Abstract

When do competing principals independently choose to share the information obtained from their privately informed agents? Information sharing affects contracting relationships within opponent organizations and induces players’ strategies to be correlated via the distortions channel. We show that principals’ incentives to share information depend on the nature of upstream externalities and the correlation of agents’ information: when externalities and correlation have opposite signs, there is a unique equilibrium in which principals share information; when externalities and correlation have the same sign, there is a unique equilibrium with no communication. In this second case, unlike with complete information, principals face a prisoners’ dilemma since they obtain a higher total payoff by sharing information.

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1 Introduction

We analyze the incentive of competing principals to share the information that they privately obtain from their exclusive agents. Information sharing agreements are widespread in real life: banks and financial intermediaries usually exchange information about borrowers; sellers often share with competitors information about demand and costs; retailers commonly report information on the downstream market to suppliers; corporations often disclose information about their management’s performance.

The economic literature has shown that information sharing agreements can emerge both to increase efficiency and to reduce competition in oligopolistic markets (Novshek and Sonnenschein, 1982; Clarke, 1983; and Vives, 1984). Pagano and Jappelli (1993) show that lenders exchange information to screen investment projects or avoid opportunistic behavior by borrowers. Lizzeri (1999) and Gromb and Martimort (2007) analyze the role of experts who acquire and disclose information to trading counterparts. Taylor (2004) and Acquisti and Varian (2005) show how sellers can use information on consumers’ purchasing history to engage in product customization and price discrimination. Strategic communication has also been analyzed in the ‘networks’ literature — see, e.g., Calvó-Armengol et al. (2009), and Hagenbach and Koessler (2010).

However, all these papers do not consider the source of the information shared by players, and model communicators as black-boxes. Hence, they are silent on the interplay between information exchange and agency conflicts within and across organizations, when organizations have to obtain information from their privately informed members. One recent notable exception is Calzolari and Pavan (2006), who study information transmission in sequential common agency and assume that principals learn information through costly contracting with common parties, and then share it with rivals. In this context, information sharing creates novel effects because information disclosed by one player affects the contractual relationships between all other players. Calzolari and Pavan (2006) focus on non-exclusive contracting, while we analyze the effects of communication when contracts are exclusive.

Exclusivity clauses are common in many real markets (e.g., Caillaud et al., 1995). Several employment relationships are, by their own nature, exclusive (e.g., because of labor natural indivisibility); supply and franchising contracts in the manufacturing industry are often exclusive (e.g., when retailers of a brand cannot distribute competing brands); procurement, regulatory and financial contracts often feature forms of exclusivity. And information sharing agreements are common in markets with exclusive deals. For instance, the growth of information intensive channels in the manufacturing industry is often seen as a mean to facilitate the dissemination of information between competing

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1 See also Gal-Or (1986), Shapiro (1986), and Raith (1996).
2 See also Bennardo et al. (2010) and Maier and Ottaviani (2009) for a model with moral hazard.
3 According to Briley et al. (1994), this seems to be the established praxis in business format franchising, where the mandatory disclosure of franchising contracts required by the Federal Trade Commission since 1979 allows firms to have free access to some of their rivals’ information.
organizations (Stern et al., 1996).  

What are the drivers of information sharing decisions in these contexts? How does information sharing interacts with rent extraction and horizontal externalities across organizations? To answer these questions, we analyze a model with two independent principals who exert production externalities on each other and delegate production to exclusive agents. Each agents is privately informed about his marginal cost of production, and costs are either positively or negatively correlated. Hence, agents’ information must be obtained by principals through the design of incentive compatible contracts. Before contracting with agents, each principal simultaneously and non-cooperatively chooses whether to commit to share this information.  

We identify the main effect of information sharing between organizations. This effect is absent with complete information and is of first-order magnitude compared to the effects of communication with complete information. The incentive of a principal to share information depends on the impact that this decision has on the opponent principal’s contract, and hence on outputs. With adverse selection within and across organizations, information sharing induces strategies to be correlated, but mainly via the distortions channel. And, because principals want to reduce these distortions, the equilibrium value of communication depends on the interaction between the nature of upstream externalities and the sign of cost correlation.  

When upstream externalities and cost correlation have the same sign — i.e., they are either both positive or both negative — there exists a unique equilibrium in dominant strategies with no communication. To see why, suppose first that upstream externalities and cost correlation are both negative. By revealing her agent’s cost, a principal induces the rival to distort her output relatively more when the first principal’s agent has a high cost. This is because costs are negatively correlated, and principals choose higher distortions in the states that are (conditionally) less likely. But, with negative externalities, this reduces the first principal’s expected profit because reaction functions are downward sloping and, hence, each principal gains from expanding (resp. reducing) her own production when the rival is inefficient (resp. efficient). Second, suppose that upstream externalities and cost correlation are both positive. By revealing her agent’s cost, a principal induces the rival to distort her output relatively more when the first principal’s agent has a low cost (because costs are positively correlated). But, this is detrimental to the first principal because, with positive externalities, reaction functions are upward sloping and, hence, each principal prefers to increase production when the rival’s output increases.  

By contrast, when upstream externalities and cost correlation have opposite signs there exists a unique symmetric equilibrium in dominant strategies where both principals share information. To see why, suppose first that upstream externalities are negative and costs are positively correlated. By
disclosing her agent’s cost, a principal induces the rival to distort her output relatively more when 
the first principal’s agent has a low cost. This increases the first principal’s expected profit because 
reaction functions are downward sloping. Second, suppose that upstream externalities are positive 
and costs are negatively correlated. By revealing her agent’s cost, a principal induces the rival to 
distort her output relatively more when the first principal’s agent has a high cost. This is beneficial 
to the first principal because reaction functions are upward sloping.

We also show that, in contrast to a situation with complete information where the equilibrium 
is always efficient, principals may face a prisoners’ dilemma when they have to obtain information 
from agents, since expected profits are always higher with information sharing. In fact, when agents’ 
information is correlated, communication between principals reduces agents’ expected rent because 
it makes an agent’s contract depend on the rival agent’s type, and a relative performance evaluation 
relaxes incentive compatibility constraints — see, e.g., Riordan and Sappington (1988). When up-
stream externalities are small, this effect outperforms the strategic effect due to correlation among 
distortions.

We also consider the possibility of implicit collusion among agents. When principals exchange 
information, the expected utility of an agent depends on his opponent’s report. Hence, it may be 
expected that an equilibrium in which principals share information and agents truthfully report their 
types is not collusion-proof, since agents may coordinate on an equilibrium in which they both lie 
about their type and obtain higher rents. However, we show that, when production externalities only 
affect upstream profits, there exists a system of transfers such that the equilibrium with information 
sharing is indeed collusion-proof, if there are no side-payments across agents.

Although we develop our arguments in a principal/agent framework, the scope of our analysis 
is broader. The results apply to any situation involving horizontal externalities between competing 
organizations, where principals deal with exclusive agents, like procurement contracting, manufac-
turer/retailer relations, executive compensations, patent licensing, and insurance or credit relation-
ships.

The paper is organized as follows. Section 2 describes the model and Section 3 analyzes the case of 
complete information. Section 4 introduces asymmetric information and characterizes the equilibrium 
outputs when no principal shares information, when both principals share information, and when 
only one principal shares information. Principals’ decisions to share information are analyzed in 
Section 5. Section 6 considers the possibility of (implicit) collusion among agents and Section 7 
concludes. All proofs are in the Appendix.

