Better the devil you know: a dynamic duopoly model with switching and transportation costs

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Abstract

We present a dynamic duopoly model in which consumers have mean-variance preferences. Due to the form of the utility function, two effects arise endogenously in the presence of information asymmetries: (a) switching cost effect, that increases the consumer’s willingness to pay for the brand that she tested before, and (b) perception effect, that increases the consumer’s willingness to pay if the signal from the tested product is good, and decreases it otherwise.

This approach differs from the classical one with exogenous switching cost because of the presence of the quality of the matching effect. With the switching cost effect only, we observe an increasing pattern of prices. On the contrary, our model allows for a decreasing pattern of prices in equilibrium if the perception effect offsets the switching cost effect. This result matches with the empirical evidence from the car industry in the US.

Keywords: switching cost, sticky prices, risk aversion, duopoly, inference, captive customers, pricing strategies.

JEL: D43, D82, D83, L13.

1 Introduction

It is easy to think of situations in which consumers face switching costs. If you want to use a different operating system in your computer, you would have to invest some time in understanding how it works; that is, you are facing a learning cost. If you want to move your savings from your old bank to a new one, you would have to pay the cancellation fee; that is, you are facing a transaction cost.

The definition of switching cost is unspecific: it is simply the cost that a consumer has to pay in order to switch from her current supplier to a different one\(^1\). Notice that it does not provide any information about the origin of the cost. But independently of the origin, whenever a consumer faces a switching cost, products that were ex-ante homogeneous become ex-post heterogeneous.

\(^1\) Cabral, 2008; Farrel and Klemperer, 2007.
Because of this, the consumer is partially forced to continue using the product she initially selected.

The conventional wisdom states that, if we consider only two periods in markets with switching costs, the firms will exploit their captive consumers and will charge high prices in the second period: due to the switching costs, the demand of the second period will become more inelastic and the market will be less competitive (harvesting effect). Firms anticipate this effect and compete fiercely in prices in the first period to attract as many consumers as possible (investment effect).

Nevertheless, the empirical evidence sometimes does not fit with this prediction. The figure below shows the evolution of the real prices of four cars, belonging to different price ranges, in the US during the last 14 years. Consumers have large brand loyalty rates in this market, but we observe a decreasing pattern of prices. This pattern cannot be explained by technological improvement, competition increase or demand decrease, but some reports show that the consumers consider that cars should be of higher quality. The point of the paper is to construct a model flexible enough to accommodate this evidence.

The origin of the switching costs is obscure in many situations. Whereas learning and transaction costs have been deeply studied in the literature, the “intrinsic motives” that lead to brand loyalty – specially in the markets of experience goods – have received little theoretical attention in Economics. We offer an explanation and hypothesize that switching costs arise endogenously from informational reasons instead of from affective ties. We use a model of two periods to analyze the equilibrium prices and market shares in each period and to

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2 Sources: Motortrend and Econstats.

3 Notice that consumers of consecutive years are not the same people. Usually consumers buy a new car after 7 or 8 years: that is why we approached this situation by using a model of two periods.

4 Supporting data about this statement are provided in the Appendix.
determine which consumers are partially informed and which are fully informed in the second period. Because of the analytical form of the consumers’ utility, two effects arise: the switching cost effect always increases the willingness to pay in the second period, whereas the perception effect between the features of the good and the individual preferences may increase or decrease the willingness to pay. Since it is possible that the perception effect offsets the switching cost effect in the second period, the model predicts that in this case the firms could set a first-period price larger than the second-period equilibrium prices.

Specifically, we construct a model with informational asymmetries. Our framework is a duopoly Hotelling model with the firms located at the extremes of the unit interval and a continuum of consumers uniformly distributed along it. We assume that firms compete in prices and live during the two periods, but that there is a different generation of consumers in each period – however, they are connected in the sense that the second generation has access to the information gathered by the first generation. The perception effect and the switching cost effect come from the mean-variance preferences of the consumers. Notice that the switching cost effect appears if consumers are risk-averse and if they do not have the same amount of signals (in other words, the same amount of information) for each brand.

In the first period, all the agents are ignorant about how well a certain brand matches the consumers’ preferences and therefore prices cannot be informative signals. Agents have no information at all and market shares are determined by the realization of a random variable, since we assume sticky prices. After consumption, consumers of one brand obtain a new signal that will be known for both firms but not directly observed by the consumers of the rival brand (although second-period prices will be informative signals for some consumers in equilibrium); with the new information, all consumers update their beliefs. In the second period, consumers may have different information depending on the previous market outcome and firms have different initial conditions – so they follow different pricing strategies.

Let us summarize the main conclusions. In the second period, only the firm with the largest first-period market share will take advantage of the higher willingness to pay of its previous customers, who will be partially informed in equilibrium. Nevertheless, the model does not predict that the dominant firm sets a price larger than the price of its rival: the perception effect has also to be taken into account. In the first period, both firms set the same price, and this price is decreasing in the risk-aversion parameter.

The model fills two gaps. First, it proposes an endogenous explanation for the switching costs that has nothing to do with the affective ties. Second, since the model differentiates the perception effect and the switching cost effect, it complements the classical intuition of the investment and harvesting effect: it may happen that the dominant firm cannot harvest after the investment.
2 Related literature

Maybe due to the broad definition, switching costs have been studied from different perspectives. When modeling, authors have considered either total commitment in the second period (infinite switching costs), or partial commitment (finite switching costs).

Among the papers considering infinite switching costs, the classic reference is Beggs and Klemperer (1992). The authors take a duopoly market and analyze the evolution of prices and market shares in an infinitely-repeated game, assuming switching costs and fully rational agents. The main result is that steady-state prices are larger with switching costs. With fully rational agents, consumers know that low prices today mean high prices tomorrow: they are less sensitive to price cuts and it leads to lower firm incentives to decrease the price. This reinforces the fact that rational firms give more weight to exploit old consumers than to attract new ones. Our conclusions differ because of two reasons: the consideration of two periods and the differentiation among the switching cost effect and the quality of the matching effect. With two periods, we cannot obtain the typical dynamics of the infinite games (one firm exploits its captive consumers whereas its rival cuts the price to attract as many new consumers as possible). With the two effects we separate, it is not clear how the demand is going to be in the second period: the switching cost makes it more inelastic, but the quality of the matching can affect the demand in the opposite direction.

Among the papers considering finite switching costs, we could mention Farrell and Shapiro (1988) or Klemperer (1987a). Nevertheless, the paper conceptually closest to ours is Villas-Boas (2006). We both endogenize one central explanation for switching costs: "the uncertainty about the quality of untested brands" (Klemperer, 1995). The idea is that consumers learn something about their own preferences for a certain brand after experiencing it. But whereas Villas-Boas assumes that learning is perfect, we assume that consumers acquire a signal to update their beliefs. Additionally, we differentiate two elements in the utility of the consumers: the expected quality of the matching and the variance effect. As Beggs and Klemperer (1992), Villas-Boas analyzes an infinitely-repeated game in a duopoly market with fully rational agents. Despite switching costs in Villas-Boas are endogenous instead of exogenous, he finds the same results as Beggs and Klemperer with the same underlying intuition; therefore, the previous discussion about the reasons for obtaining different conclusions also applies here.

