Market Dominance and Search Quality in the Oligopolistic Search Engine Market

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Abstract

We incorporate the choice of quality improving innovations by a search engine platform in an asymmetric two-sided model of oligopolistic internet search engine market. This extension of a two-sided market model in Armstrong (2006) provides a convenient tool for the joint analysis of market structure, rate of innovation, pricing, quality choices, and welfare in innovative industries with network effects. We characterize the structure of the solution under two different R&D cost regimes in both symmetric and asymmetric settings performing comparative statics exercise with respect to the magnitude of network effects, discrepancies in cost structures and product differentiation. Based on this characterization, we analyze what issues the dominance in the search engine market raises for antitrust policy. One of the main findings shows that in asymmetric settings, substantial differences in installed data base of users between competing search engines may lead to negative impact on users’ welfare. While leveled playing field, where dominant search engine shares data on clicking behavior with weaker competitors improves both users’ and advertisers’ welfare.

JEL Classification: L4, C7, L5, L82, L86, L96

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1 Introduction

The search engine market is relatively young, however both industrial organization and law and economics are confronted with a number of important questions related to the rapid growth of online search, its concentration and its increasing importance for modern society. In this paper we analyze the implications of market dominance and concentration in the search engines market for different players (such as users, search engines and advertisers) and different market outcomes (such as prices charged to advertisers, the quality of search results and welfare).

The markets for search-based and online advertising have a number of specific features that set them apart from most markets. These features include network effects, double-sidedness, and high levels of R&D and innovation. Network effects often play an important role in analyzing competition in R&D intensive markets. Network effects present opportunities for enhanced consumer welfare, but also can create the potential for competitive harm and increased barriers to entry.\(^1\) Hence, the potential interplay between network effects and innovation incentives in the search market must be examined.\(^2\) In this project we modify the existing models of two-sided markets by e.g. Armstrong (2006) or Armstrong and Wright (2007) to jointly analyze pricing, R&D efforts and investments into quality improvements by competing platforms as well as welfare implications of these strategic choices in the presence of possible network effects. Such an extension allows us to analyze interplay between innovation and competition incentives in the search engine market and in the two-sided markets in general.

The second innovation of our paper is that we characterize the solution of the proposed two-sided market model also allowing for heterogeneity of platforms under different R&D investment cost specifications. We analyze the situations where platforms may differ in terms of the size of the network effects (e.g. installed user base). This provides a convenient tool for explicit analysis of market dominance and the role of market leaders in two-sided markets, which has not been done in earlier theoretical studies. We define the dominance in relation to the share of users as well as the size and scale of network effects or relative costs of quality improvements. More specifically, the source of dominance may originate either from better search technology (e.g. 'know how' on web-sites ranking implemented by Google in 2001), size of the network effects (e.g. bigger installed user base), or better matching technology for ads (e.g. exploiting the scale in search or more efficient use of previous search queries in order to match the new once). The theoretical debate on the role of dominant firms

\(^1\) See Evans (2008), Manne and Wright (2011) or Lianos and Motchenkova (2013) for the detailed discussion of the size and the direction of the impact of network effects.

\(^2\) See e.g. Economides (2010) or Larouche (2009).
(market leaders) in multi-sided markets is still limited: most of the literature on multi-sided markets (with exception of Etro (2011)) is focused on monopolistic pricing and symmetric competition between platforms, not on competition between a potentially dominant platform and weaker engines. In this paper we provide some new insights on modeling asymmetries in multi-sided markets.

The structure of the search engine market and its pricing/quality strategies have a number of distinctive features, which have been identified in e.g. Pollock (2010). The search engine acts as a platform intermediating between content providers, users, and advertisers. Closely related to this structure of connections between agents is the associated pricing structure, where users/searchers enjoy the service for free, advertisers are required to pay strictly positive prices for search engine services\(^3\), and content providers are subsidized by the search engine. These features of the search engine markets call for applications of two-sided markets models as has already been recognized in Devine (2008), Evans (2010), Jeon et al. (2012), or Halaburda and Yehezkel (2011). While a positive price is only set for one of the three groups (i.e. advertisers), quality competition plays nevertheless an important role with regard to the relation between search engines and users and between search engines and content providers, by the intermediary of users (the better a search engine is, the more users it will attract and thus the more valuable it will be for content providers).

Furthermore, to the difference of other two sided platforms, search engines detain an important amount of information about their users and advertisers (the "map of commerce", see Spulber (2009)). Utilizing this information allows search engines to increase the relevance of their advertisements, and increased relevance means increased value to those who wish to advertise. Hence, the quality of matching and the quality and the relevance of search results are valued not only by users of the search engine, but also by advertisers. These arguments imply that the quality of the search and the relevance of the search results play a crucial role for both consumers and advertisers. Next, the fact that asymmetric search engines would utilize this accumulated users’ information differently could extend market size asymmetries to asymmetries in the size of network effects.

In pursuit of quality improvements search engines invest heavily in technology improvements. Search engines are R&D intensive and the market generally displays high levels

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\(^3\)The majority of Google’s income comes from sponsored links paid by the featured organization. The amount of Google’s charges is calculated according to a Vickrey second price keyword auction, adjusted by ‘quality factors’ and conducted through Google’s AdWords platform. The rest of Google’s income comes from selling advertisements in designated spaces in third-party websites, through its AdSense application. In general, prices paid by advertisers depend on the ‘quality score’ and calculated on ‘per click’ basis (see e.g. Lianos and Motchenkova (2013)). In the current paper, similar to Armstrong (2006), we implement fixed price charged to each advertiser. Extensions with ‘click fee’ and ‘quality score’ are postponed to further research (see also Appendix and section 5 on robustness checks).
of innovation (see e.g. Devine (2008)). Furthermore, according to Pollock (2010), search engines display many of the characteristics of natural monopolies, as their cost structure involves important fixed costs, such as hardware, support, updates, monitoring, but almost zero marginal costs on both the user and advertiser side of the market. Such cost structure reinforces the tendency of internet search market to concentration. According to recent data, in the US, Google had a market share of 66.2%, Yahoo of 16.4%, and Bing of 11.8%. In the UK, just as in many other European countries, Google had a market share of 90.83%, Yahoo of 3.21%, and Bing of 3.12%. See Pollock (2010) or Argenton and Prüfer (2012) for more detailed overviews. The basic conclusion is that a single firm (Google) is emerging to dominate the market at least in the US and in Europe. The threat of domination becomes even stronger in the search engine market, since it can result not only in excessive pricing for advertisers, but also in reduction of quality of search results, which harms both advertisers and users.

This paper analyzes these specific features of the search engine market from an economic perspective and incorporates the analysis of quality improving capital investments in a two-sided model of oligopolistic internet search engine market. There we analyze the interplay between network effects and incentives to innovate. We also evaluate possible consequences of dominant platform’s strategies on users’ and advertisers’ welfare that turn out to depend on the size of network effects, differences in the installed user base, and the degree of product differentiation.

The implications we obtain are similar to Argenton and Prüfer (2012), though we use a significantly different framework. In simple oligopoly settings they observe that monopolization of the search engine market has negative effects on the expected average search quality, the rate of innovation, consumer surplus, and total welfare. They find that there is a strong tendency towards market tipping and, subsequently, monopolization, with negative consequences for economic welfare. As a remedy they propose to require search engines to share their data on previous searches. Presumably, this would level the playing field in the quality dimension. In our model, which is a modification of Armstrong (2006) and Armstrong and Wright (2007) approach, we endogenize both pricing and quality decisions on both sides of the platform. We provide characterizations of the solutions of the extended asymmetric model of competition in two-sided market for different structures of R&D cost.

Argenton and Prüfer (2012) use a simple model of beauty contest, simplifying along several dimensions. We obtain some similar results though using a significantly different framework. In addition Argenton and Prüfer (2012) start with a triopoly and then compare it to duopoly, whereas we can easily generalize our findings to the case of n engines. Also, our analysis allows us to do a richer set of comparative statics to capture the impact of asymmetries on pricing, quality, users’ welfare and markets shares of the platforms. Our framework also allows to incorporate the impact of product differentiation and to provide new insights on the decisions of users and advertisers to join different platforms.
function and analyze legal antitrust issues arising in the search engine market. Our results are complementary to results of the oligopoly model in Argenton and Prüfer (2011), where only quality choices are endogenized.

Results of our paper are also related to the findings in Etro (2011). His analysis suggests that a platform that has reached dominance in search advertising can have an incentive to limit services to consumers to be more aggressive in the competition for advertisers or to exploit its scale in search to build barriers to entry and to adopt price discrimination through opaque click-weighted auctions to manipulate pricing for sponsored links. We also show that in some cases the dominant platform may not have sufficient incentives to invest in quality improvements. Our conclusions in section 4.2 also highlight that the gap in the size of network effects between dominant and weaker search engine may have a negative impact on users’ and advertisers’ utility. Another novelty of our analysis is that we also show that for plausible parameter values (partly borrowed from Jeon et al. (2012)) the quality of search is also increasing in the initial installed base of the weaker engine. The intuition for this is that it simply forces the stronger engine to compete harder for the customers. In other words, our approach allows us to find a direct economic links to policy prescriptions from the outcomes of the competition which also accounts for the quality of search results.

The structure of the paper is as follows. We begin in section 2 with literature review and overview of the legal issues. In section 3 we discuss some specific features of search engine market and introduce a model of oligopolistic internet search engine. In section 4 we provide characterizations of the solutions of the extended two-sided market model for different structures of R&D cost function and analyze possible distortions caused by the presence of the dominant platform. Section 5 concludes. There we argue that the evidence on increasing concentration and the theoretical results in the paper suggest that some form of intervention is needed in order to level the playing field in the search engine market and avoid possible harm to users and advertisers.