2 The Model

Players and Payoffs. There are two (female) principals, $P_1$ and $P_2$, and two (male) exclusive agents,
\(A_1\) and \(A_2\), who produce outputs \(q_1\) and \(q_2\), respectively. All players are risk neutral. \(P_i\)’s utility is

\[V_i(q_i, q_j, t_i) = S(q_i, q_j) - t_i, \quad i, j = 1, 2,\]

where \(t_i\) is the monetary transfer paid by \(P_i\) to \(A_i\). We assume that

\[S(q_i, q_j) = \kappa + \beta q_i - q_i^2 + \delta q_i q_j.\]

This quadratic surplus function is commonly used in the literature on information sharing (e.g., Raith, 1996; Vives, 2000, Ch. 8). Hence, \(P_i\)’s surplus is strictly concave in \(q_i\). The parameter \(\delta\) measures the magnitude of strategic complementarity \((\delta > 0)\) or substitutability \((\delta < 0)\) between outputs. A positive \(\delta\) implies that principals’ reaction functions are upward sloping; a negative \(\delta\) implies that principals’ reaction functions are downward sloping. We assume that \(\delta\) is small, so that expected profits can be computed through a Taylor expansion around \(\delta = 0\).

\(A_i\)’s utility is

\[U_i(t_i, q_i, \theta_i) = t_i - \theta_i q_i, \quad i = 1, 2,\]

where \(\theta_i\) is \(A_i\)’s marginal cost of production. We assume that agents enjoy limited liability — i.e.,

\[U_i(t_i, q_i, \theta_i) \geq 0 \quad \forall (t_i, q_i, \theta_i).\]

This standard hypothesis of the screening literature implies that \(P_i\) cannot use information about \(\theta_j\) to leave \(A_i\) with no rent (Bertoletti and Poletti, 1996).

**Information.** The parameter \(\theta_i \in \Theta \equiv \{\theta, \bar{\theta}\}\) is private information to \(A_i\); it can be learned by \(P_i\) only through a revelation mechanism, and by \(P_j\) and \(A_j\) only if \(P_i\) chooses to share information.

The vector of random variables \((\theta_1, \theta_2)\) is drawn from a joint cumulative distribution function with:

- \(\Pr(\theta, \bar{\theta}) = \nu^2 + \alpha;\)
- \(\Pr(\theta, \bar{\theta}) = \Pr(\bar{\theta}, \theta) = \nu (1 - \nu) - \alpha;\)
- \(\Pr(\bar{\theta}, \bar{\theta}) = (1 - \nu)^2 + \alpha.\)

The parameter \(\alpha\) measures the correlation between \(\theta_1\) and \(\theta_2\) — i.e., \(\Pr(\theta, \bar{\theta}) \Pr(\bar{\theta}, \bar{\theta}) - \Pr(\theta, \bar{\theta})^2 = \alpha.\) Hence, \(\alpha > 0\) (resp. \(<\)) indicates positive (resp. negative) correlation between agents’ marginal costs, while agents’ costs are uncorrelated when \(\alpha = 0\). It follows that \(\Pr(\theta) = \nu, \Pr(\bar{\theta}) = 1 - \nu\) and, using Bayes rule, \(\Pr(\theta|\theta) = \nu + \frac{\alpha}{1 - \nu}, \Pr(\bar{\theta}|\theta) = \nu - \frac{\alpha}{1 - \nu}, \Pr(\theta|\bar{\theta}) = 1 - \nu - \frac{\alpha}{1 - \nu}, \) and \(\Pr(\bar{\theta}|\bar{\theta}) = 1 - \nu + \frac{\alpha}{1 - \nu}.\)

To ensure that probabilities are not negative, we assume that: (i) \(\nu (1 - \nu) \geq \alpha\) if \(\alpha \geq 0,\) and (ii) \(\min \{(1 - \nu), \nu\} \geq \sqrt{\alpha}\) if \(\alpha < 0.\) We also assume that \(\beta > \bar{\theta} > \theta > 0\) and that \(\Delta \theta \equiv \bar{\theta} - \theta\) is small to ensure that outputs are positive in all states of the world.
Communication. Principals can share the information obtained from their agents. Following Vives (1984) and Raith (1996), we assume that principals follow an “all-or-none” sharing rule: they either fully commit to disclose their agents’ costs, or they keep this information secret.\textsuperscript{6} Once a principal commits to share information, she cannot renegotiate this decision after learning her agent’s costs — see, e.g., Vives (2000, Ch. 8) and Raith (1996) for a similar approach.\textsuperscript{7}

The information transmitted by a principal is verifiable. More precisely, a report made by $A_i$ to $P_i$ can be credibly shared with $P_j$, and then transmitted by $P_j$ to $A_j$ — i.e., there is no moral hazard on the principals’ side.

Contracts. Principals design contracts to obtain information from their agents and decide how much to produce. Given that principals commit to deterministic disclosure policies before contracting with agents, we can use the Revelation Principle and consider direct deterministic mechanisms in which $A_i$ sends a private message $m_i \in \Theta$ about his cost to $P_i$. Contracts are secret: $P_j$ and $A_j$ observe neither the contract between $P_i$ and $A_i$, nor $A_i$’s report to $P_i$.\textsuperscript{8}

When $P_j$ does not share her information about $\theta_j$ — i.e., $A_j$’s report $m_j$ — $P_i$ offers to $A_i$ a mechanism

$$\{t_i(m_i), q_i(m_i)\}_{m_i \in \Theta},$$

which maps $m_i$ into a monetary transfer $t_i(m_i)$ and an output $q_i(m_i)$. When, instead, $P_j$ shares her information about $\theta_j$, $P_i$ offers a mechanism

$$\{t_i(m_i, m_j), q_i(m_i, m_j)\}_{(m_i, m_j) \in \Theta^2},$$

in which the transfer and the output are also contingent on $m_j$.

Since by assumption agents must obtain a non-negative utility in each contractible state: if $P_j$ does not share information, $A_i$’s utility must be non-negative for all $m_i$; if $P_j$ shares information, $A_i$’s utility must be non-negative for all $(m_i, m_j)$.

Timing. The timing of the game is as follows:

$t = 0$. Principals simultaneously and independently choose whether to share information.

$t = 1$. Agents privately observe their costs, and principals’ information sharing decisions become common knowledge.

\textsuperscript{6}In our model, there is no scope for (deterministic) type-contingent disclosure policies; e.g., when a principal commits to only revealing her agent’s type when the agent has a low cost. This is because, with only two types, an unraveling argument implies that this policy is equivalent to full disclosure of the agent’s cost.

\textsuperscript{7}Ziv (1993) shows that, without commitment, there is no information sharing in equilibrium.

\textsuperscript{8}In a regulatory environment, Iossa and Stroffolini (2012) argue that, by making procurement contracts public, regulators may signal information about demand or costs to potential competitors of regulated firms. Secret contracts rule out signaling in our model.
$t = 2$. Principals contract with agents.

$t = 3$. Communication, if any, takes place.

$t = 4$. Agents produce and payments are made.

**Equilibrium concept.** The equilibrium concept is Perfect Bayesian equilibrium (PBE). We assume that agents have passive beliefs — i.e., when an agent is offered a contract different from the one he expects in equilibrium, he does not revise his beliefs about the contract offered to the other agent (e.g., Caillaud et al., 1995, and Martimort, 1996).

### 3 Complete Information

First assume that costs are common knowledge within each hierarchy — i.e., each principal observes her agent’s cost, but not the rival agent’s cost. In this case, regardless of principals’ communication choices, agents obtain no rents. Hence, the two hierarchies act as vertically integrated firms and our model is similar to the one in Shapiro (1986), who analyzes information sharing between firms competing à la Cournot.

We denote by $q^*(\theta_i)$ the equilibrium output of $A_i$ when both principals do not share information; by $q^*(\theta_i, \theta_j)$ the equilibrium output of $A_i$ when both principals share information; and by $q_i^*(\theta_i)$ and $q_j^*(\theta_j, \theta_i)$ the equilibrium outputs of $A_i$ and $A_j$, respectively, when $P_i$ does not share information and $P_j$ shares information.

