers, competing to provide a single unit to a buyer. Suppliers are heterogeneous in costs and quality. In particular, supplier \( i \) has cost \( c_i \), which is only known to the supplier \( i \). Each \( c_i \) is taken from a common distribution \( F(c) \) on \([c, \bar{c}]\). The quality component does not enter the profit function by the supplier, so the profit of supplier \( i \) if he wins the contract at price \( p \) is given by:

\[
\pi_i(p, c_i) = p - c_i
\]
The quality of each supplier has two separate components. First, there is a commonly-known (vertical) quality component \( q \in [q, \overline{q}] \) for each supplier. The common distribution of the \( q \)'s is given by \( G(q) \). For example, in the procurement of a customer designed application specific circuit (ASIC), all suppliers satisfy the necessary technical requirements (given by \( q \)). Some suppliers might have a superior technology, which is commonly known, where the additional value to the buyer is denoted by \( q - \overline{q} \). For notational convenience, let \( Q_i = \max(q_1, q_2, \ldots, q_n) - q_i \) represent supplier \( i \)'s vertical quality disadvantage relative to the best competitor in the pool.

There are different ways to model horizontal differences among suppliers. For example, let \( \alpha_i \) be buyer’s incremental cost of dealing with supplier \( i \) relative to dealing with the buyer’s most preferred supplier, and let all the \( \alpha \)'s be the private information of the buyer. Assume \( \alpha \) is independently distributed according to some commonly-known distribution, with finite support \( \alpha_i \in [0, \overline{\alpha}] \). Then the utility of the buyer, if she awards the contract at price \( p \) to supplier \( i \), is

\[
    u(p, q_i, \alpha_i) = q_i - p - \alpha_i
\]

The parameter \( \alpha \) measures how far the private preferences of the buyer enter her surplus and \( \alpha \) can be quite small: Consider for example the sourcing of a display for a new mobile phone. The overall value of the contract might be several hundreds of millions in US dollars, which is captured by the term \( q \). Individual observable differences between the suppliers – e.g. one firm is known to be the technology leader – are in the range of ten million US dollar, captured by the differences in \( q \). Unobservable preferences by the buyer, i.e. a preference for a particular provider whose engineers speak English fluently, might differ in the size of several hundred thousand US dollars. These are captured by the term \( \alpha \).

But \( \alpha \) might also be large relative to the differences in quality: Consider a company recruiting a marketing agency. All potential firms provide the necessary quality \( q \) for the planned
marketing campaign. The differences in $q_i$ capture features such as better technical skills, special equipment, and so on. In the end, however, the decision will be strongly influenced by the specific preference parameter - how the board of the firm likes the marketing company, its people, their ideas and creativity. This is captured by the $\alpha$ term.

2.2 Binding Auctions

2.2.1 Binding Auctions without Announcement of $\alpha$

If vertical quality $q$ is common knowledge and the buyer’s preferences $\alpha_l$ not known by the bidders, the buyer can conduct a binding auction in which the bidder with the lowest vertical quality-adjusted bid is guaranteed to win. Such an auction is feasible, because the terms $q_l$ have already been monetized and suppliers are aware of them.³

Therefore, as a benchmark, we consider a vertical quality-adjusted binding auction and a quality adjusted binding auction. The rules of the corresponding open-bid reverse auction are then as follows. Each bidder $i$ submits a price bid $b_i$ which is then adjusted by $Q_l$ to represent the bidder $i$'s vertical quality penalty relative to the highest vertical quality bidder in the pool. The highest allowable bid, the reservation price, is $R$. During the auction bidders observe full vertical quality-adjusted price feedback - they see all $b_i + Q_l$'s that have been submitted. They can place new bids that must be lower than the lowest current standing vertical quality-adjusted bid to become the leading bid. The bidder with the lowest vertical quality-adjusted bid is the leading bidder in the auction and would win the auction if it were to stop at this point. The auction ends when there are no new bids placed for a certain amount of time. The price the buyer pays is equal to the (unadjusted) price bid (e.g. $b_i$).

³ A commonly used way to monetize $q_i$ is to set up a bonus/malus system, which quantifies differences between suppliers with respect to the different dimensions, e.g. quality, payment terms, technical criteria, and so on.
Under this rule, it is a dominant strategy for each supplier to keep lowering his bid as long as he is not currently winning the auction, until \( b_l = c_l \). Thus the auction ends when the bidder with the second lowest vertical quality-adjusted costs (with the second lowest \( c_l + Q_l \)) exits the auction. The bidder with the lowest vertical quality-adjusted costs wins the auction. If bidder \( i \) with vertical quality \( q_l \) wins, and bidder \( j \) with the second lowest vertical quality-adjusted bid exited at \( c_j \), then the bid of bidder \( i \) satisfies \( b_l + Q_l = c_j + Q_j \), and thus the price the buyer pays is equal to \( b_l = c_j + Q_j - Q_l \). As \( Q_j - Q_l = q_i - q_j \), the utility of the buyer is then:

\[
u = (q_i - b_l) - \alpha_i = (q_j - c_j) - \alpha_i,
\]

where \( q_j - c_j \) is the second highest cost-adjusted quality, which we denote by \((q - c)^{(2)}\). As the distribution of \( \alpha_i \) is independent of costs and quality realization, the expected buyer surplus is

\[E[(q - c)^{(2)}] - E[\alpha] \]

The buyer obtains in expectation the second highest cost-adjusted quality, and the expected value of the horizontal quality parameter.