Typically, in the two-period models with finite switching costs, first-period prices are lower than the prices of the one-shot game due to the intensity of the competition for market share. In our model with different generations of consumers in each period and fully rational firms, prices in the first period are lower than prices of the one-shot game only if consumers are sufficiently risk averse. Otherwise, first period is less competitive in a market with switching costs than in a market without them. It happens because firms anticipate that, if having a bad experience with the good is very likely, the effect of the expected quality of the matching would be stronger than the effect of the switching costs.
in the second period if consumers are not sufficiently risk averse: firms would not be able to set large prices in the second period and they optimally react by charging larger prices in the first period. Nevertheless, the result of less competition in the first period in markets with switching costs is known in the literature: Klemperer (1987b) obtains it when firms and consumers are both rational. The intuition behind is that consumers anticipate perfectly the future effects of the switching costs in the prices of the second period; therefore, the demand of the first period is less sensitive to price cuts and this effect offsets the procompetitive effect of competition for market share. But to the best of our knowledge, this is the first paper that results in less competition in the first period with switching costs and two generations of consumers that are informationally linked (we will explain that generation of the second period have access to the information owned by the generation of the first period).

Empirical papers also have considered the relationship between available information and consumption of experience goods. Ledesma et al. (2005) argue that moral hazard may arise in the tourism market due to asymmetrical information in favor of sellers, who cut the quality to increase their profits. The authors test if reputation, understood as a set of signals about the true quality, is relevant for repetition by using data from the tourism market in Tenerife. Nevertheless, moral hazard is not a problem in our framework because sellers cannot choose the quality in the classical sense: the consumer values the quality of the matching, but she learns something about her own preferences only after consuming the good. The trade-off faced by the consumer comes up because she may prefer the brand she has more information about, although it does not match her preferences perfectly.

More widely, the paper also contributes to the literature of information and signaling. Caminal and Vives (1996) analyze a duopoly in which consumers have to infer the quality differential $q$ of the two goods. When firms are ignorant about this $q$, consumers believe that a high market share is an informative signal of high quality. The result is that firms try to signal-jam the inference process by competing fiercely in prices to maximize the market shares. Therefore, prices are not informative signals, whereas market shares are. In our model, prices are informative in the second period for a segment of consumers whereas first-period market shares do not add any information to the problem: they come from a random component that affects the utility of the consumers but it is irrelevant to infer the expected quality of the matching with a certain brand.

3 The model

Consider a two-period duopoly model such that firms $A$ and $B$ are located at points 0 and 1, respectively, and they both keep their locations. Firms live during the entire game and compete in prices in each period (discrimination or commitment are not allowed).

We assume a continuum of consumers uniformly distributed along the unit interval in each period –that is, two generations: parents and children. The par-
ents do not internalize the future utility of their children when making their con-
sumption choices; nevertheless, the two generations are informationally linked:
after consumption, the parents obtain a signal about how well the good matches
with their preferences, and this signal is communicated to children. We assume
that families live at the same "house" (location) during the entire game, so chil-
dren pay the same transportation cost as their parents.

Since we want to explain the markets of experience goods, we model a game
in which consumers do not have any information before acquisition (this is, their
information set is empty). We consider that how well a certain product matches
with the consumer preferences, \( X_j \), is unknown for all the agents—consumers learn something about their own preferences after consumption,
and prices are not revealing in the first period—. Consumers obtain a signal
\( Z_j \) after consumption, that is the same for all consumers of the
same brand and known for both firms. Communication among consumers is not
allowed.

The signals are discrete random variables that can take two values: \( Z_j = \{0, 1\} \). Their respective meanings are bad matching and good
matching, \( Z_j \) are i.i.d.

To simplify, we assume that the qualities of the matching \( X_A \) and \( X_B \) are
also i.i.d. and can take the realizations \( \{0, 1\} \). The next assumption states how
the unconditional and conditional probabilities are:

\begin{assumption}
the probabilities of having a good and a bad matching when
consumers have no information are equal:
\[ p(X_j = 1) = p(X_j = 0) = \frac{1}{2} \quad \forall j = \{A, B\} \]

But the probability of having a good (bad) matching increases when a good
(bad) signal is obtained. Concretely, for \( 0 < \varepsilon < 1/2 \),
\[ p(X_j = 1|Z_j = 1) = \frac{1}{2} + \varepsilon; \quad p(X_j = 0|Z_j = 1) = \frac{1}{2} - \varepsilon \]
\[ p(X_j = 1|Z_j = 0) = \frac{1}{2} - \varepsilon; \quad p(X_j = 0|Z_j = 0) = \frac{1}{2} + \varepsilon \]
\end{assumption}

Given the previous probabilities, we can define the following means and
variances:
\[
E[X_j] = 1p(X_j = 1) + 0p(X_j = 0) = \frac{1}{2}
\]
\[
E[X_j|Z_j = 1] = 1p(X_j = 1|Z_j = 1) + 0p(X_j = 0|Z_j = 1) = \frac{1}{2} + \varepsilon > E[X_j]
\]
\[
E[X_j|Z_j = 0] = 0 = 1p(X_j = 1|Z_j = 0) + 0p(X_j = 0|Z_j = 0) = \frac{1}{2} - \varepsilon < E[X_j]
\]

\footnote{Notice that we are implicitly assuming that the learning is not perfect after consumption
and that the preferences of parents are children are the same.}

\footnote{We could also interpret this with perfect learning but with different, although correlated, preferences of parents and children.}

\footnote{This is equivalent to assume that firms and consumers do not know the quality fo the good.}
\[
\begin{align*}
\text{Var}[X_j] &= p(X_j = 1) [1 - E[X_j]]^2 + p(X_j = 0) [0 - E[X_j]]^2 = \frac{1}{4} \\
\text{Var}[X_j|Z_j = 1] &= 1 = E \left[ X_j^2 | Z_j = 1 \right] - (E[X_j|Z_j = 1])^2 = \frac{1}{4} - \varepsilon^2 < \text{Var}[X_j] \\
\text{Var}[X_j|Z_j = 0] &= 0 = E \left[ X_j^2 | Z_j = 0 \right] - (E[X_j|Z_j = 0])^2 = \frac{1}{4} - \varepsilon^2 < \text{Var}[X_j]
\end{align*}
\]

Notice two things: first, the expected quality of the matching given a good (bad) signal is larger (smaller) than if there is no information; second, the effect of having more information is to reduce the conditional variance, independently of the value of the signal: this is the source of the switching cost, and means that the consumer values in a positive way to face less uncertainty with respect to the quality of the matching.

More precisely, the timing of the game and the information provision is as follows:

- **Period 1**
  - Qualities of the matching \(X_A\) and \(X_B\) are realized.
  - Firms fix prices \((p_{1A}, p_{1B})\) simultaneously.
  - The random variable\(^7\) \(\varphi_{1A} - \varphi_{1B} = q_1 \sim U[-Q_1, Q_1]\) is realized.
  - Each consumer buys the brand \(j\) that maximizes her instant utility and obtains the signal \(Z_j\).

- **Period 2**
  - Firms fix prices \((p_{2A}, p_{2B})\) simultaneously, knowing \(q_1, Z_A\) and \(Z_B\).
  - If possible, first-period consumers of brand \(j\) infer the signal \(Z_{-j}\) through current prices and update their beliefs.
  - Each consumer buys the brand that maximizes her instant utility and the game ends.

Prior to solve the game, let us explain in detail the elements of the consumer’s utility and of the profit function of the firms.