Let $s_i$ be the information upon which $P_i$ conditions the contract offered to $A_i$. Hence, $s_i = \theta_i$ if $P_j$ does not share information, and $s_i = (\theta_1, \theta_2)$ if $P_j$ shares information. Abusing notation, let $\tilde{q}_i(s_i)$ be $P_i$’s equilibrium output, given all possible communication decisions.\(^9\) We start by describing two properties of expected output and profit.

**Lemma 1** Regardless of principals’ communication decisions, $P_i$’s expected profit is

$$V_i^* = \kappa + \left(\frac{\mathbb{E}_{s_i} [\tilde{q}_i (s_i) | \theta_i]}{\text{average } \tilde{q}_i(s_i)}\right)^2 + \mathbb{E}_{s_i} [\tilde{q}_i (s_i)] - \mathbb{E}_{s_i} [\tilde{q}_i (s_i) | \theta_i]^2, $$

and the expected output is $q^* \equiv \frac{\beta - \theta}{2 - \delta} - \frac{1 - \nu}{2 - \delta} \Delta \theta$.

In equilibrium, principals obtain higher profit if production increases or becomes more volatile, because the indirect profit function is convex. Moreover, since outputs are linear in costs, information sharing decisions do not affect expected output — see, e.g., Vives (2000).

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9See Pagnozzi and Piccolo (2012) for a discussion of the role of beliefs with private contracts.

10Hence, $\tilde{q}_i(s_i) = q^*(\theta_i)$ if both principals do not share information; $\tilde{q}_i(s_i) = q^*(\theta_i, \theta_j)$ if both principals share information; and $\tilde{q}_i(s_i) = q_i^*(\theta_i)$ and $\tilde{q}_j(s_j) = q_j^*(\theta_j, \theta_i)$ if $P_i$ does not share information and $P_j$ shares information.
Therefore, when choosing whether to share information, each principal simply maximizes the volatility of her own output (given the opponent’s communication choice). The reason is that, by allowing $P_j$ to learn $\theta_i$, $P_i$ can influence the distribution of $P_j$’s equilibrium output, and therefore her own output volatility, because reaction functions are linear. Hence, ceteris paribus, if $\tilde{q}_j (s_j)$ becomes more volatile, the variance of $\tilde{q}_i (s_i)$ also increases.

**Proposition 1** Suppose that $\delta \neq 0$. With complete information: if $\alpha > -\nu (1 - \nu)$, there is a unique equilibrium in dominant strategies in which both principals share information; if $\alpha < -\nu (1 - \nu)$, there is a unique equilibrium in dominant strategies in which no principal shares information.

If $\delta = 0$ or $\Pr(\bar{\theta}, \bar{\theta}) = 0$, with complete information principals obtain the same payoff regardless of whether they share information or not.

Hence, with complete information principals share information only when agents’ costs are positively or not too negatively correlated. To see this, suppose that $P_j$ commits to disclose $\theta_j$, and consider $P_i$’s incentive to reveal $\theta_i$. Sharing information has both a direct and an indirect effect on the equilibrium distribution of outputs. First, allowing $P_j$ to condition her contract on $\theta_i$ expands the set of contingencies upon which $A_j$’s output can be conditioned, thus increasing the volatility of output: the direct effect. Second, revealing $\theta_i$ also affects the correlation among outputs. If costs are positively correlated, there is a higher probability of either state $(\theta, \theta)$ or state $(\bar{\theta}, \bar{\theta})$, where both firms produce the same output, which is either very large or very small when principals share information. Hence, the indirect effect increases volatility too. By contrast, if costs are negatively correlated, the most likely states are $(\bar{\theta}, \bar{\theta})$ and $(\theta, \theta)$, where outputs are more concentrated around their mean when principals share information. Hence, the indirect effect reduces volatility.

On balance, with positive or not too negative correlation between costs, there is an equilibrium with communication because the direct effect of information sharing dominates the indirect effect. By contrast, the indirect effect is stronger than the direct effect with a negative and large correlation between costs. Of course, with no externality ($\delta = 0$) $P_i$’s payoff does not depend on $P_j$’s output, and learning $\theta_j$ does not affect $P_j$’s strategy. The same is true when costs are perfectly correlated — i.e., when $\Pr(\bar{\theta}, \bar{\theta}) = 0$ — since in this case the output volatility is the same irrespective of whether principals share information.

We now consider the effect of information sharing on total principals’ profit and efficiency. The next proposition shows that total principals’ profits are higher with information sharing than without information sharing if and only if each principal has an incentive to unilaterally share information. Moreover, since agents’ information rents are only transfers among players, maximizing principals’ profit is equivalent to maximizing efficiency.

**Proposition 2** With complete information, principals’ information-sharing decisions always maximize efficiency and total principals’ profit.
Our results are consistent with the general analysis of information sharing in oligopoly by Raith (1996), who only considers independent or positively correlated types. However, while Shapiro (1986) argues that information sharing unambiguously increases firms’ profits with substitutes, Propositions 1 and 2 show that this is not necessarily true with negative correlation, if \( \alpha \) is negative and large in absolute value.

4 Asymmetric Information

Suppose now that agents are privately informed about their costs. Before sharing information, principals must learn their agents’ costs through contracting and, hence, they must give agents an information rent in order to screen types. In order to minimize this rent, principals distort outputs away from efficiency. Of course, distortions depend on whether principals share information, which in turn affects the strategic interaction between principals and agents and, therefore, the value of communication.

Since information sharing decisions are public, we characterize equilibrium contracts in the following three subgames: no communication — i.e., when principals do not share information; bilateral information sharing — i.e., when both principals disclose their private information; and unilateral information sharing — i.e., when only one principal shares information.

4.1 No Communication

Suppose that principals do not share information. In a separating equilibrium, \( P_i \) offers a contract that satisfies the following incentive and participation constraints

\[
\begin{align*}
U_i(\theta_i) &\geq t_i(m_i) - \theta_i q_i(m_i) \quad \forall (\theta_i, m_i) \in \Theta^2, \\
U_i(\theta_i) &\geq t_i(\theta_i) - \theta_i q_i(\theta_i) \geq 0 \quad \forall \theta_i \in \Theta.
\end{align*}
\]

As usual, only the incentive constraint of the efficient type and the participation constraint of the inefficient type matter (see, e.g., Laffont and Martimort, 2002). Hence, letting \( q^e(j) \) be \( A_j \)'s output in a (symmetric) separating equilibrium, \( P_i \) solves the following problem

\[
\begin{aligned}
\max_{\{q_i(), U_i()\}} \Bigg\{ & \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) \left[ S(q_i(\theta_i), q^e(\theta_j)) - \theta_i q_i(\theta_i) \right] - \sum_{\theta_i} \Pr(\theta_i) U_i(\theta_i) \Bigg\}, \\
\text{subject to} & \begin{cases}
U_i(\theta) \geq U_i(\overline{\theta}) + \Delta \theta_i(\overline{\theta}), \\
U_i(\overline{\theta}) \geq 0.
\end{cases}
\end{aligned}
\]
Since at the optimum both constraints bind, $P_i$’s problem is

$$\max \left\{ \sum_{\theta_i} \Pr (\theta_i) \sum_{\theta_j} \Pr (\theta_j | \theta_i) [S (q_i (\theta_i), q_j (\theta_j)) - \theta_i q_i (\theta_i)] - \nu \Delta \theta q_i (\bar{\theta}) \right\}. $$

Because agents’ costs are correlated, $P_i$’s beliefs about $\theta_j$ depend on $A_i$’s report. Unlike in the complete information case, however, $P_i$ must now grant a rent $\Delta \theta q_i (\bar{\theta})$ to $A_i$, in order to induce him to reveal his marginal cost.