### 2.2.1 Binding Auctions with Announcement of \( \alpha \)

If the buyer communicates the horizontal qualities \( \alpha_i \) to all bidders and monetizes not only the vertical but also the horizontal quality differences, he can conduct a binding auction in which the bidder with the lowest quality-adjusted bid is guaranteed to win. The buyer’s expected profit is then given by

\[E[(q - c - \alpha)^{(2)}] \]

Revealing private preferences \( \alpha_i \) (if that is possible at all) can lead to a larger or lower revenue compared to a binding auction which only adjusts for vertical quality \( q \). The reason is that the
optimal revenue maximizing mechanism (Che, 1993) discriminates against non-price attributes in order to make price competition tougher. Thus it can be better not to reveal $\alpha_l$ at all.

2.3. Non-binding dynamic auctions: General framework

Now consider a non-binding auction, which is commonly used in practice. The auction works exactly the same way as the open-bid binding auction in terms of the bidder feedback and the ending rule, but after the auction ends, the buyer is not obligated to award the contract to the bidder with the lowest quality-adjusted bid $b_l + Q_l$, but may instead award the contract to a different bidder, taking his preferences $\alpha_l$ into account.

The fundamental difference between the non-binding auction and its binding counterpart is that bidders generally do not have the dominant strategy unless they know for certain that they are either winning or losing. A bidder $j$ only knows for certain that he is losing when his quality-adjusted bid $b_j + Q_j$ is more than $\bar{\alpha}$ above the current lowest quality-adjusted bid. On the other hand a bidder $i$ knows for certain that he is winning when his quality-adjusted bid $b_i + Q_i$ is more than $\bar{\alpha}$ below the next lowest quality-adjusted bid. A bidder, who knows that he is winning, has the dominant strategy to not lower his bid. While a bidder who knows that he is losing, has the dominant strategy to lower his bid, as long as the bid is still larger than his costs.

Let us call the lowest standing quality-adjusted bid $B = \min \{b_1 + Q_1, b_2 + Q_2, \ldots, b_n + Q_n\}$.

A bidder $i$ whose quality-adjusted bid is within $\bar{\alpha}$ of $B$, $B - \bar{\alpha} \leq b_i + Q_i \leq B + \bar{\alpha}$, does not know his winning status, and thus does not have a dominant bidding strategy. In general, his strategy will depend on his beliefs about the other suppliers’ future actions. A bidder who believes that lowering the bid would lead to an outright bidding war is less likely to lower his bid than a bidder who merely expects competitors to lower their bids by a small amount.
The (collusive) equilibrium we analyze has the following structure. All suppliers bid initially very high in a way such that the probability of winning for every supplier is the same. When one supplier lowers his bid to increase his probability of winning, those suppliers whose probability of winning is decreased will follow suit and lower their bids as well. This makes the initial deviation unattractive and thus collusion can be sustained.

In the most general formulation the bidding behavior off the equilibrium path, i.e. if bidders deviate from colluding on high prices, is complex. In order to facilitate the analysis, we set the information structure such that if someone lowers his bid in order to increase his probability of winning, the probability of winning for at least one other supplier falls to zero. Thus this supplier has a weakly dominant strategy to lower his bid as well. In this case collusion is not only a Nash equilibrium, but also a perfect Bayesian equilibrium.

2.4 Non-binding dynamic auction: Specific model

We now consider a special case in which we can characterize the conditions for a perfect Bayesian collusive equilibrium to exist. The buyer has one preferred supplier, but the suppliers do not know the identity of this supplier. Let \( \alpha > 0 \) be the additional cost the buyer incurs when she has to deal with a non-preferred supplier. Let \( \tau_i = 0 \) if \( i \) is the buyer’s preferred supplier, and 1 otherwise, and assume that \( \tau_i \)'s are the buyer’s private information. The buyer knows who her preferred supplier is, but suppliers do not, and since suppliers are ex ante symmetric, each supplier \( i \) believes that the probability that \( \tau_i = 0 \) is \( 1/N \). Denote \( q^{max} = \max \{q_1, q_2, \ldots, q_n\} \) and \( q^{min} = \min \{q_1, q_2, \ldots, q_n\} \) and by \( \Delta_q = q^{max} - q^{min} \) the spread in (vertical) quality. It is also assumed that bids must be in multiples of the minimum bid decrement \( \epsilon \), where \( \epsilon \) is sufficiently small. Discreteness of prices is used to insure that there are no ties. This is achieved by assuming that \( \alpha \) is not a multiple of \( \epsilon \).
Before specifying the equilibrium formally, one definition is necessary. Let $b_{-i}$ be a vector of bids of all suppliers apart from supplier $i$. If supplier $i$ were to bid $b_i$ and the bidding would stop at this point, then the probability for supplier $i$ of obtaining the contract is given by