Formally, the utility of the consumer located at \(x_i\) at period \(t\) when she buys brand \(j\) is:

\[
E[X_j | I_{t,j}] - \rho \text{Var}[X_j | I_{t,j}] - p_{t,j} - \tau x_j + \varphi_{t,j}
\]

where \(I_{t,j}\) is the information set about brand \(j\) at period \(t\); \(\rho > 0\) is the risk-aversion coefficient; \(p_{t,j}\) is the price of brand \(j\) at period \(t\), \(\tau > 0\) is the transportation cost; \(\varphi_{t,j}\) is the realization of \(\varphi_{1,j}\) and \(\varphi_{2,j} = 0\) (thus, \(\varphi = 0\)).

Notice that goods are not vertically differentiated in the first period since there is no information. Then, in the first period \(I_{1,A} = I_{1,B} = \emptyset\) and this

\(^7\) We will see later that it is an element of horizontal differentiation.
leads to $E[X_A] = E[X_B] = 1/2$ and $Var[X_A] = Var[X_B]$. In the second period, consumers use the signal $Z_j$ derived from first-period direct consumption (the signal will be different for first-period consumers of $A$ and for first-period consumers of $B$). If prices are revealing in equilibrium for a certain consumer, she infers $Z_{-j}$. We will say that she is a "fully informed consumer" because she will have the same number of signals for each brand; in particular, $I_{2A} = \{Z_A\}$ and $I_{2B} = \{Z_B\}$. If prices are not revealing in equilibrium for a certain consumer, she does not have additional information. We will say that she is a "partially informed consumer" and her information sets will be $I_{2,j} = \{Z_j\}$ and $I_{2,-j} = \emptyset$. This difference in the amount of information plus the risk-aversion of the consumers is the source of the switching cost. Notice that the fully informed consumers do not face any switching cost, whereas the partially informed consumers do.

First-period prices do not reveal any information (because the signals $Z_j$ and $Z_{-j}$ are unknown before consumption) and everybody perceives that the two brands match consumer's preferences in the same way. Nevertheless, the market does not share equally in the first period (with probability one) due to the realization of the continuous random variable, $q_1$, that affects the consumer's utility but that is not a signal for inferring the quality of the matching.

For a better understanding, think about the next example: consumers know that there are two brands of chocolate ice cream, Wen&Perry's and Hasten-Daps. In the first period, firms fix prices and newspapers reveal that Hasten-Daps does not use recycled materials for their containers (this is $q_1$). Since we consider sticky prices\(^8\), Hasten-Daps cannot react until the second period. Consumers check in the Internet the ingredients written on the labels, but the information is complex due to the large amount of components and consumers do not obtain more information about the quality of the matching. But they are committed with the environment and they will penalize Hasten-Daps, although the materials used for its containers have nothing to do with the sweetness of the ice cream. Each consumer buys the brand that provides the largest utility, without taking into account the future consequences of her current choice, and the first period finishes.

In the second period, each consumer has a derived-from-consumption signal (if she bought brand $j$ in the first period, she has the signal $Z_j$). Remember that consumers cannot communicate among themselves, but both firms know $Z_A$ and $Z_B$ before setting prices\(^9\). Since consumers know $q_1$, market shares from the first period are not informative signals in the second period; first-period consumers of brand $j$ would infer $Z_{-j}$ only if second-period prices are revealing –we will see that this is the case only for one segment of the market.

Mathematically, the indifferent consumer at period $t$ is located at $\hat{x}_t$:

$$E[X_A | I_{t,A}] - \rho Var[X_A | I_{t,A}] - p_{t,A} - \tau \hat{x}_t + q = E[X_B | I_{t,B}] - \rho Var[X_B | I_{t,B}] - p_{t,B} - \tau (1 - \hat{x}_t)$$

\(^8\)For a discussion about this assumption, please check the Conclusions.

\(^9\)This can be justified because firms have more resources to discover all the available information in the market.
Firms solve an intertemporal problem because they are rational (therefore, we proceed by backward induction). Taking \( \delta = 1 \), assuming zero production costs and denoting the demand of brand \( j = \{A, B\} \) at period \( t \) by \( D_{tj} \), the objective functions are:

\[
\begin{align*}
\pi_{2j} &= p_{2j}D_{2j}(p_2) \\
\pi_{1j} &= p_{1j}D_{1j}(p_1) + E\pi_{2j}(p_1)
\end{align*}
\]

### 3.1 Equilibrium of the second period

We impose full market coverage in every period\(^\text{10} \) (\( D_{tA} + D_{tB} = 1 \ \forall t \)) and non-negligible transportation costs:

**Assumption 2:** considered transportation costs are \( \tau > \overline{\tau} \), where \( \overline{\tau} \) is defined in the appendix.

This assumption rules out single suppliers and equilibrium prices less or equal than zero in the second period.

Out-of-equilibrium beliefs are defined below:

**Assumption 3:** if consumers observe out-of-equilibrium prices, they will infer the lowest possible value for the signal (zero in this model).

Let us denote the first-period market share of brand \( A \) by \( \overline{l_1} \) (remember that it is given in the second period). Since we assumed \( Q_1 \) large enough, firms may face three different situations in the second period: \( A \) supplied to the entire market (\( \overline{l_1} \geq 1 \)), \( B \) supplied to the entire market (\( \overline{l_1} \leq 0 \)) or both \( A \) and \( B \) supplied to the market (\( \overline{l_1} \in (0, 1) \)). In the first two cases, all consumers have the same derived-from-consumption signal and they only differ in their locations; in the last case, consumers who tasted brand \( A \) know \( Z_A \), consumers who tasted brand \( B \) know \( Z_B \) and they still differ in their locations (consumers belonging to different groups are not marginally different).

Let us define the following differences:

\[
\begin{align*}
\mbox{ }m_a &= E[X_A|Z_A] - E[X_B] \leq 0 \\
\mbox{ }v_a &= \mbox{Var}[X_B] - \mbox{Var}[X_A|Z_A] = \varepsilon^2 > 0 \\
\mbox{ }m_b &= E[X_B|Z_B] - E[X_A] \geq 0 \\
\mbox{ }v_b &= \mbox{Var}[X_A] - \mbox{Var}[X_B|Z_B] = \varepsilon^2 > 0
\end{align*}
\]

The next proposition establishes the second-period equilibrium prices when a single firm supplied to the entire market in the first period:

**Proposition 1** If a single firm supplied to the entire market in the first period, it sets a price positively dependent on the risk aversion coefficient in the second period.

In particular, \( A \) is the single supplier if \( \overline{\tau} \geq \tau - p_{1B} + p_{1A} \); and \( B \) is the single supplier if \( \overline{\tau} \leq -\tau + p_{1B} + p_{1A} \).

\(^{10}\)For a discussion about this assumption, please check the Conclusions.
The second-period equilibrium prices when \( A \) was the single supplier in the first period are

\[
p_{2A}^* = \tau + \frac{1}{3}m_a + \frac{1}{3}\rho v_a, \quad p_{2B}^* = \tau - \frac{1}{3}m_a - \frac{1}{3}\rho v_a
\]

and the second-period equilibrium prices when \( B \) was the single supplier in the first period are

\[
p_{2A}^* = \tau - \frac{1}{3}m_b - \frac{1}{3}\rho v_b, \quad p_{2B}^* = \tau + \frac{1}{3}m_b + \frac{1}{3}\rho v_b
\]

**Proof.** In the Appendix. ■

**Corollary 2** The first-period single supplier does not supply to some of its captive consumers in the second period.

**Proof.** In the Appendix. ■

The intuition is simple: since \( j \) was the single supplier in the first period, all the agents know \( \{Z_j\} \) and there is only one marginal consumer in the second period. All consumers are partially informed and \( j \) takes advantage of the switching cost: its second-period equilibrium price depends on \( \rho \) in a positive way. But due to the size of the transportation costs, the most profitable option for \( j \) is to set a second-period equilibrium price such that some captive consumers choose the rival brand (notice that the equilibrium price set by \( j \) is larger than the price that would induce to all consumers to choose again the brand \( j \)). Notice that prices cannot reveal \( Z_{-j} \) because that information has not been disclosed.