The necessary and sufficient first-order conditions for $P_i$’s problem are\(^{11}\)

$$\sum_{\theta_j} \Pr (\theta_j | \bar{\theta}) S_1 (q^e (\bar{\theta}), q_j (\theta_j)) = \bar{\theta},$$

(1)

and

$$\sum_{\theta_j} \Pr (\theta_j | \bar{\theta}) S_1 (q^e (\bar{\theta}), q_j (\theta_j)) = \bar{\theta} + \frac{\nu}{1 - \nu} \Delta \theta.$$  

(2)

Therefore, a low-cost agent produces the efficient output that equalizes the (expected) marginal benefit for the principal to the marginal cost, while a high-cost agent produces an inefficiently low output for rent extraction reasons.

Recall from Section 3 that $q^* (\theta_i)$ is agent $A_i$’s efficient output when principals know their agent’s cost and do not share information.

**Proposition 3** Suppose that principals do not to share information. In the unique symmetric separating PBE, outputs are

$$q^e (\bar{\theta}) = q^* (\bar{\theta}) - \frac{\delta \nu (1 - \nu - \alpha)}{(2 - \delta) (2 \nu (1 - \nu) - \alpha \delta)} \Delta \theta, \quad q^e (\bar{\theta}) = q^* (\bar{\theta}) - \frac{2 \nu - \delta (\nu^2 + \alpha)}{(2 - \delta) (2 \nu (1 - \nu) - \alpha \delta)} \Delta \theta.$$  

Moreover,

- $q^e (\bar{\theta}) - q^e (\bar{\theta}) > 0$ for $\Delta \theta \neq 0$ and $\nu \neq 0$;

- Expected output is downward distorted — i.e., $q^e \equiv \sum_{\theta_i} \Pr (\theta_i) q^e (\theta_i) < q^*.$

Despite being set with an efficient rule, the output of a low-cost agent still features a distortion with respect to the output with complete information: $q^e (\bar{\theta}) > q^* (\bar{\theta})$ if $\delta < 0$, and $q^e (\bar{\theta}) < q^* (\bar{\theta})$ if $\delta > 0.$\(^{12}\) The reason is that a principal expects the rival’s output to be distorted downward, and this distortion affects her own output in the low cost state through the upstream externality: since $A_j$ produces a lower output when he has a high cost, if goods are substitutes (resp. complements)

\(^{11}\)We will denote by $S_i (q_i, q_j)$ the partial derivative of $S (q_i, q_j)$ with respect to $q_i$.

\(^{12}\)This two-way distortion has also been analyzed by Cella and Etro (2010).
$P_i$ responds by producing a higher (resp. lower) output than with complete information when $A_i$ has a low cost. By contrast, when there is no externality ($\delta = 0$) the output in the low-cost state is efficient. Finally, expected output is lower under asymmetric information than under complete information because, with privately informed agents, outputs in the high-cost state are downward distorted for rent extraction reasons.

Consider now the equilibrium relationship between $\delta$ and $\alpha$ when principals do not share information.

**Lemma 2**  
\[
\text{sign} \left( \frac{\partial [q^e(\theta) - q^e(\bar{\theta})]}{\partial \alpha} \right) = \text{sign} \, \delta.
\]

The effect of an increase in cost correlation on $q^e(\theta) - q^e(\bar{\theta})$ — the output difference between a low-cost and a high-cost agent — depends on the sign of $\delta$. The intuition is the following. A higher $\alpha$ implies that, when one agent’s cost is high, his opponent’s cost is more likely to be high. Hence, if $\delta < 0$, it is less profitable for a principal to distort the output of a high-cost type, thus increasing $q^e(\theta) - q^e(\bar{\theta})$, because with strategic substitutes each principal prefers to produce more when her rival produces less. By contrast, if $\delta > 0$, a higher $\alpha$ increases $q^e(\theta) - q^e(\bar{\theta})$ because, with strategic complements, principals prefer to jointly increase (resp. reduce) production when both their agents have low (resp. high) costs.

### 4.2 Bilateral Information Sharing

Suppose now that both principals share information. Consider a pure-strategy, symmetric, separating equilibrium in which agents truthfully report their types to principals, who then share this information. Since an agent does not know his rival’s cost when he reports his cost, the incentive and participation constraints are

\[
\left\{ \begin{array}{l}
\sum_{\theta_j} \Pr(\theta_j|\theta_i) U_i(\theta_i, \theta_j) \geq \sum_{\theta_j} \Pr(\theta_j|\theta_i) t_i(m_i, \theta_j) - \theta_i q_i(m_i, \theta_j) \quad \forall (m_i, \theta_i) \in \Theta^2, \\
\sum_{\theta_j} \Pr(\theta_j|\theta_i) U_i(\theta_i, \theta_j) \geq 0 \quad \forall \theta_i \in \Theta.
\end{array} \right.
\]

Moreover, the limited liability constraint is

\[
U_i(\theta_i, \theta_j) = t_i(\theta_i, \theta_j) - \theta_i q_i(\theta_i, \theta_j) \geq 0 \quad \forall (\theta_i, \theta_j) \in \Theta^2.
\]

Clearly, when this constraint is satisfied, the participation constraint is also satisfied.

As usual, the relevant limited liability constraints is that of the high-cost type

\[
U_i(\bar{\theta}, \theta_j) \geq 0 \quad \forall \theta_j \in \Theta,
\]

(3)
while the relevant incentive constraint is that of the low-cost type

\[
\sum_{\theta_j} \Pr(\theta_j|\overline{\theta}) U_i(\overline{\theta}, \theta_j) \geq \sum_{\theta_j} \Pr(\theta_j|\overline{\theta}) \left[ t_i(\overline{\theta}, \theta_j) - \theta q_i(\overline{\theta}, \theta_j) \right]
\]

\[
\Leftrightarrow \sum_{\theta_j} \Pr(\theta_j|\overline{\theta}) U_i(\overline{\theta}, \theta_j) \geq \sum_{\theta_j} \Pr(\theta_j|\overline{\theta}) U_i(\overline{\theta}, \theta_j) + \Delta \theta \sum_{\theta_j} \Pr(\theta_j|\overline{\theta}) q_i(\overline{\theta}, \theta_j).
\]

(4)

Therefore, letting \( q^e(\theta_i, \theta_j) \) denote the equilibrium output, \( P_i \) solves

\[
\max_{\{q_i(\theta_i), U_i(\theta_i)\}} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) \left[ S(q_i(\theta_i, \theta_j), q^e(\theta_j, \theta_i)) - \theta_i q_i(\theta_i, \theta_j) - U_i(\theta_i, \theta_j) \right],
\]

subject to (3) and (4).

At the optimum, the transfer \( t_i(\overline{\theta}, \theta_j) \) is such that the high-cost type obtains no rent regardless of his opponent’s cost — i.e., \( t_i(\overline{\theta}, \theta_j) = \overline{\theta} q_i(\overline{\theta}, \theta_j) \forall \theta_j \) — and the incentive constraint (4) is binding — i.e.,

\[
\sum_{\theta_j} \Pr(\theta_j|\overline{\theta}) U_i(\overline{\theta}, \theta_j) = \Delta \theta \sum_{\theta_j} \Pr(\theta_j|\overline{\theta}) q_i(\overline{\theta}, \theta_j).
\]

(5)

Hence, \( P_i \)’s optimization problem is

\[
\max_{\{q_i(\theta_i), U_i(\theta_i)\}} \left\{ \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) \left[ S(q_i(\theta_i, \theta_j), q^e(\theta_j, \theta_i)) - \theta_i q_i(\theta_i, \theta_j) \right] - \nu \Delta \theta \sum_{\theta_j} \Pr(\theta_j|\overline{\theta}) q_i(\overline{\theta}, \theta_j) \right\}.
\]

(6)

Notice that, although each principal can condition her contract on her opponent’s cost, agents still earn an information rent, because they must obtain a non-negative utility in every contractible state.