$$P(b_i^t, b_{-i}^t, Q) = \text{Prob}(q_i - \alpha \tau_i - b_i > q_j - \alpha \tau_j - b_j \; \forall j \neq i)$$

$$= \text{Prob}(Q_i + \alpha \tau_i + b_i < Q_j + \alpha \tau_j + b_j \; \forall j \neq i)$$

where $Q = (Q_1, Q_2, \ldots, Q_n)$ denotes the (observable) vector of individual quality adjustment factors (recall that $Q_i = q^{\text{max}} - q_i$). Note that from the point of view of supplier $i$, both $\tau_i$ and all $\tau_j$ are random variables.

We now describe the following collusive equilibrium bidding strategy:

- $b_i^1 = R$, all bidders start bidding at the reservation price of $R$.
- For bidder $i$, if $P(b_i^t, b_{-i}^t, Q) \geq \frac{1}{n}$ then $b_{i}^{t+1} = b_i^t$.
- If $P(b_i^t, b_{-i}^t, Q) < \frac{1}{n}$ then $b_{i}^{t+1} = \max\{c_i, b^*(b_{-i}^t, Q)\}$, where $b^*(b_{-i}^t, Q)$ is the maximum bid $b$ which satisfies $P(b, b_{-i}^t, Q) \geq \frac{1}{n}$.

If bidding starts at $t = 1$ with all bidders bidding $R$, and for some bidder $i$, the probability of winning is below $\frac{1}{n}$ then at $t = 2$, bidder $i$ sets his bid $b^*(b_{-i}^1, Q)$ so as to barely outbid the bidder with the highest quality $q$ (lowest quality adjustment $Q$) in the event that $i$ turns out to be the preferred supplier. Since the bidding is done in increments of $\epsilon$, and since the quality adjustment of the highest quality bidder is equal to zero, this implies for the bid of bidder $i$:

$$R + \alpha > b^*(b_{-i}^1, Q) + Q_i$$

and therefore $b^*(b_{-i}^1, Q)$ is the price on the price grid which lies in the region

$$(R + \alpha - Q_i - \epsilon, R + \alpha - Q_i)$$

\footnote{\text{The} \; b^*(b_{-i}^1, q) \; \text{exists, as the optimization is done over a finite set of possible bids.}}
If all suppliers behave according to this bidding strategy, then the auction will end with all bidders placing bids close to $R$.

### Example

Consider the following example. Three bidders have costs and qualities as shown in Table 1. Assume that $R = 100$, the costs are $c \sim U(0,100)$, $\alpha = 5.01$ and $\epsilon = 1$.

<table>
<thead>
<tr>
<th>Bidder</th>
<th>$q$</th>
<th>$Q$</th>
<th>$c$</th>
<th>$c + Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>79</td>
<td>0</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>75.5</td>
<td>3.5</td>
<td>50</td>
<td>53.5</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
<td>9</td>
<td>60</td>
<td>69</td>
</tr>
</tbody>
</table>

*Table 1. Costs and qualities in an example.*

It is useful to start by calculating the probability of winning for each bidder if this bidder tries to outbid the other bidders. For bidder $i$, the probability of winning the auction outright is the probability that both competitors cannot follow his bids:

$$Pr_i(Win) = \prod_{j \neq i} \text{Prob}(c_i + Q_i < c_j + Q_j - \alpha) = \prod_{j \neq i} [1 - F_j(c_i + Q_i - Q_j + \alpha)]$$

E.g., bidder 1 can outbid his rivals if his costs 25 are smaller than $c_2 + Q_2 - \alpha$ and smaller than $c_3 + Q_3 - \alpha$. Then bidder 1’s chance of outbidding the opponents is given by $Pr_1(Win) \approx 0.58$. A similar calculation for bidder 2 (3) gives $Pr_2(Win) \approx 0.21$ ($Pr_3(Win) \approx 0.08$). Bidder 1 wins if bidder 2 has costs higher than 26.51 and bidder 3 higher than 21.01. He then has to pay the lower quality-adjusted costs minus $\alpha$. Since the exact calculation is cumbersome, we only report results. If bidder 1 wins, the average price he gets paid is 50.3. Bidder 2 (3) receives an average price of 65.1 (69.2) in case of winning. Multiplying this with the probability of winning and taking costs into account we get the

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$E[II_i|\text{ wins}] = E[\min\{\min\{c_j + Q_j, c_k + Q_k\}, R\}|c_i + Q_i + \alpha < \min\{c_j + Q_j, c_k + Q_k\}]$
expected profit from competition. For bidder 1 the expected profit from competition is 14.7, for bidder 2 it is 3.2 and 0.8 for bidder 3.