2 Overview of the Legal Issues and Related Literature

The motivation for this paper is derived from recent attention of the leading antitrust authorities on a market for online advertising, which is dominated by Google at the global level. Recent investigation by the European Commission identifies four concerns where Google business practices may be considered as abuses of dominance.⁵ Firstly, in its general search results on the Web, Google displays links to its own vertical search services differently than

it does for links to competitors. European Commission expresses concerns that such display practices may result in preferential treatment compared to those of competing services. Secondly, there is a concern that Google may be copying original material from the websites of its competitors and using that material on its own sites without their prior authorization. Hence, Google may appropriate the benefits of the investments of competitors. This in turn could reduce competitors’ incentives to invest in the creation of original content for the benefit of internet users. The third concern relates to agreements between Google and partners on the websites of which Google delivers search advertisements. The agreements result in de facto exclusivity requiring them to obtain all or most of their requirements of search advertisements from Google, thus deferring competing providers of search advertising intermediation services. Commission fourth concern relates to restrictions that Google puts to the portability of online search advertising campaigns from its own platform to the platforms of competitors. There is a threat that Google imposes contractual restrictions on software developers which prevent them from offering tools that allow the seamless transfer of search advertising campaigns across AdWords and other platforms for search advertising.

In a companion paper Lianos and Motchenkova (2013) we identified main competition issues that summarize the above discussion of the recent concerns outlined by the European Commission. Main competition issues coming from the concentration of the search engine market are strategies reducing multi-homing, leveraging, and exploitative practices. All of them may lead to increased asymmetries and enhanced dominant position of the leading search engine. In the formal analysis section we address the resulting oligopolistic market in the presence of asymmetric network effects and cost asymmetries.

Next, an important issue, which arises in connection to analyzing abuse of market power by the dominant firms, is the determination of the relevant market. There is a number of articles on determination of the relevant market for online advertising, which highlight the differences between online and traditional advertising and also between displayed and search-based advertising, within the class of online advertising. The recent examples are Ratliff and Rubinfield (2010), Etro (2011), Evans (2009), or Goldfarb and Tucker (2011). See also French Competition Authority report (2010). These references imply that there is no substitutability between online and traditional advertising and only limited substitutability between displayed and search-based advertising. Hence, search-based advertising can be considered as a separate market. That’s why in this paper we concentrate on the market for search-based advertising, where two-sided aspect and innovation incentives aimed at increasing quality and relevance of search results play a crucial role. The efficient technology of matching adds on the one side to search queries by users on the other side is essential only for the search-based advertising segment of the market. Search-based advertising is
facilitated by the search platform and, actually, can only exist on the basis of such platform. Hence, for the subsequent analysis in this paper we will assume that the relevant market is on-line search-based advertising market.

Further, the existing theoretical literature focuses mainly on the advertising side of search engines (see e.g. Edelman et al. (2007), Varian (2007), Ellison and Ellison (2004), Chen and He (2006), or Athey and Ellison (2011)). They view internet search engine as some form of improved ‘yellow-pages’. Given the two-sided nature of search and its similarity to ‘yellow-pages’, the obvious analytical tools to use would be those developed in the literature on two-sided markets (see e.g. Rochet and Tirole (2003, 2006), Caillaud and Jullien (2003), Armstrong (2006), Armstrong and Wright (2007), Gomes (2010), or Weyl (2010)). We contribute to this literature by taking into account the importance of quality improving capital investments (or innovation efforts) by the platform. This point will be central to our analysis and it differentiates our analysis from much of the existing literature. The issue of quality and innovations has not been addressed in the theoretical literature on oligopolistic two-sided markets so far. Moreover, there are very few attempts to model a search engine as a two-sided platform (exceptions are Etro (2011), Halaburda and Yehezkel (2011) and Jeon et al. (2012)).

Another stream of the literature looks at the importance of the quality of information provided by the search engine, but does not take into account its two-sidedness and alleged network externalities (see e.g. Pollock (2010) and White (2008)). The approach we take is also very different from Pollock (2010) and White (2008), while it still emphasizes the importance of quality considerations for the search engine market. Turning to our approach, it should be stressed again that the two primary groups a search engine sits between are users and advertisers. In this market surplus is created when these two groups interact. In addition, also cross-group externalities are present, and the benefit enjoyed by a member of one group depends upon how well the platform does in attracting agents from the other group. Furthermore, in internet search engine market with end users on one side and the advertisers on the other side, the quality of the search results is important for both sides. As was mentioned above, we extend the standard two-sided markets models by incorporating these quality decisions by the platforms.

Another important innovation of our paper is characterization of the solution of the asymmetric model of competition in two-sided markets. We analyze both pricing and quality decisions of competing firms in the presence of dominant firm in a two-sided search engine market model. The theoretical debate on the role of dominant firms (market leaders) in

\[^6\text{We implemented similar innovation in an earlier paper by Lianos and Motchenkova (2013) for analysis of quality improving incentives for a monopolistic internet search engine.}\]
multi-sided markets is still limited: most of the literature on multi-sided markets (with exception of Etro (2011)) is focused on monopolistic pricing and symmetric competition between platforms, not on competition between a potentially dominant platform and weaker engines. In this paper we incorporate this feature and provide new insights stemming from plausible asymmetries in multi-sided markets.\footnote{It should also be stressed that our model applies also to any ads financed two-sided service where quality of matching between customers’ search queries and ads is essential. These types of models are missing in the literature on two-sided markets so far.}

3 The Model of the Internet Search Market

3.1 Structure of the Search Engine Market

As has been discussed in the introduction, the search engine market has certain distinctive features related to structure, costs and pricing, which should be taken into account when building a theoretical model.

Firstly, the structure of the search engine market has a multi-sided aspect in which the search engine acts as a platform intermediating between content providers, users/searchers, and advertisers. Secondly, search engines do not directly charge users for their service but supply it for free. Hence, in our framework we assume that search engines cannot set prices for users (whether positive or negative) but rather are constrained to price at zero, so that $p_U = 0$. In addition, we do not model explicitly the content providers’ side of the market. But rather implicitly incorporate them into the search engine technology through additional cost component.\footnote{Similar to Pollock (2010) we assume that the pool of material made available by content providers is available to all search engines and, as such, content providers can be ignored as (strategic) agents allowing us to focus solely on the other three types (users, advertisers, and platform (or search engine) itself).}

Advertisers are required to pay strictly positive prices for search engine services, so that $p_A > 0$.\footnote{Our approach to modeling advertisers’ side of the market is simplified compared to Edelman et al (2007), Varian (2007), Ellison and Ellison (2004), Chen and He (2006), Athey and Ellison (2011), or White (2008). Since the primary aim of our project is to concentrate on the impact of network effects and quality improving innovation efforts, we believe the advertisers’ side of the market can be modeled using the general approach in Armstrong (2006).} Advertisers also value the quality of the search engine. However, contrary to users the marginal cost of serving one additional advertiser is strictly positive, so that $f_U = 0$ and $f_A > 0$. This reflects the cost of, for example, signing the contract, assisting, or arranging the auction procedures for each particular advertiser.

Further, the important feature of the search engine market relates to technology and costs. In particular, search engines are R&D intensive and the market generally displays high levels of innovation. This innovation usually occurs within a particular software environment that determines the type of engineers (and specific skills) required. These specific skills...
may be scarce and very costly. In addition, considerable investment efforts are necessary for supporting, monitoring, and sponsoring content providers. This implies that running a search engine service is highly capital intensive. We will denote these investments (or innovation efforts) as $k$ – quality improving innovation efforts.\(^\text{10}\) Both of these types of cost, whether related to R&D or the development and maintenance of service infrastructure and content, will be modeled as an increasing function of quality improving innovation efforts $F(k)$, with $F'(k) > 0$ for all $k \in [0, \infty)$. The marginal cost of serving one additional user is assumed to be very low and will be set equal to zero, i.e. $f_U = 0$.

### 3.2 The Model

In this sub-section we present the two-sided search engine market model, which is a modification of Armstrong (2006) and Armstrong and Wright (2007) framework. Suppose there is a unit measure of agents in group-$A$ and a unit measure of agents in group-$U$. We will refer to the group-$A$ agents as advertisers and group-$U$ agents as users. Suppose also that there are two platforms (search engines), $i = 1, 2$. They each offer a service to the two groups. Following the existing literature, we assume each agent values the number of agents from the other group with whom he can interact, but not the number of agents from his own group.\(^\text{11}\) In addition, and this is one of our main innovations, we assume that agents on both sides (both users and advertisers) value the quality and the relevance of search results or the advancement of the search technology offered by a particular engine, which we denote by $k^i, i = 1, 2$. Further, we assume multi-homing advertisers, i.e. advertisers can join either platform 1, platform 2, or both platforms if they multi-home. Similar to Jeon et al. (2012), users are restricted to single-homing.\(^\text{12}\)

On the user side of the market platforms differ in a standard Hotelling manner. They are located at either end of a unit interval and users are located uniformly along the unit interval. Users incur a "transport cost" $tx$ of travelling a distance $x$ to the platform(s) they use, $t \geq 0$. An agent located at $x$ on the unit interval incurs a transport cost $tx$.

\(^\text{10}\)In Argenton and Prufer (2012) search engine quality is defined as 'overall accuracy of search results'. Argenton and Prufer (2012) also provide results of the recent surveys where this accuracy has been estimated for several leading search engines. We adopt a similar definition of quality in this paper.

\(^\text{11}\)In particular, in the solution section we assume one-sided network effects, such that only advertisers obtain benefit $\alpha_An$ by participating in a market which allows them to interact with $n$ users, while users are only interested in the quality and relevance of the content and do not derive extra utility from being exposed to ads. Similar assumption is adopted in Jeon et al. (2012). However, our proposed algorithm and characterization of the asymmetric numerical solution can also be extended to capture different functional forms of utilities.