Now, consider that both firms supplied to the market in the first period. We will name **Condition (a)** to the following inequality

\[
\max \left\{ \frac{1}{6\tau} \left( 3\tau + \varepsilon + \rho \varepsilon^2 \right), \frac{1}{6\tau} \left( 3\tau + 6\varepsilon - 2\rho \varepsilon^2 \right) \right\} < l_1 < 1
\]

and **Condition (b)** to the inequality below

\[
0 < l_1 < \min \left\{ \frac{1}{6\tau} \left( 3\tau - \varepsilon - \rho \varepsilon^2 \right), \frac{1}{6\tau} \left( 3\tau - 6\varepsilon + 2\rho \varepsilon^2 \right) \right\}
\]

The next proposition establishes the second-period equilibrium prices when both firms supplied to the market in the first period:

**Proposition 3** If both firms supplied to the market in the first period,

1. The equilibrium in which both firms supply to all their previous consumers at the same time in the second period, charging prices positively dependent on the risk-aversion coefficient, happens if \( q_1 = (6p_{1A} - 6p_{1B} + m_a - m_b)/6 \). Since \( q_1 \) is a continuous random variable, this event happens with probability equal to zero.
2. If Condition (a) holds, there exists an equilibrium such that \( A \) sets a second-period equilibrium price positively dependent on \( \rho \). Furthermore, prices are revealing signals for the captive consumers of \( B \). In particular,

\[
p_{2A}^* = \tau + \frac{1}{3}m_a + \frac{1}{3}\rho v_a, \quad p_{2B}^* = \tau - \frac{1}{3}m_a - \frac{1}{3}\rho v_a
\]

3. If Condition (b) holds, there exists an equilibrium such that \( B \) sets a second-period equilibrium price positively dependent on \( \rho \). Furthermore, prices are revealing signals for the captive consumers of \( A \). In particular,

\[
p_{2A}^* = \tau - \frac{1}{3}m_b - \frac{1}{3}\rho v_b, \quad p_{2B}^* = \tau + \frac{1}{3}m_b + \frac{1}{3}\rho v_b
\]

Proof. In the Appendix. ■

**Corollary 4** In the equilibria proposed in **Proposition 3**, the firm with the largest first-period market share does not supply to all its captive consumers in the second period.

Proof. In the Appendix. ■

Notice that, if some consumers tasted brand \( A \) and others tasted brand \( B \), there is a signaling problem since firms can reveal information through prices.

Signaling problems are characterized by multiplicity of equilibria: we have constructed an equilibrium such that there are fully informed consumers, partially informed consumers, and where the prices reveal the information truthfully. Nevertheless, we are not claiming that this is the only equilibrium of the game.

The intuition is as follows: if both firms supplied to the market in the first period, there is a marginal consumer in each one of the captive segments. If first-period market shares were equal, both firms would supply only to their previous consumers at the same time in the second period. However, first-period market shares coincided only if \( q_1 \) took a certain value: since \( q_1 \) is continuous, this event happens with zero probability.

If first-period market shares were different enough, the firm with the largest market share takes advantage of the switching cost and sets a price positively dependent on \( \rho \). But due to the non-negligible transportation costs, it sets a price to induce some of its captive consumers to choose the rival brand (that is, its last attracted consumer is the marginal consumer of its own captive segment). Furthermore, the dominant brand sets an equilibrium price that is not revealing for its captive consumers but that is revealing for the captive consumers of the rival. For the parametric conditions stated in the Appendix, the rival (firm with the scarce market share) supplies to the rest of the market and does not want to deviate.

In summary, both firms supply to the market in the equilibrium of the second period but second-period market shares are different from first-period market shares.

The graph represents the Case 2 of the Proposition 3, where the second-period market share of \( A \) is pink and second-period market share of \( B \) is blue:
3.2 Equilibrium of the first period

Recall that the objective function of firm \( j \) in the first period is

\[
\pi_{1j} = p_{1j} D_{1j}(p_1) + E \pi_{2j}(p_1)
\]

Since there is no information before consumption, there is a single marginal consumer in the first period located at

\[
l_1 = \frac{1}{2\tau} (p_{1B} - p_{1A} + \tau + q_1)
\]

But \( q_1 \) realizes after the firms have set the prices. Therefore, firms take its expected value and solve the maximization problem with

\[
D_{1A} = l_1^E = \frac{1}{2\tau} (p_{1B} - p_{1A} + \tau)
\]

\[
D_{1B} = 1 - l_1^E = \frac{1}{2\tau} (p_{1A} - p_{1B} + \tau)
\]

The probability of having two suppliers in the second period is independent of first-period prices due to the uniform distribution of \( q_1 \). Hence, this part can be ignored in the first-period maximization problem.

Explicitly, the objective functions of the firms are

\[
\pi_{1A} = \frac{1}{2\tau} p_{1A} (p_{1B} - p_{1A} + \tau) + \frac{1}{2Q_1} (p_{1A} - p_{1B} - \tau + Q_1) V_A + \frac{1}{2Q_1} (p_{1B} - p_{1A} - \tau + Q_1) W_A
\]

\[
\pi_{1B} = \frac{1}{2\tau} p_{1B} (p_{1A} - p_{1B} + \tau) + \frac{1}{2Q_1} (p_{1A} - p_{1B} - \tau + Q_1) V_B + \frac{1}{2Q_1} (p_{1B} - p_{1A} - \tau + Q_1) W_B
\]

where \( V_j \) is the expected second-period profit of firm \( j \) when \( B \) was the single first-period supplier, and \( W_j \) is the expected second-period profit of firm \( j \) when \( A \) was the single first-period supplier. By symmetry, \( V_A = W_B \) and \( V_B = W_A \).
Mathematically,

\[
V_A = \frac{1}{2} \pi_{2A}^{(0,0)} + \frac{1}{2} \pi_{2A}^{(0,1)}
\]

\[
W_A = \frac{1}{2} \pi_{2A}^{(0,0)} + \frac{1}{2} \pi_{2A}^{(1,1)}
\]

The proposition below establishes the equilibrium of the first period:

**Proposition 5** First-period equilibrium prices are

\[
p_{1A}^* = p_{1B}^* = \tau + \frac{1}{Q_1} \tau (V_A - W_A) = \tau - \frac{2\tau \rho e^2}{3Q_1}
\]

Even more, the dominant firm has to charge a second-period price lower than the first period price for \(Q_1\) large enough if \(Z = 0\) and

\[
0 < \rho < \frac{1}{\varepsilon}
\]

Conditions for no deviation are provided in the Appendix.

**Proof.** In the Appendix. ■

With respect to the first part of the proposition, both firms set the same first-period equilibrium price because they solve symmetric problems. With respect to the second part, the dominant firm will have to set a second-period price lower than the first-period price if the signal received by consumers is bad and the risk-aversion coefficient is not large enough (that is, if perception effect offsets the switching cost effect, the dominant firm cannot harvest after investing). Nevertheless, the dominant firm is always protected by the switching cost effect, and this is the reason to have first-period prices lower than the prices of the one-shot game.