If bidders follow the collusive equilibrium we described, they start bidding at 100. At this price bidder 3 has zero probability of winning, because even if he is the preferred supplier, his quality and \( \alpha \)-adjusted cost of 109 would be higher than bidder 1’s quality and \( \alpha \)-adjusted cost of 105.1. Therefore, bidder 3 must lower his bid to:

\[
b^{*}(b_{-i}^{1}, Q) \leq R + \alpha - Q_{3} = 100 + 5.1 - 9 \Rightarrow b^{*}(b_{-i}^{1}, Q) = 96.
\]

With this bid, bidder 3 would just outbid bidder 1 if bidder 3 is the preferred supplier, and each bidder has 1/3 chance of winning. Expected profit for bidder 1 is \( \frac{100 - 25}{3} = 25 \), for bidder 2 is \( \frac{100 - 50}{3} = 16 \frac{2}{3} \), and for bidder 3 is \( \frac{96 - 60}{3} = 12 \); all higher than expected profits without collusion, which is 14.7 for bidder 1, 3.2 for bidder 2 and 0.8 for bidder 3.

Equilibrium analysis

We claim that the bidding strategy as defined above constitutes an equilibrium, depending on the reservation price \( R \), the size of the buyer preference term \( \alpha \), the differences in observable quality \( \Delta q \), and the distribution of costs \( F(c) \).

We start the formal analysis by considering two bidders.

Proposition 1. For the case of two bidders and \( R \geq \bar{c} \).

i. If \( \Delta q > \alpha > 0 \) a collusive equilibrium exists if

\[
\frac{1}{2}(R - \bar{c}) \geq E[c] + \Delta q - \alpha - \bar{c}
\]

ii. If \( \alpha > \Delta q \geq 0 \) a collusive equilibrium exists if

\[
\frac{1}{2}(R - \bar{c}) \geq \int_{\bar{c} + \alpha - \Delta q}^{\bar{c}} (x + \Delta q - \alpha - \bar{c}) f(x) \, dx
\]
Proof:

If both bidders behave according to the equilibrium strategy defined above, then the bidding ends after the first or second round and the expected profit of supplier $i$ is given by:

$$\pi_i(q_i, c_i) = \frac{b_i - c_i}{2}$$

where $b_i = \min(R, R - Q_i + \alpha)$. If someone deviates from this strategy, he has to lower the price by so much that his probability of winning is increased. Otherwise he has just lowered his price without a change in the probability of winning, which cannot be optimal. This then implies that the other supplier has a zero probability of winning. This can be seen as follows: let $j^*$ be the supplier with the current smallest quality-adjusted bid, i.e. $b_j^* - q_j$ is minimal among all quality-adjusted bids. If the auction were to stop at this point, then if the other supplier has a quality-adjusted bid smaller than $b_j^* - q_j + \alpha$, he has the same probability of winning the contest. If not, i.e. if he bids strictly more than $b_j^* - q_j + \alpha$, he has a zero probability of winning. So if a deviating supplier wants to increase his probability of winning, this immediately implies that the other supplier has a zero probability of winning if the auction were to stop at this point. This supplier will, according to the equilibrium strategy, lower his bid as well. As he has probability of zero of winning if he does not react, lowering the bid is optimal.

So a deviator can only increase his probability of winning if the price is so low that the other will not follow suit anymore. This is the case if the bid is smaller than costs, in which case he will bid his costs. Suppose that $R > \max\{\bar{c}, \bar{c} + \Delta_q - \alpha\}$. The incentive to deviate are largest for a supplier with the lowest costs ($c = \bar{c}$) and the highest quality ($q = q^{\text{max}}$), i.e. if this bidder has no incentive to deviate then no bidder has. Then, if this bidder wants to outbid the other, he has to bid $\alpha - \Delta_q$ less than the other, where $\Delta_q$ is the difference in quality. Consider first the
case of $\Delta_q > \alpha$. Then the deviator can undercut the other bidder by so much that he wins for sure. Therefore, the expected profit for the deviating bidder is given by:

$$\int_{\underline{c}}^{\overline{c}} \left( x + \Delta_q - \alpha \right) dF(x) - \underline{c} = E[c] + \Delta_q - \alpha - \underline{c}$$

If, on the other hand, $\Delta_q < \alpha$, then the deviating bidder will not be able to underprice his competitor if the other bidder has low costs as well. Optimally he stops lowering his price further once the other’s bid has reached $\underline{c} + \alpha - \Delta_q$.

Thus, the expected profit from deviating is given by:

$$\int_{\underline{c} + \alpha - \Delta_q}^{\overline{c}} \left( x + \Delta_q - \alpha - \underline{c} \right) dF(x)$$

Proposition 1 has interesting implications for the existence of a collusive equilibrium. Collusion is more likely if:

- the reservation price $R$ is large, as this makes collusion profitable
- the individual observable quality differences $\Delta_q$ are small, as otherwise the supplier with the highest quality $q^{max}$ might find it optimal to deviate
- the spread in costs $E[c] - \underline{c}$ is small, as otherwise the supplier type with the lowest costs $(c)$ might find it optimal to deviate
- the individual preference component $\alpha$ is not too small, as this implies that anyone trying to undercut his competitors in order to gain a higher probability of winning must lower the price sufficiently, which makes this behavior unattractive.
Proposition 1 implies, that in industries in which the quality and costs of firms are quite similar, i.e. firms are relatively homogeneous, but individual preferences matter, collusion becomes more likely.