The symmetric equilibrium output is determined by the following necessary and sufficient first-order conditions

\[
S_1(q^e(\overline{\theta}, \theta_j), q^e(\theta_j, \overline{\theta})) = \overline{\theta} \quad \forall \theta_j \in \Theta,
\]

(7)

and

\[
S_1(q^e(\overline{\theta}, \theta_j), q^e(\theta_j, \overline{\theta})) = \overline{\theta} + \frac{\nu}{1 - \nu} \frac{\Pr(\theta_j|\overline{\theta})}{\Pr(\theta_j|\overline{\theta})} \Delta \theta \quad \forall \theta_j \in \Theta.
\]

(8)

As when principals do not share information, the output of a low-cost agent is chosen by an efficient rule, while the output of a high-cost agent is distorted for rent extraction reasons. By condition (8), this distortion increases with \( \frac{\Pr(\theta_j|\overline{\theta})}{\Pr(\theta_j|\overline{\theta})} \), which is an index of the informativeness of signal \( \overline{\theta} \) relative to signal \( \overline{\theta} \) on the rival’s cost \( \theta_j \).

Essentially, a principal imposes a higher distortion on the output of a high-cost agent when the
cost of a rival agent takes its less likely value. In fact,

\[
\frac{\Pr(\theta | \theta)}{\Pr(\theta | \overline{\theta})} - \frac{\Pr(\overline{\theta} | \theta)}{\Pr(\overline{\theta} | \overline{\theta})} = \frac{\alpha \Pr(\overline{\theta})}{\Pr(\overline{\theta} | \theta) \Pr(\overline{\theta} | \overline{\theta}) \Pr(\theta)} > 0 \iff \alpha > 0.
\]

Hence, if costs are positively correlated, the distortion of a high-cost agent’s output is larger when his opponent has a low rather than a high cost: \(q_i(\overline{\theta}, \theta) > q_i(\theta, \theta)\). This is because, when costs are positively correlated, a principal whose agent has a high cost expects the opponent’s agent to have a high cost too, and therefore requires a higher distortion when the opponent has a low cost, which is less likely. Similarly, if costs are negatively correlated, the distortion of a high-cost agent’s output is larger when his opponent has a high cost.

Recall from Section 3 that \(q^*(\theta_i, \theta_j)\) is agent \(A_i\)’s efficient output when principals know their agent’s cost and share information.

**Proposition 4** Suppose that both principals share information. In the unique symmetric PBE, outputs are

\[
q^e(\theta, \theta) = q^*(\theta, \theta), \quad q^e(\overline{\theta}, \theta) = q^*(\overline{\theta}, \theta) - \frac{\delta (\nu^2 + \alpha)}{(4 - \delta^2)(\nu (1 - \nu) - \alpha)} \Delta \theta,
\]

\[
q^e(\overline{\theta}, \theta) = q^*(\overline{\theta}, \theta) - \frac{2 (\nu^2 + \alpha)}{(4 - \delta^2)(\nu (1 - \nu) - \alpha)} \Delta \theta, \quad q^e(\theta, \theta) = q^*(\theta, \theta) - \frac{\nu (1 - \nu) - \alpha}{(2 - \delta)((1 - \nu)^2 + \alpha)} \Delta \theta.
\]

Moreover,

- \(q^e(\theta, \theta) > q^*(\theta, \theta)\) if \(\delta < 0\), and \(q^e(\overline{\theta}, \theta) < q^*(\overline{\theta}, \theta)\) if \(\delta > 0\);
- **Expected output is the same when both principals share information and when they do not communicate** — i.e., \(\sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j | \theta_i) q^e(\theta_i, \theta_j) = q^e\).

The outputs produced when both agents have a low cost are efficient; while the output produced by a high-cost agent is inefficiently low to reduce information rents — i.e., \(q^e(\overline{\theta}, \theta_j) < q^*(\overline{\theta}, \theta_j)\) \(\forall \theta_j \in \Theta\). Moreover, this distortion induces principals to also distort the quantity produced by a low-cost agent, when the rival agent has a high cost. In other words, when principals share information, there is a strategic linkage between the distortion imposed by one principal and the opponent’s cost, because the output chosen by \(P_i\) depends on \(A_j\)’s output and, hence, on the distortion chosen by \(P_j\).

The distortion in \(q^e(\theta, \theta)\) relatively to the complete information benchmark depends on the sign of \(\delta\). If \(\delta < 0\), \(P_i\) induces her low-cost agent to overproduce when \(\theta_j = \overline{\theta}\), because outputs are strategic substitutes and, hence, a principal wants to increase production when the rival produces less; if \(\delta > 0\), \(P_i\) induces her low-cost agent to underproduce when \(\theta_j = \overline{\theta}\), because outputs are strategic complements and, hence, a principal wants to reduce production when the rival produces less. Finally, expected outputs are the same when there is no communication and when both principals share information because of the linearity of outputs with respect to costs.
4.3 Unilateral Information Sharing

Suppose now that only one principal, say $P_i$, commits to share information, while $P_j$ does not. Let $q^e_i(\theta_i)$ and $q^e_j(\theta_j, \theta_i)$ be the equilibrium outputs. In this case, $P_i$’s optimization problem is

$$\max_{q_i(.)} \left\{ \sum_{\theta_i} \Pr(\theta_i) \left[ \sum_{\theta_j} \Pr(\theta_j|\theta_i) S(q_i(\theta_i), q_j^e(\theta_j, \theta_i)) - \theta_i q_i(\theta_i) \right] - \nu \Delta \theta q_i(\bar{\theta}) \right\},$$

while $P_j$’s optimization problem is

$$\max_{q_j(. . .)} \left\{ \sum_{\theta_j} \Pr(\theta_j) \left[ \sum_{\theta_i} \Pr(\theta_i|\theta_j) S(q_j(\theta_j, \theta_i), q_i^e(\theta_i)) - \theta_j q_j(\theta_j, \theta_i) \right] - \nu \Delta \theta \sum_{\theta_i} \Pr(\theta_i|\theta) q_j(\bar{\theta}, \theta_i) \right\}.$$

When choosing contracts, $P_i$ takes into account that $P_j$ will also condition $A_j$’s output on $A_i$’s report, while $P_j$ takes into account that $P_i$ will condition her contract only on $A_i$’s report.

The necessary and sufficient first-order conditions of $P_i$’s program are

$$\sum_{\theta_j} \Pr(\theta_j|\bar{\theta}) S_1(q_i^e(\bar{\theta}), q_j^e(\theta_j, \bar{\theta})) = \bar{\theta}, \quad (9)$$

and

$$\sum_{\theta_j} \Pr(\theta_j|\bar{\theta}) S_1(q_j^e(\bar{\theta}), q_j^e(\theta_j, \bar{\theta})) = \bar{\theta} + \frac{\nu}{1 - \nu} \Delta \theta, \quad (10)$$

while the necessary and sufficient first-order conditions of $P_j$’s program are

$$S_1(q_j^e(\bar{\theta}, \theta_i), q_j^e(\theta_i)) = \bar{\theta} \quad \forall \theta_i \in \Theta, \quad (11)$$

and

$$S_1(q_j^e(\bar{\theta}, \theta_i), q_j^e(\theta_i)) = \bar{\theta} + \frac{\nu \Pr(\theta_i|\bar{\theta})}{1 - \nu \Pr(\theta_i|\bar{\theta})} \Delta \theta \quad \forall \theta_i \in \Theta. \quad (12)$$

Therefore, low-cost agents produce according to an efficient rule; while both principals induce a high-cost agent to produce an inefficiently-low output for rent extraction reasons. The interpretation of this distortion is analogous to the interpretation of condition (8) in Section 4.2.