### 4 Conclusions

Empirical evidence in the markets of experience goods shows two apparently contradictory facts: consumers declare to be brand loyal to certain brands, but at the same time we observe price decreases that cannot be explained by technological improvements or competition increments. According to the standard results in the models of switching costs, we should observe an increase in prices (if firms play a finite game) or an alternance between the prices of the different brands (if firms play an infinite game). We hypothesize that switching cost arise due to informational reasons, but that this information acquisition also gives rise to a different effect (perception) that can offset the switching cost effect in some cases, accommodating the empirical evidence explained before.

We present a Hotelling model of two periods, with two firms and a continuum of consumers. Firm A is located at point 0, firm B is located at point 1, and consumers are uniformly distributed along the unit interval. Every agent
remains at her position during the game. Firms are perfectly rational, risk-neutral and fix the prices; consumers are risk-averse, purchase the goods and we consider one generation in each period—although the second generation has access to the information gathered by the first generation—.

The consumer is ignorant about how well a certain brand matches her preferences. Thereby, she acquires some signals during the game: her risk-aversion plus the difference in the number of signals for each brand are the source of the switching cost.

In the first period, firms fix prices before the realization of a random variable that affects the consumers’ utility; since react to this variable is impossible until the second period, its realization determines the market shares in the first period. In equilibrium, both firms fix the same first-period price. In the second period, the firm with the largest market share fixes a price that depends positively on the risk-aversion coefficient, whereas the price fixed by his rival depends negatively on that parameter.

The model provides an information-based reason to explain the brand loyalty observed in markets without learning costs or transaction costs. But it does not predict that captive consumers pay a price higher than informed consumers: although the firm supplying to the captive consumers fixes a price positively dependent on the risk-aversion coefficient, the expected quality of the matching is also an element of the price.

The model predicts a negative relationship between first-period prices and the risk-aversion coefficient (the partial derivative is negative). But the relationship between first-period prices and the transportation cost may be positive or negative. The partial derivative reveals that an increase in the transportation cost leads to an increase in first-period prices if and only if \( Q_1 > 2\rho \varepsilon^2 / 3 \). Since we are considering sufficiently high levels of uncertainty (that is, \( Q_1 \) large enough), we expect the relationship to be positive in most of the cases.

Concretely, the main contributions to the literature are the modelization of endogenous switching cost, the obtention of first-period equilibrium prices larger than second-period equilibrium prices for the dominant firm under certain conditions, and the obtention of asymmetries in the second period due to the lack of information faced by firms when fixing first-period prices.

About the endogeneity of the switching cost, observe that every consumer acquires the next two signals: the common prior and the specific signal derived from first-period consumption. In addition, second-period prices are informative only for past consumers of one brand: they can infer the signal derived from first-period consumption of the brand they did not taste. Informed consumers have two signals for each brand, whereas captive consumers have two signals for the brand they tasted and only one signal for the other. Notice that consumption and pricing decisions are endogenous, and so is the switching cost.

About the comparison between the prices of each period, to have larger first-period price in markets with switching costs is not new: Klemperer (1987b) obtains the same result, but with perfectly rational consumers. The intuition in his model is that "consumers’ foresight of the future effects of switching costs makes the first-period demand sufficiently inelastic to more than offset the
procompetitive effect of competition for market share”. We obtain the result with two different generations of consumers that are informationally linked and the intuition is as follows: when fixing first-period prices, firms know that the dominant one will always be protected by the switching cost, so first-period prices are decreasing in the risk aversion coefficient. But with probability $1/2$ the signal received by consumers is bad, and the perception effect can offset the switching cost effect if the risk-aversion is not large enough.

About asymmetries in the first-period market shares, they only happen because the random variable $q_1$ exists. If it would not, firms would fix the same price in the first period, the market would share equally and we would reach the symmetric solution in both periods. As it was said, firms cannot react to something that they do not know yet: when fixing prices in the first period, firms are completely symmetric; nevertheless, first-period market shares are determined by the realization of the random variable. Unless this realization takes a concrete value, which happens with probability zero because it is continuous, firms are asymmetric in the second period: first-period market shares are different with probability one and the problem of the firm with the largest market share is different to the problem of his rival.

To finish the section, it is worth to discuss the role of some assumptions maintained during the analysis. First, we have restricted the firms to fully cover the market in both periods. This assumption is crucial to obtain only one firm exploiting part of his captive customers in equilibrium; when we do not restrict the market to be fully covered, it is possible to find an equilibrium in the second period such that both firms exploit part of their captive consumers and a segment of the market is unsupplied. Second, we assume $Q_1$ big enough (technically speaking, we require $Q_1 > \max\{2V_B, \tau\}$). The purpose is to allow for corner solutions in the first period. In equilibrium, the probability to supply to the entire market is the same for each one of the firms: since the shock affects them equally, they solve a symmetric problem in the first period.

5 Appendix

EMPIRICAL EVIDENCE

In this section of the Appendix, we are providing data to support our claim that the observed decreasing pattern of prices in the US car industry is not due to technological improvements, demand decrease or competition increase. Rather, there is evidence about the coexistence of better automobiles and large rates of brand loyalty joint with decreasing consumer’s valuation.

About technological improvements, we have to differentiate between expenditures in R&D and real innovation: in the concrete case of the automotive industry, the former is usually referred to simple development (for instance, manufacturers can invest in more comfortable seats, but they do not invest in do the process more efficient and cheap), whereas the latter is referred to investment in new devices (that require a large initial investment, but that are easy and cheap to reproduce afterwards). According to Booz&Co., Toyota and
GM were in the top 10 of the R&D spenders from 2005 to 2012 (Ford was also in the ranking from 2005 to 2009); nevertheless, only Toyota is in the top 10 of the most innovative firms.

To check the demand side, we have used the NADA DATA reports to know the number of new cars sold in the United States. From 1998 to 2007, the sales were relatively stable around 17000000 of units, but with the economic crisis the number of units reached its lowest level in 2009 with 10400000 units. Nevertheless, in 2010 and 2011 the sales level increased again, being currently at 12730000 units.

![New cars sold in US (millions of units)](image)

About the level of competition, we have used again the NADA DATA reports to check the aggregated market share of the six biggest manufacturers (Chrysler, Ford, GM, Toyota, Nissan and Honda). The data show that the market share was close to 90% in 1998. Then, it decreased and was relatively stable around 86% until 2007. It decreased every year from then until the current level: 77%. In fourteen years, the aggregated market share decreased by 12.6%. But still, the six biggest manufacturers control more than 3/4 of the market.

During this period, cars have increased their quality: we measure it by the Vehicle Dependability Study (VDS) published by J.D. Power & Associates. The VDS provides the number of problems per 100 cars of a certain brand. What we observe is that there are less problems in all the brands (and therefore, in the average number of problems of the entire industry), but that firms do not change their relative position with respect to the average: those that had less (more) problems than the average in 2006 have also less (more) problems than the average in 2012. For example, the average number of problems was 227 in 2006, whereas it was 132 in 2012. Among the six biggest manufacturers, three of them had less problems than the average in 2006 (Ford, Toyota and Honda with 224, 179 and 194, respectively) and three of them had more problems (Chrysler, GM and Nissan with 232, 239 and 242). In 2012, the same three manufacturers had less problems than the average (124, 104 and 131) and the others had more problems (192, 158 and 152).