Next consider the case of three bidders. Now, if a bidder with lowest costs (and highest quality) deviates, he will first lower the price until one competitor cannot follow anymore. Suppose this happens at some \( b = c_m + Q_m - \alpha \), where \( c_m \) are the costs of the first supplier to exit, and \( Q_m \) is the quality difference of this supplier compared to the deviator (who is assumed to have the highest quality). Now, at this point the deviator has to decide whether he wants to lower his price further. This is equivalent to the situation above with two bidders, but where however the reserve price is in between the range of costs. If he stops lowering, and recalling that with his equilibrium strategy the other bidder will not lower his bid further, he will win the auction with probability \( \frac{1}{2} \) and obtain in case of winning \( b - c \). Alternatively, he could try to underbid the other competitor as well. For simplicity, assume that \( Q_j - \alpha > 0 \), where \( Q_j \) is the quality difference with regard to the remaining bidder. Then the profit of the deviating bidder if he undercuts the remaining bidder as well is given by:

\[
E[c | c \leq b] = c + Q_j - \alpha
\]

In case this expression is smaller (larger) than \( \frac{b - c}{2} \), the bidder will decide to stop (not stop) lowering his bid further.

Remarks:

1. The general expression for the incentives of a deviator to break the collusion is complicated, as it depends on the distribution of costs and qualities of the other bidders. It is well possible, that a deviator starts to undercut, and stops at some point if he realizes that the others are still in the contest.
2. This dynamic also implies that other colluding equilibria are possible. It is easy to create examples where some bidders would in equilibrium lower their bid somewhat below the reservation price, and then start to collude. Suppose all bidders have costs of either 0 or 10, each with probability \( \frac{1}{2} \). There are no quality differences, reserve price \( R \) is equal to 10 and \( \alpha = 0.5 \). Price steps are \( \epsilon = 1 \). Then a bidder with costs 0 might lower the price to 9 and stop there. By doing this, he will avoid the competition of those bidders with costs of 10, but he will still collude with those with costs of 0. For example in the case of four bidders, collusion at 10 would give a profit of \( \frac{10}{4} = 2.5 \). If a bidder with costs of 0 lowers the price to 9, his profit is given by

\[
\left( \frac{1}{2} \right)^3 \times \frac{9}{4} + 3 \times \left( \frac{1}{2} \right)^3 \times \frac{9}{3} + 3 \times \left( \frac{1}{2} \right)^3 \times \frac{9}{2} + \left( \frac{1}{2} \right)^3 \times 9 = \frac{135}{32} \geq 2.5.
\]

While the general dynamics for \( n > 2 \) bidders and arbitrary distribution of costs and quality can become very complex, the analysis simplifies if costs are distributed uniformly and quality differences do not exist.

**Proposition 2.** With uniformly distributed costs, no quality differences and \( \alpha > 0 \), collusion is an equilibrium.

The proof is relegated to the Appendix. Interestingly, in the case of costs being uniformly distributed and observable quality differences being absent, collusion is an equilibrium independent of the number of bidders. Increasing the number of bidders \( n \) has two opposing implications for the stability of collusion. Having more bidders decreases the gain from sticking to high prices, as the probability of winning (which is equal to \( 1/n \)) is lowered. On the other hand, more bidders make it less likely that by lowering the price one will succeed in pricing the others out of the market.
From the buyer's point of view, if she cannot fully reveal her preferences $\tau_i$ prior to the auction, there is a trade-off between using an auction with commitment and without. In the former case, price competition will be stronger, while in the latter the preferences can be better accounted for in the selection of the supplier. Formally, the expected profit in a quality-adjusted auction with commitment is given by:

$$\Pi^c = E[(q - c)^2] - \frac{n-1}{n} \alpha$$

while the expected profit in a non-binding auction is given by:

$$\Pi^{nc} = E[(q - \tau \alpha)^1] - R$$

To simplify the comparison, consider again the case where $\Delta_q = 0$, i.e. all suppliers provide the same (vertical) quality. Then $q$ can be taken out of the expectation operator, the $\tau$ of the winner is always 0, and all bidders in the non-binding auction bid $R$. The difference between the two procurement mechanisms is given by:

$$\Pi^{nc} - \Pi^c = \frac{n-1}{n} \alpha - (R - E[c^{(2)}])$$

A non-binding dynamic contest has the negative effect of larger prices $\tau$, but at the same time leads to a better fit of the preferences $\frac{n-1}{n} \alpha$. A binding auction in which also the buyer's preferences $\alpha_i$ are monetized will always lead to better results than a non-binding bidding contest.