\(^\text{12}\)These assumptions seem to be satisfied in practice, where advertisers normally contract several search engines to maximize market coverage. While users have one favorite (most convenient) search engine, with which they have experienced best (taste specific, habit specific, or most relevant) search results.
when joining platform 1 and a transport cost $t(1 - x)$ when joining platform 2. Possible interpretations of the transport cost in case of search engine include costs of installing the search engine browser, or the initial set-up costs that users face while learning about a new engine.\footnote{Google likes to argue that "competition is one click away". However, it is highly contestable whether users can actually leave as easily as Google suggests: Popular web browsers Firefox and Chrome strongly favor Google. In the mobile context, Android offers Google similar lock-in. And even on non-Google mobile platforms, Google serves 95% of searches due to defaults which systematically direct users to Google. At the same time syndication contracts assure Google exclusive long-term placement on most top web sites. This implies both high switching costs for the existing users as well as high likelihood that new users are bound to flow to Google. (See http://www.benedelman.org/news/092011-1.html)} Similar to Armstrong and Wright (2007) or Jeon et al. (2012) we concentrate on the case where one side views the platforms as homogenous, while the other views the platforms as heterogenous. On the users’ side platforms are horizontally differentiated for two different reason. First, they differ in terms of the way they generate search results for a given query. They may have different databases, use different algorithms for search and different ways to display search results. Second, they offer different services as portals. For the remainder of the paper we assume that in the two-sided search engine market, which involves users and advertisers, advertisers view the competing platforms as more or less homogenous (controlling for the size of the network benefits), while users have preferences for using one particular platform over the other. Hence, in the formal model we set the degree of product differentiation $t_A = 0$ and $t_U = t > 0$.

The utilities of agents are determined in the following way: if the platform $i$ attracts $n_U^i$ and $n_A^i$ members of the two groups, the utilities of group-$U$ agents and group-$A$ agents are given by the following expressions.

The utility of a group-$U$ agent located at $x \in [0, 1]$ when she joins platform 1 is given by

$$u_U^1(k^1, p_U^1, n_A^1) = \alpha_U n_A^1 + k^1 - p_U^1 - tx. \quad (1)$$

When the same agent subscribes to platform 2, she obtains utility

$$u_U^2(k^2, p_U^2, n_A^2) = \alpha_U n_A^2 + k^2 - p_U^2 - t(1 - x). \quad (2)$$

The utility of a group-$A$ agent when she joins platform 1 is given by

$$u_A^1(k^1, p_A^1, n_U^1) = \alpha_A n_U^1 + k^1 - p_A^1. \quad (3)$$

Finally, the utility of a group-$A$ agent when she joins platform 2 is given by

$$u_A^2(k^2, p_A^2, n_U^2) = \alpha_A n_U^2 + k^2 - p_A^2. \quad (4)$$

where $p_U^i$ and $p_A^i$, $i = 1, 2$, are platforms’ prices to the two groups. Recall that $p_U^i = 0$, $i = 1, 2$, are set to zero, since users are served for free.\footnote{Similar assumption is employed in e.g. Jeon, Jullien, and Klimenko (2012).} While $p_A^i$, $i = 1, 2$ are assumed
to be positive.\textsuperscript{15} The parameter $\alpha_U$ measures the benefit a group-$U$ agent (user) enjoys from interacting with each group-$A$ agent (advertiser). $\alpha_A$ measures the benefit a group-$A$ agent (advertiser) obtains from interacting with each group-$U$ agent (user). The variable $k^i$, $i = 1, 2$, denotes the quality improving innovation efforts. Expressions in (1)-(4) describe how utilities are determined on each platform $i = 1, 2$, as functions of the numbers of agents who participate on each platform ($n^j_i$), network externalities ($\alpha^j_i$), prices charged by each platform ($p^j_i$), and the amount of quality improving innovation investments incurred by the search engine ($k^i$), which is platform specific, but not agent specific.$^{16}$

Turning to the cost side, we assume that both platforms incur positive per-agent cost $f^i_A$, $i = 1, 2$ for group $A$ (advertisers) and costs of quality improving capital investments $F(k^i)$, with $F'(k^i) > 0$ for all $k \in [0, \infty)$, $i = 1, 2$.\textsuperscript{17} A per-agent cost $f^i_U$ for group $U$ (users) is assumed to be zero, $f^i_U = 0$, $i = 1, 2$. Therefore, the search engine $i$’s profit is given by

$$\pi^i(k^i, p^i_A) = n^i_A(p^i_A - f^i_A) - F(k^i), \ i = 1, 2. \quad (5)$$

Platforms simultaneously choose prices and the level of $k^i$, and after observing prices and quality characteristics advertisers simultaneously decide which platform(s) to join.

For subsequent analysis we employ a set-up, where network effects are present only on advertisers side\textsuperscript{18}, i.e. $\alpha_A > 0$, $\alpha_U = 0$. Advertisers value the presence of users eyeballs (potential consumers), while the externality on users’ side is assumed to be zero. It largely seems to be the case in the search engine market that only one side (advertisers) cares about platform performance on the other side. Users mainly do not care about the amount of advertising on the other side of the search engine and only interested in the content and quality and relevance of search results. That’s why for the analysis in this section we restrict

\textsuperscript{15}Again, similar to Jeon, Jullien, and Klimenko (2012), we assume that each platform charges a positive subscription fee to advertisers. Actually, Google’s advertising fee is per click, which can be incorporated in our model as a multiplicative function of the number of users $n^i_U$ and the quality of the matching technology $k^i$ (e.g. $f^i k^i n^i_U \sim f^i k^i x$), which enters the profit function of each platform with the positive sign. However, this would make it problematic to derive closed form solutions even for the symmetric case in the current framework. Numerical analysis of both symmetric and asymmetric cases can be adapted to incorporate this extension. We postpone this extension to future research (see also section robustness checks).

\textsuperscript{16}We assume here that quality improving efforts (investments) map one-to-one to realized quality of the search engine, which is valued by users and advertisers. In general, the results of the model would go through for any increasing mapping from $k^i$ to quality.

\textsuperscript{17}Having $F(k)$ an increasing function of $k$ seems to be consistent with S-shaped returns to scale in the search engine market discussed in Etro (2011b). However, the approach to model the impact of quality improving efforts, $k^i$ and $F(k^i)$, can be improved. For example, the cost of quality improving efforts can be increasing not only with $k^i$ but also with $n^i_U$, since it might be more difficult to manage the engine when more queries are running. Then $k^i$ and $n^i_U$ should enter additively the cost function. Again, for the purpose of tractability of the current model we leave this extension to future research.

\textsuperscript{18}We leave the detailed analysis of the case with two-sided network effects to the future research. Preliminary calculations show that qualitative results and policy implications of one-sided network effects case will not change if two-sided network effects are introduced.
our attention to the case of one-sided network effects, i.e. $\alpha_U = 0$. In the setting with $\alpha_A > 0$, $\alpha_U = 0$ and $p_U^1 = p_U^2 = 0$ the system in (1)-(4) will be rewritten as follows

$$u_U^1(k^1, p_U^1, n_A^1) = k^1 - tx$$

$$u_U^2(k^2, p_U^2, n_A^2) = k^2 - t(1 - x)$$

$$u_A^1(k^1, p_A, n_U^1) = \alpha_A n_U^1 + k^1 - p_A^1$$

$$u_A^2(k^2, p_A, n_U^2) = \alpha_A n_U^2 + k^2 - p_A^2,$$

while each search engine’s profit function is given by (5).

In order to check robustness of the results in the two subsequent sub-sections we will consider two different functional forms of the cost technology for quality improving efforts. The search engine cost of quality improving capital investments $F(k)$ is modelled as either linear or quadratic function of $k$, i.e. $F(k^1) = \lambda k^1$ and $F(k^2) = \frac{\lambda (k^2)^2}{2}$, $i = 1, 2$.\(^{19}\) We assume that users single-home, while advertisers can also multi-home. Two search engines compete for market share within each group (users and advertisers). Following Armstrong and Wright (2007), to analyze the users’ choice of platform we adopt the Hotteling model of product differentiation. Assuming that the users’ market is covered this implies that the number of users participating in platforms 1 and 2 are given by expressions (10) and (11), respectively.

$$n_U^1 = x = \frac{1}{2} + \frac{k^1 - k^2}{2t}$$

$$n_U^2 = 1 - x = \frac{1}{2} + \frac{k^2 - k^1}{2t}$$

Advertisers are assumed to be heterogeneous in their fixed costs of joining each platform. Similar to Jeon et al. (2012) we assume that they will join platform $i'$ as long as their resulting profit, $\alpha_A n_U^i + k^i - p_A^i$, exceeds the fixed cost of joining the search engine. We adopt the assumption that the fixed cost of an advertiser who joins the search engine $i$ is distributed with constant density $f = 1$. This implies that the mass of advertisers who join platforms 1 and 2 are determined by (12) and (13), respectively.

$$\alpha_A n_U^1 + k^1 - p_A^1 - n_A^1 = 0$$

$$\alpha_A n_U^2 + k^2 - p_A^2 - n_A^2 = 0$$

\(^{19}\)Results and policy implications appear to be highly dependent on the structure of the cost technology.
4 Price / Quality Competition in Two-sided Market