The automotive sector has very large rates of brand loyalty. According to Experian Automotive, in 2011 Ford registered a brand loyalty of 46.5% of the buyers, whereas both Toyota and Honda had 40% of buyers returning. In 2012, Toyota had a brand loyalty of 47.3%, Ford of 46% and Honda of 43%.
All the previous ingredients work in the direction of increasing the car prices. A possible reason for the decreasing pattern of prices appears in the Car-Brand Perception Survey conducted by the Consumer Reports National Research Center. The survey shows the "Overall Brand Perception", that is an index calculated as the total number of times that a particular make was mentioned as exemplary across all seven categories (safety, quality, performance, design, technology and environmentally friendly), divided by the total unaided awareness of the brand. We have chosen two different years, 2008 and 2012, to compare the numbers. Toyota, Ford and Honda were the highest-ranked brands in both years, but only Ford experienced an increment in its score: Ford moved from 112 to 121, but Toyota moved from 189 to 131 and Honda from 146 to 94. The decrease in the value of the index is a common pattern for most of the brands.

**FORMAL PROOFS**

We only consider transportation costs $\tau > \bar{\tau}$, such that

$$\bar{\tau} = \max \left\{ \frac{1}{3} (-m_a - \rho v_a), \frac{1}{3} (m_a + \rho v_a), \frac{1}{3} (-m_b - \rho v_b), \frac{1}{3} (m_b + \rho v_b) \right\}$$

**Proposition 1:** If a single firm supplied to the entire market in the first period, it sets a price positively dependent on the risk aversion coefficient in the second period. In particular, the second-period equilibrium prices when $A$ was the single supplier in the first period are

$$p_{2A}^* = \tau + \frac{1}{3} m_a + \frac{1}{3} \rho v_a, \quad p_{2B}^* = \tau - \frac{1}{3} m_a - \frac{1}{3} \rho v_a$$

**Proof.** When all consumers tasted brand $A$, all the agents know $Z_A$. Then,

$$U_{2A} = E[X_A|Z_A] - \rho Var[X_A|Z_A] - p_{2A}^* - \tau x_i$$
$$U_{2B} = E[X_B|\emptyset] - \rho Var[X_B|\emptyset] - p_{2B}^* - \tau (1 - x_i)$$

And the marginal consumer is located at

$$l_i^A = \frac{1}{2\tau} (\tau + m_a + \rho v_a - p_{2A}^* + p_{2B})$$

Hence, the second-period demands are

$$D_{2A} = l_i^A$$
$$D_{2B} = 1 - l_i^A$$

and the profit functions are

$$\pi_{2A} = p_{2A}D_{2A} = p_{2A} \frac{1}{2\tau} (m_a + \rho v_a + \tau + p_{2B} - p_{2A})$$
$$\pi_{2B} = p_{2B}D_{2B} = p_{2B} \left(1 - \frac{1}{2\tau} (m_a + \rho v_a + \tau + p_{2B} - p_{2A})\right)$$
Since profit functions are concave, we take first-order conditions to calculate the reaction functions. Solving the system, we obtain the equilibrium prices:

\[ p_{2A}^* = \tau + \frac{1}{3} m_a + \frac{1}{3} \rho v_a, \quad p_{2B}^* = \tau - \frac{1}{3} m_a - \frac{1}{3} \rho v_a \]

To prove the Corollary 2, we just need to plug the equilibrium prices into the demand function \( D_{2A}^* \):

\[ D_{2A}^* = \frac{1}{2\tau} (m_a + \rho v_a + \tau + p_{2B}^* - p_{2A}^*) = \frac{1}{6\tau} (3\tau + m_a + \rho v_a) \]

Given non-negligible transportation costs, \( D_{2A}^* < 1 \).

The proof is analogous for \( l_1 \leq 0 \).

**Proposition 3.1:** The equilibrium in which both firms supply to all their previous consumers at the same time in the second period, charging prices positively dependent on the risk-aversion coefficient, happens with zero probability.

**Proof.** When both firms supplied to the market in the first period and they want to attract simultaneously all their previous consumers in the second, prices cannot be revealing about the signal of the rival. Then, the marginal consumer in each segment is defined by:

\[ l_i^A = \frac{1}{2\tau} (m_a + \rho v_a + \tau + p_{2B} - p_{2A}) \]

\[ l_i^B = \frac{1}{2\tau} (-m_b - \rho v_b + \tau + p_{2B} - p_{2A}) \]

For both firms to attract all their previous consumers in the second period, it has to be that

\[ \Gamma_1 = l_i^A = l_i^B \iff m_a + \rho v_a = -m_b - \rho v_b \iff \rho = \frac{-(m_a + m_b)}{v_a + v_b} \]

If \( m_a + m_b > 0 \), then \( \rho < 0 \) and it is impossible (by assumption, \( \rho > 0 \)). Otherwise, we may have an equilibrium if \( \rho = \tilde{\rho} \). Now, we proceed to rule out this possibility.

Let us rename

\[ m_a + \tilde{\rho} v_a = -m_b - \tilde{\rho} v_b = C \]

Take the equilibrium candidate \( (p_{2A}^*, p_{2B}^*) \), such that \( p_{2B}^* = p_{2A}^* + 2\tau \tilde{\Gamma}_1 - \tau - C \), and consider the following deviations:

(a) \( p_{2A}^{\text{D}} = p_{2A}^* + \xi \), with \( \xi > 0 \). Deviation is profitable if \( \xi < p_{2A}^* - 2\tau \tilde{\Gamma}_1 \), and we can find a value for \( \xi \) if \( p_{2A}^* < 2\tau \tilde{\Gamma}_1 \).

(b) \( p_{2B}^{\text{D}} = p_{2A}^* - \eta \), with \( \eta > 0 \). Deviation is profitable if \( \eta < 2\tau \tilde{\Gamma}_1 - p_{2A}^* \), and we can find a value for \( \eta \) if \( 2\tau \tilde{\Gamma}_1 < p_{2A}^* \).

For now, we cannot reject the case of \( p_{2A}^* = 2\tau \Gamma_1 \) and \( p_{2B}^* = 4\tau \Gamma_1 - \tau - C \).

Considering analogous deviations for \( B \), we cannot reject the case of \( p_{2A}^* = -4\tau \Gamma_1 + 3\tau + C \) and \( p_{2B}^* = 2\tau (1 - \Gamma_1) \).
Then, to have an equilibrium we need
\[ 2\tau_1 - 4\tau l_1 + 3\tau + C \quad \text{and} \quad 2\tau (1 - l_1) = 4\tau l_1 - \tau - C. \]

Both equations are actually the same one, and the solution is
\[ l_1 = \frac{1}{2} + \frac{C}{6\tau}. \]

Nevertheless, \( l_1 \) is a continuous random variable and the probability of taking the realization \( 1/2 + C/6\tau \) is zero.

**Proposition 3.2:** If \( 1 > l_1 > \max \left\{ \frac{1}{6\tau} (3\tau + \epsilon + \rho \epsilon^2), \frac{1}{6\tau} (3\tau + 6\epsilon - 2\rho \epsilon^2) \right\} \), the prices truthfully reveal the signal \( Z_A \) in equilibrium.