Since $P_j$ is able to choose outputs as a function of both agents’ costs, she has a competitive advantage relative to $P_i$ because she can impose a higher distortion in the states that are (conditionally) less likely. However, as shown in Section 5, this does not necessarily harm $P_i$.

**Proposition 5** Suppose that $P_i$ shares information while $P_j$ does not. In the unique symmetric
PBE, outputs are

\[
q_i^e(\theta) = q^e(\theta, \theta) - \frac{\delta}{4 - \delta^2} \Delta \theta, \quad q_i^e(\bar{\theta}) = q_i^e(\theta) - \frac{2}{(1 - \nu)(4 - \delta^2)} \Delta \theta,
\]

\[
q_j^e(\theta, \theta) = q^e(\theta, \theta) - \frac{\delta^2}{2(4 - \delta^2)} \Delta \theta, \quad q_j^e(\theta, \bar{\theta}) = q_j^e(\theta, \theta) - \frac{\delta}{(1 - \nu)(4 - \delta^2)} \Delta \theta,
\]

\[
q_j^e(\bar{\theta}, \theta) = q_j^e(\theta, \theta) - \frac{\nu}{2(\nu(1 - \nu) - \alpha)} \Delta \theta, \quad q_j^e(\bar{\theta}, \bar{\theta}) = q_j^e(\bar{\theta}, \theta) - \frac{1 - \nu}{2(\alpha + (1 - \nu)^2)} \Delta \theta.
\]

Moreover,

- \( q_i^e(\theta) > q_i^e(\bar{\theta}); q_j^e(\theta, \theta) > q_j^e(\bar{\theta}, \theta); \) and \( q_j^e(\theta, \bar{\theta}) > q_j^e(\bar{\theta}, \bar{\theta}); \)

- \( q_j^e(\theta, \theta) > q_j^e(\bar{\theta}, \theta) \) if \( \delta > 0, \) and \( q_j^e(\theta, \theta) < q_j^e(\bar{\theta}, \theta) \) if \( \delta < 0; \)

- Expected output is the same for both hierarchies and it is equal to the expected output without communication and when both principals share information — i.e., \( \sum_{\theta_i} Pr(\theta_i) q_i^e(\theta_i) = \sum_{\theta_j} Pr(\theta_j) \sum_{\theta_i} Pr(\theta_i|\theta_j) q_j^e(\theta_j, \theta_i) = q^e. \)

The intuitions for the distortion imposed by principals to the outputs of low-cost and high-cost agents are the same as the ones discussed after Proposition 3 and Proposition 4. Since expected outputs are always the same regardless of principals’ communication decisions, sharing information only induces principals to reallocate output distortions across states.

5 Do Principals Share Information?

Consider now principals’ decision to share information at time 1. Principals’ profits when they choose to share information (I) or not to share information (N) are

<table>
<thead>
<tr>
<th></th>
<th>( P_1 )</th>
<th>( I )</th>
<th>( P_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( V^e_I )</td>
<td>( V^e_{I,N} )</td>
<td>( V^e_{N,I} )</td>
</tr>
<tr>
<td>N</td>
<td>( V^e_{I,N} )</td>
<td>( V^e_N )</td>
<td>( V^e_N )</td>
</tr>
</tbody>
</table>

where \( V^e_I \) and \( V^e_N \) are principals’ profits when they both share information and when they do not share information, respectively; \( V^e_{I,N} \) is a principal’s profit when she shares information but her opponent does not; and \( V^e_{N,I} \) is a principal’s profit when she does not share information but her opponent
does. An equilibrium where both principals share information exists if and only if \( V_e^c \geq V_{N,I}^e \), an equilibrium with no communication exists if and only if \( V_N^e \geq V_{I,N}^e \).

The incentive to share information depends on the effect of principals’ communication decisions on rivals’ behavior. With information sharing, a principal’s output depends on the opponent agent’s cost; hence information sharing makes distortions correlated and increases principals’ profit if it ‘softens’ competition. Essentially, while outputs are independently distributed with no communication, with information sharing principals may wish to coordinate distortions for strategic reasons: a correlated distortions effect.

**Proposition 6** When agents are privately informed about their marginal costs:

- If \( \delta \neq 0 \) and \( \delta \alpha \leq 0 \), there is a unique symmetric equilibrium in dominant strategies in which both principals share information.

- If \( \delta \alpha > 0 \), there is a unique symmetric equilibrium in dominant strategies in which no principal shares information.

- If \( \delta \alpha = 0 \), there are two symmetric, payoff equivalent, equilibria: one with information sharing and one without information sharing.

The incentive for a principal to disclose her agent’s cost depends on how this information affects the rival’s output. When \( \delta = 0 \) communication has no effect because there is no strategic interaction between principals; hence disclosing information does not affect a rivals’ output. By contrast, if goods are substitutes (resp. complements), and sharing information induces rivals to reduce (resp. increase) output in the most likely states, then each principal prefers to share information about her agent’s cost. Hence, when \( \delta \neq 0 \), the impact of the correlated distortions effect on the incentive to share information depends on the signs of \( \delta \) and \( \alpha \).

\( P_i \) prefers to share information about \( \theta_i \) if and only if \( \delta \alpha < 0 \). The reason is as follows. Suppose first that \( \alpha > 0 \) — i.e., costs are positively correlated. When \( P_i \) reveals information about \( \theta_i \), she induces \( P_j \) to distort the output of her high-cost agent relatively more (i.e., to produce less) when \( A_i \)'s cost is low (and hence \( A_i \) produces more) and relatively less (i.e., to produce more) when \( A_i \)'s cost is high (and hence \( A_i \) produces less), because the first case is less likely when costs are positively correlated. This increases \( P_i \)'s profits when \( \delta < 0 \) — i.e., with strategic substitutes — because \( P_i \) prefers to produce less (resp. more) when her rival produces more (resp. less); while it reduces \( P_i \)'s profit when \( \delta > 0 \) — i.e., with strategic complements — because principals prefer to produce positively-correlated outputs.

Suppose now that \( \alpha < 0 \) — i.e., costs are negatively correlated. By revealing \( \theta_i \), \( P_i \) induces \( P_j \) to distort the output of her high-cost agent relatively more (i.e., to produce less) when \( A_i \)'s cost is high (and hence \( A_i \) produces less) and relatively less (i.e., to produce more) when \( A_i \)'s cost is low (and hence \( A_i \) produces more), because the first case is less likely when costs are negatively
correlated. This increases $P_i$'s profit when $\delta > 0$ — i.e., with strategic complements — because principals prefer to produce positively-correlated outputs; while it reduces $P_i$’s profits when $\delta < 0$ — i.e., with strategic substitutes — because $P_i$ prefers to produce less (resp. more) when her rival produces more (resp. less).

Notice that the correlated distortion effect is of first-order magnitude relative to the effects of information sharing with complete information between principals and agents, where only the sign and magnitude of the correlation parameter affects the value of communication.\(^{13}\)

The equilibrium characterized in Proposition 6 are in dominant strategies — e.g., when $\delta \alpha < 0$ and information sharing is an equilibrium, each principal strictly prefers to share information regardless of what the other principal does. This implies that there is no equilibrium in mixed strategies where principals randomize between sharing and not sharing information.

The next proposition compares equilibrium expected profits when both principals share information and when they both do not share information.

**Proposition 7** Principals’ expected profits are higher when they both share information than with no communication, while agents’ expected rents are higher with no communication than when both principals share information.

Hence, when agents are privately informed about their costs, principals and agents have opposing preferences regarding information sharing. The reason is that, since costs are correlated, communication creates an informational externality that reduces the information rent that a principal has to pay to her agent. This is because, when agents’ contracts are contingent on the rivals’ types, cost correlation generates a relative performance evaluations effect that relaxes incentive compatibility and makes information acquisition less costly for principals. For $\delta$ small this effect outperforms the strategic effect due to correlated distortions, because upstream externalities are negligible relative to cost of information acquisition.