4.1 Linear Technology: Characterization of the Solution

We start our analysis of linear technology case by characterizing symmetric equilibrium. In symmetric case it is possible to obtain closed form analytical solution. Asymmetric oligopoly model, even under linear technology regime, does not have a tractable closed form analytical solution. However, numerical analysis provided in Appendix 2 confirms that main insights of the symmetric case also hold in asymmetric oligopoly setting. Two main insights from the analysis of the linear technology case are summarized in propositions 1 and 2. They imply that price / quality choices by competing search engines depend on the degree of product differentiation, size of network effects and possible cost advantages. Also it seems that the dominant search engine does not have sufficient incentives to invest in quality improvements even in the presence of potential (but weaker) competitors.\footnote{These results may raise some concerns for antitrust policy. However, the solution of linear cost case only possesses the properties of the saddle point equilibrium. There is no global maximum equilibrium under linear cost technology. Hence, conclusions of the linear cost case should be interpreted carefully. For the sake of completeness we present complete characterization of both symmetric and asymmetric solutions in both linear and convex technology regimes, as these are the two main functional forms analyzed in the literature.}

Analysis of the FOCs of the profit functions in (5) under linear technology implies the following symmetric equilibrium result with $p_M = P_M = Q_M$ and $k_M = k_1M = k_2M$, where $M$ stands for multi-homing.\footnote{For detailed derivations see Appendix 1.}

**Proposition 1** Under linear technology the symmetric saddle point equilibrium exists, where platforms will serve both sides of the market with advertisers multi-homing and users single-homing. The price to users is $p_U = 0$. The equilibrium price to advertisers is given by

$$p_M = f_A + \frac{2t}{\alpha + 2t}.$$ 

The equilibrium quality improving innovation efforts are given by

$$k_M = f_A + \frac{4t}{\alpha + 2t} - \frac{\alpha}{2}.$$ 

Comparative statics of the symmetric equilibrium shows:

$$\frac{\partial p_M}{\partial t} > 0, \frac{\partial p_M}{\partial f_A} > 0, \frac{\partial p_M}{\partial \lambda} > 0, \frac{\partial p_M}{\partial \alpha} < 0,$$

$$\frac{\partial k_M}{\partial t} > 0, \frac{\partial k_M}{\partial f_A} > 0, \frac{\partial k_M}{\partial \lambda} > 0, \frac{\partial k_M}{\partial \alpha} < 0.$$ \hspace{1cm} (14)
This implies that symmetric competing search engines will charge higher prices to advertisers when there is higher degree of product differentiation, higher costs of serving advertisers, higher costs of quality improving capital investments, or when advertisers benefit less from network effects. On the other hand, in an oligopoly with high degree of product differentiation there will be no negative effect on the investments in quality of search results. Higher fixed costs, lower cost efficiency, and lower advertisers’ benefits from network effects will also imply enhanced symmetric equilibrium quality of the search results.

The analysis of asymmetric equilibria (where $\alpha_A, f_A, or \lambda$ can differ between engines, i.e. $\alpha_A^1 \neq \alpha_A^2, f_A^1 \neq f_A^2, or \lambda^1 \neq \lambda^2$) is quite cumbersome and it is very hard to obtain closed form analytical solutions for equilibrium prices and quality improving investments. Before we state the results, let us discuss possible relationships between the size of the above mentioned parameters and the degree of domination of the search engine market by a single firm (such as e.g. Google). The dominant firm can benefit from the scale economies and stronger network effects. Hence, it will have lower costs of serving advertisers (i.e. lower $f_A$), lower costs of quality improving capital investments (or higher cost efficiency, i.e. lower $\lambda$ due to experience, e.g. learning by doing effect). The dominant platform may also allow advertisers benefit more from network effects, implying higher $\alpha_A$ (due to e.g. bigger installed user base or better matching technology for ads (by e.g. exploiting more efficient use of previous search queries in order to match the new once)). Now, if we compare the outcomes in the asymmetric equilibrium through the FOCs specified in the Appendix 1, the conjecture of Proposition 2 follows immediately. The results of this proposition are also confirmed by numerical analysis provided in Appendix 2.

**Proposition 2** Under the linear technology in the asymmetric saddle point equilibrium we have $p_A^{1M} > p_A^{2M}$ and $k_A^{1M} > k_A^{2M}$ if 
\[
\begin{align*}
\lambda^1 &> \lambda^2 \\
\alpha_A^2 &> \alpha_A^1 \\
f_A^1 &> f_A^2
\end{align*}
\]

This proposition has a number of policy implications. Given that the dominant search engine (platform 2) can be characterized by lower $\lambda$, lower $f_A$, and higher $\alpha_A$, it will offer lower quality of the search results in any of the three model variations described in Proposition 2 compared to the weak (or less cost efficient) search engine (like e.g. Bing). Hence, the impact of the market domination on the resulting quality is clearly negative. The impact on the prices charged to advertisers is similar, but appears to be welfare improving. Dominant (more efficient) platform will charge lower prices than non-dominant due to greater cost savings and stronger network effects.

The comparison of the strategies chosen by the dominant (more cost efficient) and non-dominant (possibly less cost efficient or weaker) platform reveals the trade-off between en-
hancement of the quality of the search results and prices charged to advertisers. Dominant search engine may choose lower prices accompanied by the lower quality of the search results in order to keep its market share. While weaker non-dominant search engines (like e.g. Bing or Yahoo) will charge higher prices to advertisers, but at the same time will offer higher quality of search results in order to increase their market share by attracting more users and, consequently, more advertisers. In this case, higher price charged to advertisers is not a problem from the antitrust policy point of view, since it simply reflects higher marginal costs and smaller network effects in the competitive oligopoly equilibrium for these weak search engines. However, the absence of the incentives for the dominant search engine to invest in quality improvements even in the presence of potential, but weaker, competitors should raise some concerns.

These results may raise some concerns for antitrust policy. However, as discussed above the solution of linear cost case only possesses the properties of the saddle point equilibrium. There is no global maximum equilibrium under linear cost technology. Hence, conclusions of the linear cost case should be interpreted carefully. In general, the results of this two-sided oligopoly model appear to be highly dependent on the R&D cost structure. In the next subsection we present complete characterization of both symmetric and asymmetric solutions under the convex technology regime.

4.2 Convex Technology: Characterization of the Solution

Our analysis of convex technology case is divided into two parts. We start out by characterizing symmetric equilibrium. In symmetric case it is possible to obtain closed form analytical solution. Similarly to linear case, asymmetric oligopoly model does not have a tractable closed form analytical solution. However, numerical analysis developed in Appendix 5 allows obtaining fairly general conclusions for the asymmetric oligopoly setting with convex R&D cost for a range of policy relevant parameter values. Main insights from the analysis of the convex technology case are summarized in Propositions 3, 4, and 5 and figures 1 - 9 below. One of the main findings shows that in asymmetric settings, substantial differences in the size of network effects and, especially, lower installed data base of users on the weaker engine may lead to negative impact on users’ welfare. While leveled playing field, where dominant search engine shares data on clicking behavior with weaker competitors improves both users’ and advertisers’ welfare.
4.2.1 Symmetric Competition

Analysis of the FOCs of the profit functions in (5) under convex technology \( F(k^i) = \lambda \left( k^i \right)^2 \), \( i = 1, 2 \) implies the following symmetric equilibrium result with \( p^M_A = p^1_M = p^2_M \geq 0 \) and \( k^M = k^{1M} = k^{2M} \geq 0 \), where again \( M \) stands for multi-homing.\(^{22}\)

**Proposition 3** Under the convex technology there are two types of the symmetric equilibria:

The symmetric global maximum equilibrium exists if \( \lambda > \frac{1}{2} \) and \( \alpha_A \geq 2f_A \).

The symmetric saddle point equilibrium exists if \( 0 \leq \lambda < \frac{1}{2} \) and \( \alpha_A \leq 2f_A \).

Under the both types of equilibria platforms serve both sides of the market with advertisers multi-homing and users single-homing. The price to users is \( p_U = 0 \). The equilibrium price to advertisers is given by

\[
p^M_A = f_A + \frac{\lambda(\alpha_A - 2f_A)}{2(2\lambda - 1)}.
\]

The equilibrium quality improving innovation effort is given by

\[
k^M = \frac{\alpha_A - 2f_A}{2(2\lambda - 1)}.
\]

Comparative statics analysis of the symmetric saddle point equilibrium, which arises when \( 0 \leq \lambda < \frac{1}{2} \), has policy implications that coincide with the implications of the linear technology case above.\(^{23}\) Hence, in the setting where cost advantages due to scale economies are substantial (which correspond to low \( \lambda \) parameter) we can identify possible detrimental effect of excessive dominance on incentives to innovate in the search engine market.\(^{24}\)

Comparative statics of the symmetric global maximum equilibrium (which arises when \( \lambda > \frac{1}{2} \)) shows:

\[
\begin{align*}
\frac{\partial p^M_A}{\partial \alpha_A} & > 0, & \frac{\partial p^M_A}{\partial \lambda} & < 0, & \frac{\partial p^M_A}{\partial f_A} & - \text{ambiguous}. \\
\frac{\partial k^M}{\partial \alpha_A} & > 0, & \frac{\partial k^M}{\partial \lambda} & < 0, & \frac{\partial k^M}{\partial f_A} & < 0
\end{align*}
\]

(15) (16)

This implies that symmetric competing search engines can charge higher prices to advertisers when the size of network effects is higher. Also substantial scale effects (lower \( \lambda \)– or lower costs of quality improving capital investments) may be to the disadvantage of the advertisers. The results of the impact of changes in costs of serving advertisers are ambiguous.

\(^{22}\) For detailed derivations see Appendix 3.

\(^{23}\) See expression (25) in Appendix 3 for details.