**Proof.** If the condition on the market share holds and the derived-from-consumption signals are \((Z_A, Z_B) = (1, 1)\) or \((Z_A, Z_B) = (1, 0)\), the equilibrium prices will depend on \( Z_A = 1 \) and will be
\[ p^*_2A(1) = \tau + \frac{1}{3} \epsilon + \frac{1}{3} \rho \epsilon^2; \quad p^*_2B(1) = \tau - \frac{1}{3} \epsilon - \frac{1}{3} \rho \epsilon^2 \]

If the condition on the market share holds and the derived-from-consumption signals are \((Z_A, Z_B) = (0, 1)\) or \((Z_A, Z_B) = (0, 0)\), the equilibrium prices will depend on \( Z_A = 0 \) and will be
\[ p^*_2A(0) = \tau - \frac{1}{3} \epsilon + \frac{1}{3} \rho \epsilon^2; \quad p^*_2B(0) = \tau + \frac{1}{3} \epsilon - \frac{1}{3} \rho \epsilon^2 \]

Given the out-of-equilibrium beliefs, neither firm can reveal the realization of \( Z_B \). Therefore, we have to check four deviations:

(1) Given the true realizations \((Z_A, Z_B) = (1, 1)\) or \((Z_A, Z_B) = (1, 0)\), the observed prices in the market are
\[ p^*_2A(1) = \tau + \frac{1}{3} \epsilon + \frac{1}{3} \rho \epsilon^2; \quad p^*_2B(0) = \tau + \frac{1}{3} \epsilon - \frac{1}{3} \rho \epsilon^2 > p^*_2B(1) \]

Notice that the consumers who tasted \( A \) previously are not cheated. Previous consumers of \( B \) receive two opposite signals: let us check the most favorable case for firm \( B \) and assume that all those consumers believe that \( Z_A = 0 \).

Since the deviation price set by firm \( B \) is larger, it is going to lose some consumers in the segment of people who tasted \( A \) previously. Specifically, the new marginal consumer on that segment locates at
\[ \hat{x}^{A,D_1}_i = \frac{1}{6\tau} (3\tau + 3\epsilon + \rho \epsilon^2) \]

About the new marginal consumer on the segment of people who previously tasted \( B \), notice that would be different depending on the realization of \( Z_B \):

If \( Z_B = 1 \), then \( \hat{x}^{B,D}_i = \frac{1}{6\tau} (3\tau - 6\epsilon - 2\rho \epsilon^2) \)

If \( Z_B = 0 \), then \( \hat{x}^{B,D}_i = \frac{1}{6\tau} (3\tau - 2\epsilon^2 \rho) \)
Since \( \bar{\upsilon}_1 > \max \left\{ \frac{1}{\pi \varepsilon} \left( 3\tau + \varepsilon + \rho \varepsilon^2 \right), \frac{1}{\pi \varepsilon} \left( 3\tau + 6\varepsilon - 2\rho \varepsilon^2 \right) \right\} \), the firm \( B \) keep all the consumers who previously tasted it. Then, it attracts less people but charges a larger price. The profits when deviates are

\[
\pi^{D_1}_{2B} = p^{D_2}_{2B}(0) \left[ 1 - \bar{x}_i^{A,D_1} \right] = \frac{1}{18\pi} \left( 9\tau^2 - 6\tau\varepsilon^2\rho - 6\tau\varepsilon + \varepsilon^4\rho^2 + 2\varepsilon^3\rho - 3\varepsilon^2 \right)
\]

Deviation is profitable if and only if

\[
\pi^{D_1}_{2B} > \pi^*_{2B} \Leftrightarrow -3\varepsilon^2 > \varepsilon^2, \text{ which is impossible.}
\]

(2) Given the true realizations \((Z_A, Z_B) = (1, 1)\) or \((Z_A, Z_B) = (1, 0)\), the observed prices in the market are

\[
p^{D_A}_{2A}(0) = \tau - \frac{1}{3}\varepsilon + \frac{1}{3}\rho\varepsilon^2 < p^*_{2A}(1); \quad p^*_{2B}(1) = \tau - \frac{1}{3}\varepsilon - \frac{1}{3}\rho\varepsilon^2
\]

Again, the previous consumers of \( A \) are not cheated and the previous consumers of \( B \) receive two opposite signals. Let us assume that all the people who tasted \( B \) believes that \( Z_A = 0 \). Since the deviation price is lower than the equilibrium price, the firm \( A \) increases its demand in the segment of people who previously tasted \( A \). Concretely,

\[
\hat{x}_i^{A,D_2} = \frac{1}{6\tau} \left( 3\tau + 3\varepsilon + \rho\varepsilon^2 \right)
\]

As before, the new marginal consumer on the segment of people who previously tasted \( B \) depends on the realization of \( Z_B \):

If \( Z_B = 1 \), then \( \hat{x}_i^{B,D} = \frac{1}{6\tau} \left( 3\tau - 6\varepsilon - 2\rho\varepsilon^2 \right) \)
If \( Z_B = 0 \), then \( \hat{x}_i^{B,D} = \frac{1}{6\tau} \left( 3\tau - 2\varepsilon^2\rho \right) \)

Since \( \bar{\upsilon}_1 > \max \left\{ \frac{1}{\pi \varepsilon} \left( 3\tau + \varepsilon + \rho \varepsilon^2 \right), \frac{1}{\pi \varepsilon} \left( 3\tau + 6\varepsilon - 2\rho \varepsilon^2 \right) \right\} \), the firm \( A \) does not gain any demand among the consumers who previously tasted \( B \). Then, it attracts more people but charges a lower price. The profits when deviates are

\[
\pi^{D_2}_{2A} = p^{D_2}_{2A}(0) \left[ \hat{x}_i^{A,D_2} \right] = \frac{1}{18\pi} \left( 9\tau^2 + 6\tau\varepsilon^2\rho + 6\tau\varepsilon + \varepsilon^4\rho^2 + 2\varepsilon^3\rho - 3\varepsilon^2 \right)
\]

Deviation is profitable if and only if

\[
\pi^{D_2}_{2A} > \pi^*_{2A} \Leftrightarrow -3\varepsilon^2 > \varepsilon^2, \text{ which is impossible.}
\]

(3) Given the true realizations \((Z_A, Z_B) = (0, 1)\) or \((Z_A, Z_B) = (0, 0)\), the observed prices in the market are

\[
p^*_{2A}(0) = \tau - \frac{1}{3}\varepsilon + \frac{1}{3}\rho\varepsilon^2; \quad p^*_{2B}(1) = \tau - \frac{1}{3}\varepsilon - \frac{1}{3}\rho\varepsilon^2 < p^*_{2B}(0)
\]
The previous consumers of $A$ are not cheated and the previous consumers of $B$ receive two opposite signals. Let us assume that all the people who tasted $B$ believes that $Z_A = 1$.