Since agents’ information rents are only a transfer among players and do not affect efficiency, an implication of Propositions 6 and 7 is that, while under complete information principals’ decisions regarding information sharing always maximizes total principals’ total profit and efficiency, uninformed principals may inefficiently choose not to share information, even though they would obtain higher total profit by sharing information.

**Corollary 8** Principals’ decision not to share information when $\delta \alpha > 0$ is inefficient.

Hence, when cost correlation and production externalities have the same sign, principals face a prisoners’ dilemma, since they have an incentive not to share information, even if they would jointly benefit from coordinating on information sharing.

\(^{13}\)In the proof of Proposition 2 we show that, under complete information, only terms of second-order magnitude matter in signing the difference between expected profits with and without information sharing.
6 Agents’ Collusion

In our analysis, we have assumed that, when $A_i$ makes a report to $P_i$, he believes that $A_j$ makes a truthful report to $P_j$. However, with information sharing the expected utility of an agent is affected by his opponent’s report, because his principal’s payoff depends on it. Hence, an equilibrium in which both principals share information, and each agent tells the truth expecting the rival to do the same, may not be collusion-proof. In other words, agents may wish to coordinate on an equilibrium in which they both lie in order to obtain higher rents at the expense of principals.

In this section, we consider the possibility of collusion among agents, assuming that agents cannot make side payments.\footnote{The reason is that, if we consider agents’ collusion enforced through side transfers, there is no a priori reason to exclude side transfers among principals. But, in this case, the analysis would be equivalent to Laffont and Martimort (2000) where a single principal — i.e., the coalition formed by $P_1$ and $P_2$ — deals with two colluding agents with correlated types — i.e., $A_1$ and $A_2$.} Let $t^e(\theta, \theta)$ be the equilibrium transfer paid to $A_i$ when both principals share information. In order for agents to prefer to truthfully report their costs rather than both lie, it must be that an efficient agent prefers to truthfully report his cost, rather than lie, when his efficient rival lies — i.e.,

$$t^e(\theta, \theta) - \theta q^e(\theta, \theta) > t^e(\bar{\theta}, \theta) - \theta q^e(\bar{\theta}, \theta).$$

But the equilibrium transfers with information sharing are indeterminate in some states, because agents make their reports before learning the rival’s type. Hence, the number of constraints that bind in a truthful equilibrium is smaller than the number of instruments available to principals.

Notice that the limited liability constraints of the inefficient types imply that $t^e(\bar{\theta}, \theta) = \bar{\theta} q^e(\bar{\theta}, \theta)$, for all $\theta$. Hence, (13) rewrites as

$$t^e(\theta, \theta) - \theta q^e(\theta, \theta) > \Delta \theta q^e(\bar{\theta}, \theta).$$

This implies that agents have no incentive to collude if principals — actually even only one of them — implement a transfer $t^e(\theta, \theta)$ such that: (i) agents still tell the truth when rivals are expected to do so; (ii) limited liability constraints are satisfied in all states; (iii) inequality (14) is satisfied.

**Proposition 9** There is a system of transfers such that the equilibrium characterized in Proposition 4 is robust to the threat of implicit collusion. This system of transfers satisfies agents’ limited liability and is such that

$$t^e(\theta, \theta) = \bar{\theta} q^e(\theta, \theta) \quad \forall \theta \in \Theta,$$
$$t^e(\theta, \theta) = \theta q^e(\theta, \theta),$$
$$t^e(\theta, \theta) = \theta q^e(\theta, \theta) + \Delta \theta \frac{\Pr(\theta|\theta)}{\Pr(\theta|\theta)} q^e(\theta, \theta) + \Delta \theta q^e(\bar{\theta}, \theta).$$

Implicit collusion is therefore not an issue in our model. Of course, this is partly due to the fact that there are no production externalities across agents — see, e.g., Martimort (1996) for a model
with this feature. In this setting communication not only creates informational externalities among
agents, but it also generates direct production externalities that may increase agents’ incentives to
jointly misreport their types.

7 Conclusions

In order to explore the effects of the exchange of information among complex organizations, we
have considered two principals who produce externalities on each other, and independently choose
whether to share the information they obtain through contracts with exclusive and privately informed
agents. Principals’ incentive to share information depends on the output distortions generated to
induce agents to reveal their information, because information sharing induces these distortions to
be correlated. Therefore, the benefit of communication depends on the interaction between the type
of production externalities and the sign of cost correlation. When production externalities and cost
correlation have the same sign, there is a unique equilibrium with no communication; when production
externalities and cost correlation have opposite signs, there is a unique equilibrium in which both
principals share information. In contrast to the case in which agents have no private information,
where the equilibrium outcome is always efficient, principals may face a prisoners’ dilemma when
agents have private information about their costs. Our results are robust to the threat of (implicit)
collusion among agents.
Appendix

Proof of Lemma 1. With complete information, principals fully extract their agents’ rents. We characterize the equilibrium outputs in the three possible cases: (i) both principals share information; (ii) no principal shares information; (iii) only one principal shares information.

No information sharing. When principals do not share information, the (symmetric) equilibrium output is

$$q^*(\theta_i) = \arg\max_{q_i} \sum_{\theta_j} \Pr(\theta_j|\theta_i) [S(q_i(\theta_i), q^*(\theta_j)) - \theta_i q_i(\theta_i)] \quad \forall \theta_i \in \Theta,$$

and the equilibrium transfer, $t^*(\theta_i)$, is such that

$$U_i(\theta_i) = 0 \quad \Rightarrow \quad t^*(\theta_i) = \theta_i q^*(\theta_i) \quad \forall \theta_i \in \Theta.$$

Hence, a symmetric equilibrium satisfies the following necessary and sufficient first-order conditions

$$\sum_{\theta_j} \Pr(\theta_j|\theta_i) S_1(q^*(\theta_i), q^*(\theta_j)) = \theta_i \quad \forall \theta_i \in \Theta,$$  (A1)

where $S_1(.)$ denotes the partial derivative of $S(q_i, q_j)$ with respect to $q_i$. Solving these conditions yields

$$q^*(\theta) = \frac{\beta - \theta}{2 - \delta} - \frac{\delta (1 - \nu) (\nu (1 - \nu) - \alpha)}{(2 - \delta) (2\nu (1 - \nu) - \alpha \delta)} \Delta \theta, \quad q^*(\bar{\theta}) = q^*(\theta) - \frac{\nu (1 - \nu)}{2\nu (1 - \nu) - \alpha \delta} \Delta \theta.$$

Bilateral information sharing. When both principals share information, the equilibrium output is

$$q^*(\theta_i, \theta_j) = \arg\max_{q_{i,\theta_j}} [S(q_i(\theta_i, \theta_j), q^*(\theta_j, \theta_i)) - \theta_i q_i(\theta_i, \theta_j)] \quad \forall (\theta_i, \theta_j) \in \Theta^2,$$

and the equilibrium transfer, $t^*_i(\theta_i, \theta_j)$, is such that

$$U_i(\theta_i, \theta_j) = 0 \quad \Rightarrow \quad t^*_i(\theta_i, \theta_j) = \theta_i q^*_i(\theta_i, \theta_j) \quad \forall (\theta_i, \theta_j) \in \Theta^2.$$

The first-order necessary and sufficient conditions are

$$S_1(q^*(\theta_i, \theta_j), q^*(\theta_j, \theta_i)) = \theta_i \quad \forall (\theta_i, \theta_j) \in \Theta^2. \quad (A2)$$