\(^{24}\) Again, this solution only possesses the properties of the saddle point equilibrium. Hence, conclusions of this case should be interpreted carefully.
In addition, lower fixed costs, higher cost efficiency, and stronger network effects will also imply enhanced symmetric equilibrium quality of the search results. Further, welfare analysis in the symmetric global maximum equilibrium implies the following result.25

**Proposition 4** Total Utility on each engine \( (TU^i, i = 1, 2) \) computed as the sum of total users’ and advertisers’ utilities is negative if the following inequality holds

\[
t > \frac{2(\alpha_A - 2f_A)(1 + 2\lambda)}{(2\lambda - 1)}.
\]

This conclusion implies that when \((t)\) degree of product differentiation is relatively high or \((\alpha_A)\) the size of network effects is relatively low the total utility of the agents on both search engines can become negative, implying that the positive effect of increase in quality of search results for users and advertisers does not outweigh the negative effect of increase in price on advertisers’ side. This indicates possible negative distortion of investment in quality improvements by the symmetric platforms when the size of the network effects is relatively low.

4.2.2 Asymmetric Competition with Dominant Platform

The analysis of asymmetric equilibria (where \(\alpha_A, f_A,\) or \(\lambda\) can differ between engines, i.e. \(\alpha_A^1 \neq \alpha_A^2, f_A^1 \neq f_A^2,\) or \(\lambda^1 \neq \lambda^2\)) is quite cumbersome. The resulting closed form analytical solutions for \(p_A^{1M}, k_A^{1M}, p_A^{2M},\) and \(k_A^{2M}\) in the asymmetric convex technology case are extremely nonlinear with respect to parameters. However, numerical analysis and characterization of the resulting asymmetric solution (see Appendix 5) allows plotting the resulting patterns of equilibrium prices, quality improving efforts, and total welfare for each group of agents for the ranges of policy relevant parameter values.26 Main conclusions of this numerical analysis are summarized in Figures 1-8 and in Proposition 5 below.27

---

25 For detailed proof see Appendix 4.

26 Procedure we used for parameter calibration is as follows. We start by normalising the degree of product differentiation to one \((t = 1)\). Similar approximation of the degree of product differentiation parameter is adapted in Jeon et al. (2012). We’ve also experimented with slightly different values and found no qualitative difference in our results. Next, we proceeded by fixing the level of costs of serving advertisements at \(f^1 = f^2 = 0, 1\). This was done to make sure prices and qualities remain positive in the convex costs case. Furthermore, it seems plausible to assume that such costs are of little importance in real practice. To impose numerical discipline in our experiment, we incorporate stability condition (i.e. condition for global maximum identified above) and non-negativity of stable equilibrium solution condition. The conditions for non-negative global maximum solution are as follows \(\lambda_i > \left(\frac{\alpha_A^i + 1}{2}\right)^2\) and \(\alpha_A^i \geq 2f_A^i\) for \(i = 1, 2\). Respecting the stability and non-negativity conditions we simultaneously calibrate \(\alpha_A^i\) and \(\lambda_i\) for both \(i = 1, 2\). Then, for different initial \(\alpha_A^i\) (initial installed base of users on the weaker engine) we plot two-dimensional Figures 1-3 and 4-6, for low and high \(\alpha_A^i\), respectively. Further, we extend this analysis to three-dimensional Figures 7 and 8, where we allow the initial installed base of users on the weaker engine \((\alpha_A^i)\) to vary between 0, 4 and 2.

27 It should be stressed that in case of asymmetric competition under convex cost technology both saddle point and global maximum equilibria exist. The conditions for non-negative global maximum solution are
Proposition 5: Under the convex technology in the asymmetric global maximum equilibrium we have $p^2_2 > p^1_1$ and $k^2_2 > k^1_1$ if \[
\frac{\alpha^2_1}{\lambda^1_1} > \frac{\alpha^1_1}{\lambda^2_2}.
\]
Further, $p^2_2 < p^1_1$ and $k^2_2 > k^1_1$ if $f^1_1 > f^2_2$.

Moreover, Total Utility on each engine ($TU^i$, $i = 1, 2$) computed as the sum of total users’ and advertisers’ utilities is negative if $\alpha^1_1$ is sufficiently low.

Finally, the quality of search ($k^i$, $i = 1, 2$) is also increasing in $\alpha^1_A$ the initial installed base of the weaker engine.

This proposition implies that in asymmetric settings, generally dominant search engine sets both higher quality and higher price to advertisers. Our conclusion seems to be in line with available empirical evidence (see e.g. Argenton and Prüfer (2012)). In some cases, when asymmetries (advantages) are interpreted in terms of lower costs of serving advertisers, agents on the most efficient engine can also benefit from lower prices and higher quality. Furthermore, substantial differences in the size of network effects and especially lower installed data base of users on the weaker search engine may lead to negative impact on users’ welfare as well as total welfare. These results are confirmed in Figures 1-3, and 4-6. Figures 1-3 describe the situation where the initial size of the network effects on the weaker search engine $\alpha^1_A$ is relatively low ($\alpha^1_A = 0.4$). This set-up implies strictly negative average users’ utilities as well as negative total utilities on both search engines. In Figures 4-6, on the contrary, we plot an alternative situation, where the initial size of the network effects on the weaker search engine $\alpha^1_A$ is relatively high ($\alpha^1_A = 2$). The latter case can be interpreted as leveled playing field, where dominant search engine shares data on clicking behavior with weaker competitor. Clearly, this remedy improves both users’ and advertisers’ welfare as total utilities on both search engines are strictly positive and average user’s and advertiser’s utilities are substantially higher compared to cases described in Figures 1-3.

Figures 1-3 illustrate Prices and Quality choices, and Total Utilities, (Market Shares, and average utilities) on both engines in the setting where the initial size of the network effects specified in Appendix 5. For subsequent analysis in this subsection and in Appendix 5 we concentrate on the characterization of the asymmetric global maximum equilibrium.
on the weaker search engine $\alpha_A^1$ is relatively low ($\alpha_A^1 = 0.4$). Other parameter values used for calibration are specified in the Appendix 5. Engine 2 is assumed to be Dominant. The horizontal axis in the Figure 1 reflects the asymmetries in the size of network effects (ratio $\alpha^2/\alpha^1$). The horizontal axis in the Figure 2 reflects the asymmetries in the size of efficiency parameter ($\lambda^2/\lambda^1$). The horizontal axis in the Figure 3 reflects the asymmetries in the size of cost of serving advertisers ($f^2/f^1$).

[Figure 4 about here]

[Figure 5 about here]

[Figure 6 about here]

Figure 4-6 illustrate Prices and Quality choices, and Total Utilities, (Market Shares, and average utilities) on both engines in the setting where the initial size of the network effects on the weaker search engine $\alpha_A^1$ is relatively high ($\alpha_A^1 = 2$). Other parameter values used for calibration are specified in the Appendix 5. Engine 2 is assumed to be Dominant. The horizontal axis in the Figure 4 reflects the asymmetries in the size of network effects (ratio $\alpha^2/\alpha^1$). The horizontal axis in the Figure 5 reflects the asymmetries in the size of efficiency parameter ($\lambda^2/\lambda^1$). The horizontal axis in the Figure 6 reflects the asymmetries in the size of cost of serving advertisers ($f^2/f^1$).

Additional three dimensional diagrams presented in Figures 7 and 8 show that above conclusion is robust for the entire range of parameter $\alpha_A^1$ values between 0.4 and 2 and that it is possible to numerically identify the thresholds on $\alpha_A^1$ below which total utility on both search engines becomes negative. Figures 7 and 8 also imply that total utility on both engines is increasing in $\alpha_A^1$ for any possible size of initial asymmetries between two competing search engines (as measures by the ratio $\alpha^2/\alpha^1$).

[Figures 7(a) and 7(b) about here]

Figures 7(a) and 7(b) illustrate Total Utilities on both engines measured on the vertical axis. Engine 2 is assumed to be Dominant. The left axis reflects the size of asymmetries (ratio $\alpha^2/\alpha^1$), the front axis reflects the size of the initial installed base for the weaker engine ($\alpha^1$). Parameter values used for calibration are as follows $f^1 = f^2 = 0.1$, $\lambda^1 = \lambda^2 = 10$, $t = 1$.

[Figures 8(a) and 8(b) about here]
Figures 8(a) and 8(b) illustrate Average User Utilities on both engines measured on the vertical axis. Engine 2 is assumed to be Dominant. The left axis reflects the size of asymmetries (ratio $\alpha^2/\alpha^1$), the front axis reflects the size of the initial installed base for the weaker engine ($\alpha^1$). Parameter values used for calibration are as follows $f^1 = f^2 = 0.1$, $\lambda^1 = \lambda^2 = 10$, $t = 1$.

The final result of Proposition 5 also shows that for plausible parameter values (partly borrowed from Jeon et al. (2012)) the quality of search is also increasing in the initial installed base of the weaker engine. The intuition for this is that it simply forces the stronger engine to compete harder for the customers. This conclusion is illustrated in Figures 9(a) and 9(b).

5 Extensions and Robustness Checks

TO BE COMPLETED

5.1 Click Fee

5.2 Quality Score

5.3 Alternative Utility Specifications

6 Conclusions and Policy Proposal

Conclusions of sections 4 imply that price / quality choices by competing search engines depend on the degree of product differentiation, size of network effects and possible cost advantages. The analysis of asymmetric oligopoly equilibria under linear cost technology reveals that there are situations when the dominant search engine (which normally enjoys substantial cost advantages due to economies of scale and experience, as well as stronger network effects) may not always have proper incentives to invest in quality improvements even in the presence of potential, but weaker, competitors. However, the solution of linear cost case only possesses the properties of the saddle point equilibrium. There is no global maximum equilibrium under the linear cost technology. Hence, conclusions of the linear cost case should be interpreted carefully. In general, the results of this two-sided oligopoly model appear to be highly dependent on the R&D cost structure.
The global maximum analysis of the symmetric equilibrium under convex technology implies that when degree of product differentiation is relatively high or the size of network effects is relatively low the total utility of the agents on both search engines can become negative, implying that the positive effect of increase in quality of search results for users and advertisers does not outweigh the negative effect of increase in price on advertisers’ side.