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6 This expression assumes that the winner of the auction bids $R$, which always happens when $\alpha$ is large enough relative to $\Delta_q$. If the winner instead bids $R + \alpha - Q_i$, the non-binding profit is correspondingly higher.
3. Experimental Evidence

3.1 Design of the Experiment

Like in the previous section we work with binary individual buyer preferences. That means we assume that in each auction one of the bidders is preferred and therefore $\alpha_i \in \{0, 10\}$. The other parameters in the laboratory experiment were set at $q_{\min} = q_{\max} = 150$ and thus $\Delta_q = 0$ and $c_i \sim U[0, 100]$ for all firms $i$. This setting captures the idea that the supplier specific buyer preferences are relatively small compared to the overall project size. Since all bidders have the same $q_i$, the problem is simplified, allowing participants to focus on the effect of $\alpha$.

In order to test our predictions, our design contains the following two treatments:

1. Buyer-determined auction (BDA): participants do not know the buyer’s preferences $\alpha_i$ and the buyer selects the supplier that maximizes her profit.
2. Binding auction: participants know the buyer’s decision rule and the seller with the lowest $\alpha$-adjusted bid ($p + \alpha$) is selected.

This design allows us to cleanly separate the effect of the buyer’s inability to commit herself to a clear decision rule.

In both treatments $n = 2$. We conducted each treatment with the same realizations of $c_i$ and $\alpha_i$, which we pre-generated prior to start of the experiments. This ensures that any differences in behavior we observe between the treatments are due to the factor we vary and not to different realizations of the parameters.

We used the between subjects design. The Buyer-determined auction treatment contained 5 independent cohorts, the Binding auction treatment 6. Each cohort included 6 participants for a total 66 participants in the study. All participants in our experiment were in the role of suppliers. Participants were randomly assigned to one of the two treatments. Each person participated only
one time. We conducted all experimental sessions in the Cologne Laboratory for Economic Research at the University of Cologne during the winter of 2012/13. We recruited participants using the on-line recruitment system ORSEE (Greiner, 2004). Earning cash was the only incentive offered.

Upon arrival at the laboratory the participants were seated at computer terminals. We handed out written instructions to them and they read the instructions on their own. When all participants finished reading the instructions, we read the instructions to them aloud, to insure common knowledge about the rules of the game.

After we finished reading the instructions to the participants we started the actual game. In each session each participant bids in a sequence of 28 auctions, the first three auctions were practice periods that made participants more familiar with the situation. We used random matching, that we kept the same within each cohort. At the beginning of each round the six participants in a cohort were divided into three groups of two bidders according to the pre-specified profile matching protocol. Each pair of bidders competed for the right to sell a single unit to a computerized buyer.

We programmed the experimental interface using the zTree system (Fischbacher, 2007). The screen included information about the subject’s cost $c_i$ and the reserve price $R = 150$. Bidders could also observe all bids placed in real time.

At the end of each round we revealed the same information in all conditions. This information included the bids and the $\alpha_i$’s of all bidders and the winner in that period’s auction. The history of past winning prices and horizontal quality adjustment $\alpha$ in the session was also provided.
For each auction in each period the auction winners earned the difference between their price bids and their costs $c_1$, while the other bidders earned zero. We computed cash earnings for each participant by multiplying the total earnings from all rounds by a pre-determined exchange rate and adding it to a 2.50 € participation fee. Participants were paid their earnings from the auctions they won in private and in cash, at the end of the session.

3.2. Results of the Experiment

Figure 1. Average $\alpha$-adjusted prices over time, compared to competitive and collusive benchmarks.

Figure 1 shows the evolution of average $\alpha$-adjusted prices\(^7\) in each period. In the first periods the $\alpha$-adjusted prices in the buyer-determined auction and the binding auction are both substantially higher than the competitive benchmark but also lower than the collusive benchmark. As time goes on the $\alpha$-adjusted prices move in opposite directions and towards their re-

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\(^7\) The $\alpha$-adjusted price if given by $p + \alpha$, i.e. the buyer’s total costs.
spective predictions. While bidders collude more effectively in the buyer-determined auction treatment, bidding behavior is highly competitive after a few rounds of practice in the binding auction treatment.

Tables 1 and 2 provide quantitative support for the observations. In table 1 the row labeled “AA Price (1-25)” contains the average $\alpha$-adjusted prices over all 25 periods, and their standard deviations. In table 2 “AA Price (13-25)” represents the average $\alpha$-adjusted prices over the second half of auctions, when bidders are experienced. Recall that treatment 1 contained five and treatment 2 six independent cohorts of 6 participants – each cohort constitutes one independent observation, and standard deviations are computed based on averages for the cohorts.