Since the deviation price is lower than the equilibrium price, the firm $B$ increases its demand in the segment of people who previously tasted $A$. The new marginal consumer locates at

$$\hat{x}_i^{A,D3} = \frac{1}{6\tau} (3\tau - 3\varepsilon + \rho \varepsilon^2)$$

Nevertheless, it would be possible for $B$ to lose some consumers who previously chose its brand despite lowering the price, since they are inferring now that the matching with the other brand is good. Depending on the realization of $Z_B$,

If $Z_B = 1$, then

$$\hat{x}_i^{B,D} = \frac{1}{6\tau} (3\tau - 2\rho \varepsilon^2)$$

If $Z_B = 0$, then

$$\hat{x}_i^{B,D} = \frac{1}{6\tau} (3\tau + 6\varepsilon - 2\varepsilon^2 \rho)$$

Since $\Gamma_l > \max \{ \frac{1}{6\tau} (3\tau + \varepsilon + \rho \varepsilon^2), \frac{1}{6\tau} (3\tau + 6\varepsilon - 2\rho \varepsilon^2) \}$, the firm $B$ keeps all the demand among consumers who previously chose brand $B$. Then, it attracts more people but charges a lower price. The profits when deviates are

$$\pi_{2B}^{D3} = p_{2B}^{D}(0) \left[ 1 - \hat{x}_i^{A,D3} \right] = \frac{1}{18\tau} (9\tau^2 - 6\tau \varepsilon^2 \rho + 6\varepsilon^2 \rho + \varepsilon^4 \rho^2 - 2\varepsilon^3 \rho - 3\varepsilon^2)$$

Deviation is profitable if and only if

$$\pi_{2B}^{D3} > \pi_{2B}^* \iff -3\varepsilon^2 > \varepsilon^2,$$ which is impossible.

(4) Given the true realizations $(Z_A, Z_B) = (0, 1)$ or $(Z_A, Z_B) = (0, 0)$, the observed prices in the market are

$$p_{2A}^{D}(1) = \tau + \frac{1}{3} \varepsilon + \frac{1}{3} \rho \varepsilon^2 > p_{2A}^*(0); \quad p_{2B}^*(0) = \tau + \frac{1}{3} \varepsilon - \frac{1}{3} \rho \varepsilon^2$$

As always, the previous consumers of $A$ are not cheated and the previous consumers of $B$ receive two opposite signals. Let us assume that all the people who tasted $B$ believes that $Z_A = 1$.

Since the deviation price is larger than the equilibrium price, the firm $A$ decreases its demand in the segment of people who previously tasted $A$. The new marginal consumer locates at

$$\hat{x}_i^{A,D4} = \frac{1}{6\tau} (3\tau - 3\varepsilon + \rho \varepsilon^2)$$

Nevertheless, it would be possible for $A$ to gain some consumers among the people who previously chose the rival brand despite increasing its price, since
they are inferring now that the matching with the brand $A$ is good. Depending on the realization of $Z_B$,

If $Z_B = 1$, then $\tilde{x}_i^{B,D} = \frac{1}{6\tau} (3\tau - 2\rho\varepsilon^2)$

If $Z_B = 0$, then $\tilde{x}_i^{B,D} = \frac{1}{6\tau} (3\tau + 6\varepsilon - 2\varepsilon^2\rho)$

Since $\Gamma_1 > \max \{ \frac{1}{6\tau} (3\tau + \varepsilon + \rho\varepsilon^2), \frac{1}{6\tau} (3\tau + 6\varepsilon - 2\rho\varepsilon^2) \}$, the firm $A$ does not gain any demand among consumers who previously chose brand $B$. Then, it attracts less people but charges a larger price. The profits when deviates are

$\pi_{2A}^D = p_{2A}^D(1) [\tilde{x}_i^{A,D1}] = \frac{1}{18\tau} (9\tau^2 + 6\tau\varepsilon^2\rho - 6\tau\varepsilon + \varepsilon^4\rho^2 - 2\varepsilon^3\rho - 3\varepsilon^2)$

Deviation is profitable if and only if

$\pi_{2A}^D > \pi_{2A}^* \iff -3\varepsilon^2 > \varepsilon^2$, which is impossible.

The proof is analogous for Proposition 3.3. ■

**Proposition 5:** First-period equilibrium prices are

$p_{1A}^* = p_{1B}^* = \tau + \frac{1}{Q_1} \tau (V_A - W_A)$

Furthermore, these prices are lower than the equilibrium prices of the one-shot game, $\tau$, if and only if

$V_A - W_A < 0 \iff \rho > \bar{\rho}$

Being $\gamma$ the probability of $Z_j$ to be equal to 0, $\bar{\rho}$ is defined as

$\bar{\rho} = \frac{\tau - \varepsilon + \gamma(\tau + \varepsilon)}{\gamma\varepsilon^2 + \varepsilon(-\varepsilon - 2\tau)}$

**Proof.** Given the objective functions of the first period, we find the equilibrium prices by solving the system formed by the reaction functions:

$p_{1A}^* = \frac{1}{3Q_1} t (W_B - 2W_A + 2V_A - V_B + 3Q_1)$

$p_{1B}^* = \frac{1}{3Q_1} t (2W_B - W_A + V_A - 2V_B + 3Q_1)$

By symmetry, $V_A = W_B$ and $V_B = W_A$; hence,

$p_{1A}^* = p_{1B}^* = t + \frac{1}{Q_1} t (V_A - W_A)$

The second part of the Proposition comes from simple algebra. The expressions of $V_A$ and $W_B$ are as follows:

$V_A = \Pr(Z_B = 0)\pi_{2A}^{(0,0)} + \Pr(Z_B = 1)\pi_{2A}^{(0,1)}$

$W_A = \Pr(Z_A = 0)\pi_{2A}^{(1,0)} + \Pr(Z_A = 1)\pi_{2A}^{(1,1)}$
Making $Pr(Z_j = 0) = \gamma$ and plugging the equilibrium profits,

\[
V_A = \frac{1}{18\tau} (9\tau^2 - 5\varepsilon^2 + \varepsilon^4 \rho^2 + 12\tau\varepsilon + 6\gamma\varepsilon^2 - 4\varepsilon^3 \rho - 6\tau\varepsilon^2 \rho + 2\gamma\varepsilon^3 \rho - 6\tau\gamma\varepsilon)
\]

\[
W_A = \frac{1}{18\tau} (9\tau^2 + \varepsilon^2 + \varepsilon^4 \rho^2 + 6\tau\varepsilon + 2\varepsilon^3 \rho + 6\tau\varepsilon^2 \rho - 4\gamma\varepsilon^3 \rho - 12\tau\gamma\varepsilon)
\]

\[
V_A - W_A < 0 \iff \rho > \frac{\tau - \varepsilon + \gamma(\tau + \varepsilon)}{\gamma \varepsilon^2 + \varepsilon(-\varepsilon - 2\tau)}
\]

Finally, we provide the conditions for firms not to deviate from the proposed equilibrium. Due to the symmetry of the problem, the shapes of the first-period profits for firms $A$ and $B$ are identical. Depending on the parameter values, we may have two different shapes: the first shape corresponds to Figure 2 and the second shape corresponds to Figures 3 and 4. Conditions arise from the comparison between the profits obtained in equilibrium and the profits obtained in each one of the other parts of the function.

If $\tau \geq Q_1/2$ (first shape), equilibrium sustainability requires

\[
\frac{Q_1 (V_A - W_A)}{Q_1 - 2W_A} \leq \tau \leq \min \left\{ \frac{4Q_1^2}{4Q_1 + V_A}, \frac{4Q_1^2}{4Q_1 + W_A} \right\}
\]

If $\tau < Q_1/2$ (second shape), equilibrium sustainability requires

\[
\tau \geq Q_1 (V_A - W_A)
\]

Profit1A

Figure 2: Equilibrium if $Q_1 = 10, \ \tau = 1, \ \rho = 0.5$ and $p_{1B}^* = 0.998$
Figure 3: Equilibrium if $Q_1 = 10$, $\tau = 7$, $\rho = 0.5$ and $p_{1B}^* = 6.986$

Figure 4: No equilibrium if $Q_1 = 10$, $\tau = 9$, $\rho = 0.5$ and $p_{1B}^* = 8.982$

References


26