Asymmetric oligopoly model under convex technology identifies few cases when the exploitative practices by dominant search engines may be welfare reducing. One of the main findings shows that in asymmetric settings, substantial differences in the size of network effects and especially lower installed data base of users on the weaker engine may lead to negative impact on users’ welfare and quality of search results. While leveled playing field, where dominant search engine shares data on clicking behavior with weaker competitors improves both users’ and advertisers’ welfare.

Based on this conclusion we argue that the evidence on increasing concentration, the current characteristics of the search engine market, and the theoretical results of the paper suggest that some form of intervention is needed in order to avoid possible exploitative abuses by the dominant search engine and to prevent possible deterioration in quality and relevance of search results and possible harm to users and advertisers.

REMEDIES - TO BE COMPLETED

7 Appendix

7.1 Appendix 1: Analysis of FOCs and Hessian under Linear Cost Technology (Symmetric Equilibrium)

Given the profit functions in (5) with linear cost technology \( F(k^i) = \lambda k^i, i = 1, 2 \), we can derive the following four first order conditions:

\[
\begin{align*}
\frac{\partial \pi^1}{\partial p_A^1} &= \alpha_A \left( \frac{1}{2} + \frac{k^1 - k^2}{2t} \right) + k^1 - 2p_A^1 + f_A = 0 \\
\frac{\partial \pi^1}{\partial k_A^1} &= \frac{\alpha_A p_A^1}{2t} - \frac{\alpha_A f_A}{2t} + p_A^1 - f_A - \lambda = 0 \\
\frac{\partial \pi^2}{\partial p_A^2} &= \alpha_A \left( \frac{1}{2} + \frac{k^2 - k^1}{2t} \right) + k^2 - 2p_A^2 + f_A = 0 \\
\frac{\partial \pi^2}{\partial k_A^2} &= \frac{\alpha_A p_A^2}{2t} - \frac{\alpha_A f_A}{2t} + p_A^2 - f_A - \lambda = 0
\end{align*}
\]

The symmetric equilibrium solution is characterized by the following Hessian matrix

\[
H = \begin{pmatrix}
-2 & \frac{\alpha_A}{2t} + 1 \\
\frac{\alpha_A}{2t} + 1 & 0
\end{pmatrix}, \quad \pi^{11} = -2 \leq 0, \quad \pi^{22} = 0, \quad \det J = \left( \frac{\alpha_A}{2t} + 1 \right)^2 < 0
\]
Hence, the symmetric equilibrium solution always has the properties of saddle point equilibrium.

### 7.2 Appendix 2: Solution Linear Cost Case (Asymmetric Equilibrium)

To characterize the solution of the asymmetric two-sided oligopoly model we need to find jointly optimal actions \((p^1_A, k^1A, p^2_A, k^2A)\). We start with necessary conditions for the optimum which for completeness are rewritten below:

\[
\alpha_{1A} \left( \frac{1}{2} + \frac{k_1 - k_2}{2t} \right) + k_1 - 2p_{1A} + f_{1A} = 0 \tag{17}
\]

\[
\alpha_{1A} \left( \frac{p_{1A} - f_{1A}}{2t} \right) + p_{1A} - f_{1A} - \lambda_1 = 0 \tag{18}
\]

\[
\alpha_{2A} \left( \frac{1}{2} + \frac{k_2 - k_1}{2t} \right) + k_2 - 2p_{2A} + f_{2A} = 0 \tag{19}
\]

\[
\alpha_{2A} \left( \frac{p_{2A} - f_{2A}}{2t} \right) + p_{2A} - f_{2A} - \lambda_2 = 0 \tag{20}
\]

Note that for fixed values of the parameters: \((f_{1A}, f_{2A}, \lambda_1, \lambda_2, t)\) the above four equations define an affine mapping \(g : \mathbb{R}^4 \mapsto \mathbb{R}^4\). Without loss of generality, for qualitative analysis of the optimum we set the second platform to have higher quality and lower costs. We also normalize setting \(f_{1A} = 1\) and \(\lambda_2 = 1\). Our normalization requires that \(f_{1A} \geq 1\) and \(0 < \lambda_2 \leq 1\).

The set of actions that satisfy FOCs is simply the kernel of \(g\), \(\text{Ker}(g)\). For analytical convenience we are interested in another simplification. Namely, we wish to restrict ourselves to parameter values that guarantee that \(\text{Ker}(g)\) is a singleton. To have that we need to ensure that the image of \(g\) has full dimension. Consider the linear part of \(g\), denoted \(g'\), it’s given by:

\[
\alpha_{1A} \left( \frac{k_1 - k_2}{2t} \right) + k_1 - 2p_{1A} = 0
\]

\[
\alpha_{1A} p_{1A} + p_{1A} = 0
\]

\[
\alpha_{2A} \left( \frac{k_2 - k_1}{2t} \right) + k_2 - 2p_{2A} = 0
\]

\[
\alpha_{2A} p_{2A} + p_{2A} = 0.
\]

Matrix form of \(g'\) is:

\[
\begin{bmatrix}
-2 & \frac{\alpha_{1A}}{2t} + 1 & 0 & -\frac{\alpha_{2A}}{2t} \\
\frac{\alpha_{1A}}{2t} + 1 & 0 & 0 & 0 \\
0 & -\frac{\alpha_{1A}}{2t} & -2 & \frac{\alpha_{2A}}{2t} + 1 \\
0 & 0 & \frac{\alpha_{2A}}{2t} + 1 & 0
\end{bmatrix}
\]

22
The only restrictions we have to (jointly) impose to ensure that the solution is a single point is \( \alpha_A \neq -2t \) and \( \alpha_A \neq -t \). It follows that \( \text{Ker}(g) \) is a translation of \( \text{Ker}(g') \) in the opposite direction of the vector we crossed out from \( g \) to get \( g' \).

Rewriting expressions in (17) implies the following system:

\[
p_1^A = \lambda^1/\left(\frac{\alpha_A^1}{2t} + 1\right) + f_1^A \tag{21}
\]

\[
p_2^A = \lambda^2/\left(\frac{\alpha_A^2}{2t} + 1\right) + f_2^A \tag{22}
\]

\[
k^1 = \frac{2p_1^A + f_1^A - \alpha_A^1 \left(2 - \frac{k_1^2}{2t}\right)}{1 + \frac{\alpha_A^1}{2t}}
\]

\[
k^2 = \frac{2p_2^A + f_2^A - \alpha_A^2 \left(2 - \frac{k_1^2}{2t}\right)}{1 + \frac{\alpha_A^2}{2t}}
\]

Now we can solve for \( k^1 \) and \( k^2 \) but before we proceed we introduce new notation: \( \gamma^i = 1 + \frac{\alpha_A^i}{2t} \) so we obtain:

\[
k^2 = \left(\frac{2t\gamma^1\gamma^2}{2t\gamma^1\gamma^2 - \alpha_A^1\alpha_A^2}\right) \left(\frac{2p_1^A + f_1^A}{\gamma^2} + \frac{\alpha_A^2}{2t^2\gamma^1\gamma^2} \left(2p_1^A - f_1^A - 1\right)\right) \tag{23}
\]

\[
k^1 = \left(\frac{2t\gamma^1\gamma^2}{2t\gamma^1\gamma^2 - \alpha_A^1\alpha_A^2}\right) \left(\frac{2p_2^A + f_2^A}{\gamma^1} + \frac{\alpha_A^1}{2t^2\gamma^1\gamma^2} \left(2p_2^A - f_2^A - 1\right)\right) \tag{24}
\]

Equations (21) - (23) describe optimal actions chosen by each platform under asymmetry and linear cost technology. From these analytical expressions it is technically possible to back out the derivatives of optimal allocation of quality with respect to parameters and prices.\(^{28}\) We demonstrated possible outcomes in Figures 9, 10 and 11.\(^{29}\) These figures confirm the major implication of the proposition 2 numerically, namely that the more efficient/dominant search engine provides the users with lower quality searches and charges higher prices to advertisers.

\[\text{[Figure 9 about here]}\]

\[\text{[Figure 10 about here]}\]

\[\text{[Figure 11 about here]}\]

\(^{28}\) However, resulting formulas are highly intractable and must be evaluated numerically.

\(^{29}\) The procedure we applied used the normalization mentioned before, namely we put engine 1 to be “Bing” and engine 2 to be “Google” so that \( \alpha_A^1 \in [0.4, 2], \lambda^1 = 1, f_1^A = 0.1 \). It should also be the case that \( \alpha_A^2 \geq \alpha_A^1, f_2^A \leq f_1^A \) and \( \lambda^2 \leq \lambda^1 \). Due to normalization, we can only see what happens to optimal quality when we change ratios of network effects, costs, and efficiency parameter. The figures are arranged in such a way that dashed (Engine 1 (Bing)) and dotted (Engine 2 (Google)) lines start at the symmetric case and as we move to the right (for network effects) or to the left (for costs), Engine 2 gains advantage over Engine 1 with respect to the parameter that is being changed.
7.3 Appendix 3: Analysis of FOCs and Hessian under Quadratic Cost Technology (Symmetric Equilibrium)

Given the profit functions in (5) with quadratic cost technology \( F(k^i) = \frac{(k^i)^2}{2}, i = 1, 2 \), we can derive the following four first order conditions:

\[
\frac{\partial \pi^1}{\partial p_A} = \frac{\alpha_A}{2} + k^1 - 2p_A^1 + f_A = 0
\]

\[
\frac{\partial \pi^1}{\partial k^1} = p_A^1 - f_A - \lambda k^1 = 0
\]

\[
\frac{\partial \pi^2}{\partial p_A} = \frac{\alpha_A}{2} + k^2 - 2p_A^2 + f_A = 0
\]

\[
\frac{\partial \pi^2}{\partial k^2} = p_A^2 - f_A - \lambda k^2 = 0
\]

The equilibrium solution is characterized by the following Hessian matrix

\[
H = \begin{pmatrix}
-2 & \frac{\alpha_A}{2t} + 1 \\
\frac{\alpha_A}{2t} + 1 & -\lambda
\end{pmatrix}, \pi^{11} = -2 \leq 0, \pi^{22} = -\lambda \leq 0, \det J = 2\lambda - \left(\frac{\alpha_A}{2t} + 1\right)^2
\]

Hence, the equilibrium solution is a global maximum, when \( \lambda \geq \frac{(\frac{\alpha_A}{2t} + 1)^2}{2} \). Otherwise, it is a saddle point equilibrium.