<table>
<thead>
<tr>
<th>Treatment: AA Price (1-25):</th>
<th>BDA</th>
<th>Binding auction</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>vs Competitive benchmark (p-value)</td>
<td>0.0005</td>
<td>0.0149</td>
<td>69.90</td>
</tr>
<tr>
<td>vs Collusive benchmark (p-value)</td>
<td>0.0150</td>
<td>0.0000</td>
<td>150</td>
</tr>
<tr>
<td>$H_0$: BDA = Binding auction (t-Test)</td>
<td>0.0003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_0$: BDA = Binding auction (Mann-Whitney U)</td>
<td>0.0061</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 1. Summary of the data and simple statistical tests for all periods.**

<table>
<thead>
<tr>
<th>Treatment: AA Price (13-25)</th>
<th>BDA</th>
<th>Binding auction</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>vs Competitive benchmark (p-value)</td>
<td>0.0007</td>
<td>0.2791</td>
<td>65.16</td>
</tr>
<tr>
<td>vs Collusive benchmark (p-value)</td>
<td>0.0680</td>
<td>0.0000</td>
<td>150</td>
</tr>
<tr>
<td>$H_0$: BDA = Binding auction (t-Test)</td>
<td>0.0002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_0$: BDA = Binding auction (Mann-Whitney U)</td>
<td>0.0061</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 2. Summary of the data and simple statistical tests for later periods.**
Statistical tests we report are parametric t-tests and non-parametric Mann-Whitney U-tests using cohort averages as the unit of analysis. This approach is extremely conservative, but nevertheless it clearly indicates that prices are significantly higher in the buyer-determined than in the binding auction. Additionally, we can see from Table 2 that average prices in the binding auction are not significantly different from the competitive benchmark, and average prices in the buyer-determined auction are only weakly significantly lower than the collusive benchmark.

The different behavior in the two mechanisms is illustrated in Figure 2. We can see the distribution of all $\alpha$-adjusted prices of the buyer-determined auction and the binding auction. In the Buyer-determined auction the median of all $\alpha$-adjusted prices is 150 and 76 in the Binding auction.

Figure 2. Distribution of $\alpha$-adjusted prices.
Our data includes 30 individual participants in the Buyer-determined auction treatment and 36 in the Binding auction treatment, each individual bidding in 25 auctions, so we can take advantage of this panel structure to obtain better estimates of the dynamics of bidding behavior.

We fit, separately for the BDA and Binding Auction treatments, the following regression model:

\[
AA\ Price_{it} = \text{Constant} + \beta_{CB} \times CB + \beta_t \times (26 - t) + \eta_i + \mu_{it}
\]

Here \( t \) is the period number (1 to 25), \( AA\ Price_{i,t} \) is the \( \alpha \)-adjusted price in auction \( i \) in period \( t \). \( CB \) represents the collusive benchmark, that is the \( \alpha \)-adjusted cost of the weaker bidder.

We wish to measure the average prices at the end of the session, as well as control for any learning over time, so we use variable \((26-t)\) to measure linear trend. While the \( \alpha \)-adjusted price in the binding auction should be given by the competitive benchmark \( CB \), the price in the Buyer-determined auction should not depend on the weaker bidder’s costs at all. Including the variable \( CB \) in the BDA regression allows us to measure the extent to which bidders in the BDA condition respond to competitive pressure. Note that there are two error components in the model: one that is independent across all observations, \( \mu_i \), and one that is participant-specific, \( \eta_i \). Each error terms has a mean of zero and some positive standard deviation. This treatment of the individual effect, is the random effects model, it is used to control for individual heterogeneity. Additionally, we cluster standard errors at the cohort level, which allows us to control for any potential dependence between participants within the same cohort. Table 2 presents estimates of the model for the two treatments.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Binding Auction</th>
<th>Buyer Determined Auction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>18.58**</td>
<td>111.88**</td>
</tr>
<tr>
<td></td>
<td>(5.09)</td>
<td>(13.28)</td>
</tr>
<tr>
<td>Competitive Benchmark (CB)</td>
<td>0.703**</td>
<td>0.432**</td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.096)</td>
</tr>
<tr>
<td>26-t</td>
<td>0.816*</td>
<td>-1.212**</td>
</tr>
<tr>
<td></td>
<td>(0.328)</td>
<td>(0.217)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.299</td>
<td>0.114</td>
</tr>
</tbody>
</table>

Note: * $p<0.05$; ** $p<0.01$

Table 3. Estimates of the model; standard errors are in parenthesis.

Results in Table 3 confirm what we already see in Figure 1 and Tables 1 and 2: Average $\alpha$-adjusted prices are significantly higher under the buyer-determined auction than they are under the binding auction, and the learning trend in the two treatments is in the opposite directions (average prices increase in the BDA treatment and decrease in the binding auction treatment). And interesting finding from the regression is that the competitive benchmark is positive and highly significant in both treatments (it should be significant in the binding auction treatment but not in the BDA treatment). The competitive benchmark variable is substantially lower in the BDA treatment than in the binding auction treatment which is directionally consistent with the model, but nevertheless, the competitive pressure affects bidders in the BDA treatment—collusion is effective but not 100% successful. This response to competitive pressure in the BDA treatment also explains why average prices are below 150.