If players are symmetric, the equilibrium solution is characterized by the following system of FOCs and Hessian matrix

\[
\frac{\partial \pi^1}{\partial p_A} = \frac{\alpha_A}{2} + k^1 - 2p_A^1 + f_A = 0
\]

\[
\frac{\partial \pi^1}{\partial k^1} = p_A^1 - f_A - \lambda k^1 = 0
\]

\[
\frac{\partial \pi^2}{\partial p_A} = \frac{\alpha_A}{2} + k^2 - 2p_A^2 + f_A = 0
\]

\[
\frac{\partial \pi^2}{\partial k^2} = p_A^2 - f_A - \lambda k^2 = 0
\]

\[
H = \begin{pmatrix}
-2 & 1 \\
1 & -\lambda
\end{pmatrix}, \pi^{11} = -2 \leq 0, \pi^{22} = -\lambda \leq 0, \det J = 2\lambda - 1
\]

Hence, the symmetric equilibrium solution is a global maximum, when \( \lambda \geq \frac{1}{2} \). Otherwise, it is a saddle point equilibrium.

When \( 0 \leq \lambda < 1/2 \) and \( \alpha_A \leq 2f_A \), the symmetric saddle point equilibrium exists and is given by\(^\text{30}\)

\[
p_A^M = f_A + \frac{\lambda(2f_A - \alpha_A)}{2(1 - 2\lambda)}, \quad k^M = \frac{2f_A - \alpha_A}{2(1 - 2\lambda)}.
\]

\(^{30}\)Note that if \( 0 \leq \lambda < 1/2 \) and \( \alpha_A > 2f_A \) then non-negative solution for \( k^M \) in the symmetric saddle point equilibrium does not exist.
Comparative statics of the symmetric saddle point equilibrium shows:

\[
\begin{align*}
\frac{\partial p^M_A}{\partial f_A} &> 0, \quad \frac{\partial p^M_A}{\partial \lambda} > 0, \quad \frac{\partial p^M_A}{\partial \alpha_A} < 0 \\
\frac{\partial k^M}{\partial f_A} &> 0, \quad \frac{\partial k^M}{\partial \lambda} > 0, \quad \frac{\partial k^M}{\partial \alpha_A} < 0
\end{align*}
\] (25)

When \(\lambda > 1/2\) and \(\alpha_A \geq 2f_A\), the symmetric global maximum equilibrium exists and is given by

\[
p^M_A = f_A + \frac{\lambda(\alpha_A - 2f_A)}{2(2\lambda - 1)}, \quad k^M = \frac{\alpha_A - 2f_A}{2(2\lambda - 1)}.
\]

Comparative statics of the symmetric global maximum equilibrium shows:

\[
\begin{align*}
\frac{\partial p^M_A}{\partial \alpha_A} &> 0, \quad \frac{\partial p^M_A}{\partial \lambda} < 0, \quad \frac{\partial p^M_A}{\partial f_A} \geq 0, \text{ if } \lambda \geq 1 \\
\frac{\partial k^M}{\partial \alpha_A} &< 0, \quad \frac{\partial k^M}{\partial \lambda} < 0, \quad \frac{\partial k^M}{\partial f_A} > 0 \text{ if } 1/2 < \lambda < 1
\end{align*}
\] (26)

### 7.4 Appendix 4: Proof of Proposition 4

The equilibrium Total Utility on each symmetric search engine \(i = 1, 2\) is given by

\[
TU^i = TU_U + TU_A = xk^M + \frac{t}{2}x^2 + \alpha_Ax + k^M - p^M_A.
\]

Recall that in symmetric global maximum equilibrium \(x = 1/2\) and equilibrium price and quality are given by

\[
p^M_A = f_A + \frac{\lambda(\alpha_A - 2f_A)}{2(2\lambda - 1)}, \quad k^M = \frac{\alpha_A - 2f_A}{2(2\lambda - 1)}.
\]

This gives the following expression for Total Utility on each search engine

\[
TU^i = \frac{2(\alpha_A - 2f_A)(1+2\lambda)}{2\lambda - 1} - \frac{t}{2}x^2 - f_Ax + k^M - p^M_A.
\]

This implies that \(TU^i < 0\), when \(t > \frac{2(\alpha_A - 2f_A)(1+2\lambda)}{(2\lambda - 1)}\).

This completes the proof of Proposition 4.

### 7.5 Appendix 5: Solution Quadratic Cost Technology (Asymmetric Equilibrium)

Supplementary materials for the paper include “Technical Documentation” and MATLAB codes, which provide the baseline package upon which one can build further numerical extensions for different specifications of the asymmetric oligopoly model. The baseline package is designed in such a way that alternative structures of user’s and advertiser’s utility functions as well as alternative pricing structures (such as e.g. ‘click fee’) can be incorporated into numerical analysis. The resulting patterns of equilibrium prices, quality improving efforts, and total welfare for each group of agents can be plotted and analyzed for the ranges of policy relevant parameter values.

Note that if \(\lambda > 1/2\) and \(\alpha_A < 2f_A\) then the non-negative solution for \(k^M\) in the symmetric global maximum equilibrium does not exist.
For the analysis performed in this paper we concentrated on the linear – quadratic specification outlined in (6)-(9) and (5) as most plausible one. Further extensions should include such specificities of search market as keywords targeting, ‘click fee’ payment structure, or setting where quality is perceived differently by different groups of agents (i.e., different $k_A$ and $k_U$). These extensions are still to be implemented as robustness checks.

### 7.5.1 Calibration of Parameter Values

Procedure we used for parameter calibration is as follows. We start by normalizing the degree of product differentiation to one ($t = 1$). Similar approximation of the degree of product differentiation parameter is adapted in Jeon et al. (2012). We’ve also experimented with slightly different values and found no qualitative difference in our results. Next, we proceeded by fixing the level of costs of serving advertsises at $f^1 = f^2 = 0, 1$. This was done to make sure prices and qualities remain positive in the convex costs case. Furthermore, it seems plausible to assume that such costs are of little importance in real world. To impose numerical discipline in our experiment, we incorporate stability condition (i.e. condition for global maximum identified above) and non-negativity of stable equilibrium solution condition. The conditions for stable non-negative global maximum solution are as follows

\[
\lambda^i > \left( \frac{\alpha^i_A}{2f^i_A} + 1 \right)^2 \quad \text{and} \quad \alpha^i_A \geq 2f^i_A \quad \text{for} \quad i = 1, 2.
\]

Respecting the stability and non-negativity conditions, we simultaneously calibrate $\alpha^i_A$ and $\lambda^i$ for both $i = 1, 2$. Then, for different initial $\alpha^1_A$ (initial installed base of users on the weaker engine) we plot two-dimensional Figures 1-3 and 4-6, for low and high $\alpha^1_A$, respectively. There we plot the resulting patterns of equilibrium prices, quality improving efforts, total utilities, market shares and average utilities for each group of agents on both engines as functions of the size of the market asymmetries measured by the ratio $\alpha^2_A / \alpha^1_A \in [1, 2]$. In simpler words, our numerical analysis can be thought of as follows: we compare two cases of the relative size of product differentiation and initial installed base of users ($\alpha_i = 0, 4$ or $\alpha_i = 2$ for $t = 1$) and then investigate the impact of increasing advantage of Engine 2 with respect to installed base, costs of serving advertisers and efficiency in producing innovation.

Further, three-dimensional Figures 7 and 8 are constructed by extending the approach above with one additional dimension. Namely, allowing the initial installed base of users on the weaker engine ($\alpha^1_A$) to vary between 0, 4 and 2. This allows to numerically identify the thresholds on $\alpha^1_A$ below which total utility on both search engines becomes negative.
7.5.2 Analysis of FOCs

We are interested in finding the solution of FOCs such that the global maximum conditions are satisfied.

\[
\alpha_{1A} \left( \frac{1}{2} + \frac{k_1 - k_2}{2t} \right) + k_1 - 2p_{1A} + f_{1A} = 0 \quad (27)
\]

\[
\frac{\alpha_{1A} p_{1A}}{2t} - \frac{\alpha_{1A} f_{1A}}{2t} + p_{1A} - f_{1A} - \lambda_1 k_1 = 0 \quad (28)
\]

\[
\alpha_{2A} \left( \frac{1}{2} + \frac{k_2 - k_1}{2t} \right) + k_2 - 2p_{2A} + f_{2A} = 0 \quad (29)
\]

\[
\frac{\alpha_{2A} p_{2A}}{2t} - \frac{\alpha_{2A} f_{2A}}{2t} + p_{2A} - f_{2A} - \lambda_2 k_2 = 0 \quad (30)
\]