4. Discussion and Conclusions

We have shown that the common practice in e-procurement of using a dynamic buyer-determined multi-attribute mechanism allows suppliers to collude on high prices. Collusion can be supported because of the uncertainty in the buyer’s final decision taking process, suppliers have a chance of winning at high prices, which might be more attractive than starting a price war.
and winning at a considerably lower price (with a possibly higher probability). This reasoning can be applied to other circumstances, in which the uncertainty of the final decision allows firms to collude in the first place. All private or public tenders in which prices and conditions are negotiated, offers by competitors are displayed to the participating firms, the firms can react to these offers, and most importantly, the final decision is not based on prices only, are vulnerable to the same form of collusion as described above.

There are several ways the buyer can counteract the problem of collusive behavior. A simple one would be to communicate all $\alpha_i$ before the auction starts, and conduct a binding auction. This would resolve the uncertainty around the decision process and thus collusion would be no longer sustainable.\(^8\) The resulting format would then be similar to the binding quality adjusted auction discussed in section 2.2, taking the different $\alpha_i$ into account. However, in our experience, in practice the manager in charge of the procurement process, in particular if she is using a non binding contest, has not (at least not alone by herself) the final say on the decision on who will be awarded the contract. If other managers influence the decision, then the person in charge of the procurement process does not have the information on the exact size of $\alpha_i$. From a practical point of view, the best alternative would be to quantify the different degrees of differences between the suppliers before the auction starts. This however implies that all parties involved in the decision taken process - procurement, logistic, quality, management - have to become involved. Once these differences are quantified, they should be included in the form of a bonus malus system or a scoring rule into the auction. For example, a supplier who offers a better quality such that the expected additional costs for recalls are expected to be lower by 3%, should be given a price preference of 3% in the auction. If all different dimensions are adequately quanti-

\(^8\) Alternatively, the buyer could announce after each round a provisional winner, such that the suppliers can deduce $\alpha_i$ by themselves.
fied ex-ante, then a price auction will lead to the efficient outcome. If the buyer, however, does not succeed in getting the uncertainty out of the process, then in a dynamic buyer-determined auction collusion can prevail.

Appendix

Proof of Proposition 2:

The general model can be solved for the case which we are going to test in the experiment. Suppose that costs are uniformly distributed and there are no quality differences, i.e. $\Delta_q = 0$. Then collusion is an equilibrium.

The crucial step in the proof is to note that even if someone tries to undercut his competitors, as soon as one competitor is excluded, he will collude with the remaining bidders. Without loss of generality, we will analyze a case where one bidder has costs of $c = 0$. This is sufficient to proof that collusion is an equilibrium since it is the bidder with lowest costs who has the greatest incentive to compete, i.e. if he wants to collude every bidder wants to collude.

Consider first the case of three bidders, where a deviator with lowest costs ($c = c = 0$) has reduced his price such that one other bidder is excluded (the case discussed above). Suppose this has happened at some price $b$. If he now stops lowering his bid further and thus colludes with the remaining bidder, his profit is given by $\frac{1}{2} b$. In case he tries to undercut his competitor as well, his profit will be equal to

$$\int_{\alpha}^{b+\alpha} (x - \alpha) \left( \frac{1}{b+\alpha} \right) dx = \frac{b}{2} \left( 1 - \frac{\alpha}{\alpha+b} \right) < \frac{b}{2}.$$

There exists an extensive literature on the optimal mechanism and auction design in a multidimensional framework (among others Che, 1993; Branco, 1997; Rezende, 2004; Morand and Thomas, 2006), once these different dimensions are quantified. As a general result it is advisable for the auctioneer to use a scoring rule where however the weight on the attributes different than the price should be somewhat lower than the true weight, as this fosters price competition between the suppliers.
By induction we next argue that if there are \( m \) bidders still remaining, someone who lowers his price will stop lowering if one of the other bidders is excluded. This works similar to the argument above for three bidders. If the bidder starts colluding after the first competitor is excluded his expected profit is \( \frac{b}{m-1} \), on the other hand he could lower his bid until another bidder drops out. Then his expected profit is given by

\[
\frac{b}{m-1} - \frac{\alpha(a+b)^{m-2} - \alpha^{m-1}}{(m-1)(m-2)(a+b)^{m-2}} \leq \frac{b}{m-1},
\]

where \( f^{(m-2)}(x) \) is the probability that the highest costs out of \( m - 2 \) bidders are equal to \( x \).

As a final step in the proof, note that a bidder who contemplates deviating from the initial collusive bids where everyone bids \( R \), has to compare getting \( R \) with probability \( 1/n \), or bidding one competitor out and then obtaining the bid \( b \) that was necessary to exclude one bidder \( b \) with probability \( 1/(n-1) \). While the expected profit from collusion is \( R/n \), the profit from competing one opponent out is

\[
\frac{1}{n-1} \int_a^c (x - \alpha)f^{(n-1)}(x)dx = \frac{c}{n} - \frac{1}{n-1} \left[ \alpha - \frac{\alpha^n}{n^{n-1}} \right] \leq \frac{c}{n},
\]

therefore the proposition follows. ■

References