In a nutshell, we’re solving an affine system \( Ax = b \) where \( x = (p^{1}_{A}, k^{1}, p^{2}_{A}, k^{2}) \), matrix \( A \) is:

\[
A = \begin{bmatrix}
-2 & \frac{\alpha_{1A}}{2t} + 1 & 0 & -\frac{\alpha_{1A}}{2t} \\
\frac{\alpha_{1A}}{2t} + 1 & -\lambda_1 & 0 & 0 \\
0 & -\frac{\alpha_{2A}}{2t} & -2 & \frac{\alpha_{2A}}{2t} + 1 \\
0 & 0 & \frac{\alpha_{2A}}{2t} + 1 & -\lambda_2
\end{bmatrix}
\]

and \( b \) is:

\[
b = \begin{bmatrix}
-\frac{\alpha_{1A}}{2} - f_{1A} \\
\frac{\alpha_{1A}}{2} + f_{1A} \\
\frac{\alpha_{2A}}{2} - f_{2A} \\
\frac{\alpha_{2A}}{2} + f_{2A}
\end{bmatrix}
\]

We introduce extra notation to save on space, namely: \( \theta_i = \frac{\alpha_{iA}}{2} + f_{iA} \) and \( \gamma_i = \frac{\alpha_{iA}}{2t} + 1 \). Upon those changes we have that:

\[
A = \begin{bmatrix}
-2 & \gamma_1 & 0 & 1 - \gamma_1 \\
\gamma_1 & -\lambda_1 & 0 & 0 \\
0 & 1 - \gamma_2 & -2 & \gamma_2 \\
0 & 0 & \gamma_2 & -\lambda_2
\end{bmatrix}
\]

and

\[
b = \begin{bmatrix}
-\theta_1 \\
f_{1A} \gamma_1 \\
-\theta_2 \\
f_{2A} \gamma_2
\end{bmatrix}
\]
A necessary and sufficient condition for the set of equations to have a solution is \( \det A \neq 0 \). By the virtue of Laplace expansion, we have that:

\[
\det A = -2(-1)^3 \det \begin{bmatrix}
-2 & \gamma_1 & 1 - \gamma_1 \\
\gamma_1 & -\lambda_1 & 0 \\
0 & 0 & -\lambda_2 \\
\end{bmatrix} + \gamma_1(-1)^4 \det \begin{bmatrix}
-2 & \gamma_1 & 1 - \gamma_1 \\
\gamma_1 & -\lambda_1 & 0 \\
0 & 1 - \gamma_2 & \gamma_2 \\
\end{bmatrix}
\]

so that:

\[
\det A = -2 \left( -2\lambda_1\lambda_2 + \lambda_2\gamma_1^2 \right) - \gamma_2 \left( 2\lambda_1\gamma_2 + \gamma_1 \left( 1 - \gamma_1 \right) \left( 1 - \gamma_2 \right) - \gamma_2\gamma_1^2 \right) \tag{31}
\]

or equivalently:

\[
\det A = 2\lambda_2 \left( 2\lambda_1 - \gamma_1^2 \right) - \gamma_2 \left( 2\lambda_1\gamma_2 + \gamma_1 \left( 1 - \gamma_1 \right) \left( 1 - \gamma_2 \right) - \gamma_2\gamma_1^2 \right) \tag{32}
\]

Now, to get analytical expressions for qualities and prices we must solve Cramer rule formulas, let \( A_j \) denote matrix \( A \) with \( j \)-th column replaced with \( b \). After some tedious algebra, we obtained that:

\[
det A_1 = -2 \left( -\theta_1\lambda_1\lambda_2 + \lambda_1 \left( 1 - \gamma_1 \right) f_2\gamma_2 + \lambda_2\gamma_1^2 f_1 \right)
- \gamma_2 \left( \lambda_1\theta_1\gamma_2 + f_1\gamma_1 \left( 1 - \gamma_1 \right) \left( 1 - \gamma_2 \right) - \theta_2\lambda_1 \left( 1 - \gamma_1 \right) - \gamma_2\gamma_1^2 f_1 \right).
\]

\[
det A_2 = -2 \left( 2f_1\gamma_1\lambda_2 + f_2\gamma_1\gamma_2 \left( 1 - \gamma_1 \right) \right) - \gamma_2 \left( -2f_1\gamma_1\gamma_2 - \theta_2\gamma_1 \left( 1 - \gamma_1 \right) + \theta_1\gamma_1\gamma_2 \right)
\]

\[
det A_3 = -2 \left( -\theta_2\lambda_1\lambda_2 + \lambda_1 f_2\gamma_2^2 + \lambda_2 f_1\gamma_1 \left( 1 - \gamma_2 \right) \right)
- \gamma_1 \left( \gamma_1\theta_2\lambda_2 + \left( 1 - \gamma_1 \right) \left( 1 - \gamma_2 \right) f_2\gamma_2 - \gamma_1 f_2\gamma_2^2 - \lambda_2\theta_1 \left( 1 - \gamma_2 \right) \right)
\]

\[
det A_4 = -2 \left( 2f_2\gamma_2\lambda_1 + f_1\gamma_1\gamma_2 \left( 1 - \gamma_2 \right) \right) - \gamma_1 \left( -2f_2\gamma_2\gamma_1 - \theta_1\gamma_2 \left( 1 - \gamma_2 \right) + \theta_2\gamma_2\gamma_1 \right),
\]

so that:

\[
p_A^1 = \frac{\det A_1}{\det A} \quad (33)
\]

\[
k_1 = \frac{\det A_2}{\det A} \quad (34)
\]

\[
p_A^2 = \frac{\det A_3}{\det A} \quad (35)
\]

\[
k_2 = \frac{\det A_4}{\det A} \quad (36)
\]

From those expressions we could back out all the derivatives to perform qualitative analysis of the solution. However, resulting expressions exhibit, strong nonlinearities with respect to all the parameters and do not have clear analytical interpretation. That’s why we resort to numerical evaluation of the partial derivatives. The results of this numerical analysis (for the range of calibrated policy relevant parameter values) are summarized in figures 1-6 capturing three possible types of asymmetries and clearly confirm the main implications of Proposition 5.
8 References


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Figure 1: Prices, qualities and total utilities on both engines, under convex costs and low initial installed base on engine 1, changing $f_2^{1}$ along the horizontal axis. Parameter values used for calibration are as follows $f^1 = f^2 = 0.1, \lambda^1 = \lambda^2 = 10, t = 1$.

Figure 2: Prices, qualities and total utilities on both engines, under convex costs and low initial installed base on engine 1, changing $\alpha_2^{2}$ along the horizontal axis. Parameter values used for calibration are as follows $\lambda^1 = \lambda^2 = 10, t = 1$. 

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Figure 7(a): illustrates Total Utility on engine 1 measured on the vertical axis. The left axis reflects the size of asymmetries (ratio $\alpha^2/\alpha^1$), the front axis reflects the size of the initial installed base for the weaker engine ($\alpha^1$). Parameter values used for calibration are as follows $f^1 = f^2 = 0.1, \lambda^1 = \lambda^2 = 10, t = 1$.

Figure 7(b): illustrates Total Utility on engine 2 measured on the vertical axis. The left axis reflects the size of asymmetries (ratio $\alpha^2/\alpha^1$), the front axis reflects the size of the initial installed base for the weaker engine ($\alpha^1$). Parameter values used for calibration are as follows $f^1 = f^2 = 0.1, \lambda^1 = \lambda^2 = 10, t = 1$. 
Figure 8(a): illustrates Average User Utility on engine 1 measured on the vertical axis. The left axis reflects the size of asymmetries (ratio $\alpha^2/\alpha^1$), the front axis reflects the size of the initial installed base for the weaker engine ($\alpha^1$). Parameter values used for calibration are as follows $f^1 = f^2 = 0.1$, $\lambda^1 = \lambda^2 = 10$, $t = 1$.

Figure 8(b): illustrates Average User Utility on engine 2 measured on the vertical axis. The left axis reflects the size of asymmetries (ratio $\alpha^2/\alpha^1$), the front axis reflects the size of the initial installed base for the weaker engine ($\alpha^1$). Parameter values used for calibration are as follows $f^1 = f^2 = 0.1$, $\lambda^1 = \lambda^2 = 10$, $t = 1$. 
Figure 9(a): illustrates Quality of search results on engine 1 measured on the vertical axis. The left axis reflects the size of asymmetries (ratio $\alpha^2/\alpha^1$), the front axis reflects the size of the initial installed base for the weaker engine ($\alpha^1$). Parameter values used for calibration are as follows $f^1 = f^2 = 0.1$, $\lambda^1 = \lambda^2 = 10$, $t = 1$.

Figure 9(b): illustrates Quality of search results on engine 2 measured on the vertical axis. The left axis reflects the size of asymmetries (ratio $\alpha^2/\alpha^1$), the front axis reflects the size of the initial installed base for the weaker engine ($\alpha^1$). Parameter values used for calibration are as follows $f^1 = f^2 = 0.1$, $\lambda^1 = \lambda^2 = 10$, $t = 1$. 
Figure 9: Prices, qualities and total utilities on both engines, under \textbf{linear costs}, changing $\frac{a_2}{a_1}$ along the horizontal axis. Parameter values used for calibration are as follows $f^1 = f^2 = 0.1$, $\lambda^1 = \lambda^2 = 1$, $t = 1$, $\alpha^1 = 0.4$.

Figure 10: Prices, qualities and total utilities on both engines, under \textbf{linear costs}, changing $\frac{a_2}{a_1}$ along the horizontal axis. Parameter values used for calibration are as follows $\lambda^1 = \lambda^2 = 1$, $t = 1$, $\alpha^1 = 0.4$. 

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Figure 11: Prices, qualities and total utilities on both engines, under **linear costs**, changing $\frac{\lambda_2}{\lambda_1}$ along the horizontal axis. Parameter values used for calibration are as follows

$$f^1 = f^2 = 0.1, \quad t = 1, \quad \alpha^1 = 0.4.$$