Solving (A2), outputs in the unique equilibrium are

$$q^*(\theta, \theta) = \frac{\beta - \theta}{2 - \delta}, \quad q^*(\theta, \bar{\theta}) = q^*(\theta, \theta) - \frac{\delta}{4 - \delta^2} \Delta \theta, \quad q^*(\bar{\theta}, \theta) = q^*(\theta, \theta) - \frac{1}{2 - \delta} \Delta \theta.$$
Unilateral information sharing. Finally, suppose that one principal, say $P_i$, commits to disclose her agent’s cost, while $P_j$ does not share information. For each $\theta_i$, $P_i$’s optimization program is

$$\max_{q_i^{(.)}} \sum_{\theta_i} \Pr(\theta_j|\theta_i) S(q_i(\theta_i), q_j^*(\theta_j, \theta_i)) - \theta_i q_i(\theta_i),$$

and, for each $(\theta_i, \theta_j)$, $P_j$’s optimization program is

$$\max_{q_j(\cdot, \cdot)} [S(q_j(\theta_j, \theta_i), q_i^*(\theta_i)) - \theta_j q_j(\theta_j, \theta_i)].$$

The first-order necessary and sufficient conditions are

$$\sum_{\theta_j} \Pr(\theta_j|\theta_i) S_1(q_i^*(\theta_i), q_j^*(\theta_j, \theta_i)) = \theta_i \quad \forall \theta_i \in \Theta, \tag{A3}$$

$$S_1(q_j^*(\theta_j, \theta_i), q_i^*(\theta_i)) = \theta_j \quad \forall (\theta_i, \theta_j) \in \Theta^2, \tag{A4}$$

yielding the equilibrium outputs

$$q_i^*(\theta) = \frac{\beta - \theta}{2 - \delta} - \frac{\delta(\nu (1 - \nu) - \alpha)}{\nu (4 - \delta^2)} \Delta \theta, \quad q_i^*(\overline{\theta}) = q_i^*(\theta) - \frac{2\nu (1 - \nu) + \alpha \delta}{\nu (1 - \nu) (4 - \delta^2)} \Delta \theta,$$

$$q_j^*(\theta, \theta) = q^*(\theta, \theta) - \frac{(\nu (1 - \nu) - \alpha)\delta^2}{2 \nu (4 - \delta^2)} \Delta \theta, \quad q_j^*(\theta, \overline{\theta}) = q^*(\theta, \overline{\theta}) - \frac{(\alpha + (1 - \nu)^2)\delta^2}{2 (1 - \nu) (4 - \delta^2)} \Delta \theta,$$

$$q_j^*(\overline{\theta}, \theta) = q^*(\overline{\theta}, \theta) + \frac{(\nu^2 + \alpha)\delta^2}{2 \nu (4 - \delta^2)} \Delta \theta, \quad q_j^*(\overline{\theta}, \overline{\theta}) = q^*(\overline{\theta}, \overline{\theta}) + \frac{\nu (1 - \nu) - \alpha)(\delta^2}{2 (1 - \nu) (4 - \delta^2)} \Delta \theta.$$

Expected outputs and profits. Consider now expected outputs in the three cases. Let $q^* \equiv q^*(\theta, \theta) - \frac{1 - \nu}{2 - \delta} \Delta \theta$. Taking expectations, it follows that

$$\sum_{\theta_i} \Pr(\theta_i) q_i^*(\theta_i) = \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q_i^*(\theta_i, \theta_j) =$$

$$\sum_{\theta_i} \Pr(\theta_i) q^*(\theta_i) = \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^*(\theta_i, \theta_j) =$$

$$= \sum_{\theta_i} \Pr(\theta_i) q_i^*(\theta_i) = \sum_{\theta_j} \Pr(\theta_j) \sum_{\theta_i} \Pr(\theta_i|\theta_j) q_j^*(\theta_j, \theta_i) = q^*.$$
Using conditions (A1), (A2), (A3) and (A4), expected profits are

\[ V^*_i = \kappa + \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q_i^*(s_i)^2 \]

\[ = \kappa + \left( \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) \bar{q}_i(s_i) \right)^2 + \]

\[ + \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) \left[ \bar{q}_i(s_i) - \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) \bar{q}_i(s_i) \right]^2, \]

where the second equality follows because \( \mathbb{E}[x^2] = (\mathbb{E}[x])^2 + \mathbb{E}[x - \mathbb{E}[x]]^2 \). \( \blacksquare \)

**Proof of Proposition 1.** Let \( V^*_r \) and \( V^*_N \) be principals' expected profits when they both share information and when they do not share information, respectively. Let \( V^*_{N,I} \) be \( P_r \)'s profit and \( V^*_{I,N} \) be \( P_j \)'s profit when \( P_i \) does not share information while \( P_j \) shares information.

A symmetric equilibrium where both principals share information exists if and only if \( V^*_r \geq V^*_{N,I} \). Assuming that \( \delta \) is small but different from 0 and using a second-order Taylor approximation around \( \delta = 0 \), we have

\[ V^*_{N,I} \approx \kappa + \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^*(\theta_i, \theta_j)^2 + 2\delta \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^*(\theta_i, \theta_j) \frac{\partial q_i^*(\theta_i, \theta_j)}{\partial \delta} + \]

\[ + \delta^2 \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) \left[ q_i^*(\theta_i, \theta_j) \frac{\partial^2 q_i^*(\theta_i, \theta_j)}{\partial \delta^2} + \left( \frac{\partial q_i^*(\theta_i, \theta_j)}{\partial \delta} \right)^2 \right], \]

and

\[ V^*_r \approx \kappa + \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^*(\theta_i, \theta_j)^2 + 2\delta \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^*(\theta_i, \theta_j) \frac{\partial q^*(\theta_i, \theta_j)}{\partial \delta} + \]

\[ + \delta^2 \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) \left[ q^*(\theta_i, \theta_j) \frac{\partial^2 q^*(\theta_i, \theta_j)}{\partial \delta^2} + \left( \frac{\partial q^*(\theta_i, \theta_j)}{\partial \delta} \right)^2 \right]. \]

Using the equilibrium outputs from Lemma 1, we have

\[ \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q_i^*(\theta_i, \theta_j)^2 = \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^*(\theta_i, \theta_j)^2, \]
and
\[
\lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^*_i(\theta_i, \theta_j) \frac{\partial q^*_i(\theta_i, \theta_j)}{\partial \delta} = \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^*(\theta_i, \theta_j) \frac{\partial q^*(\theta_i, \theta_j)}{\partial \delta}.
\]

Hence,
\[
V^*_I - V^*_{N,I} \approx \delta^2 \left\{ \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^*_i(\theta_i, \theta_j) \frac{\partial^2 q^*_i(\theta_i, \theta_j)}{\partial \delta^2} - \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^*_i(\theta_i, \theta_j) \frac{\partial^2 q^*_i(\theta_i, \theta_j)}{\partial \delta^2} + \right. \\
+ \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) \left( \frac{\partial q^*(\theta_i, \theta_j)}{\partial \delta} \right)^2 - \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) \left( \frac{\partial q^*(\theta_i, \theta_j)}{\partial \delta} \right)^2 \right\} \\
\approx \frac{(\nu (1 - \nu) + \alpha) \Pr(\theta) \delta^2 \Delta \theta^2}{8 \nu (1 - \nu)}. 
\]  
(A5)

Therefore, there is a symmetric equilibrium where both principals share information if and only if \( \nu (1 - \nu) + \alpha \geq 0 \) — i.e., if \( \alpha \geq 0 \) or if \( \alpha < 0 \) and \( |\alpha| \leq \nu (1 - \nu) \).

A symmetric equilibrium where principals do not share information exists if and only if \( V^*_N \geq V^*_{I,N} \). Assuming that \( \delta \) is small but different from 0 and using a second-order Taylor approximation around \( \delta = 0 \), we have
\[
V^*_{I,N} \approx \kappa + \lim_{\delta \to 0} \sum_{\theta_j} \Pr(\theta_j) q^*_j(\theta_j)^2 + 2 \delta \lim_{\delta \to 0} \sum_{\theta_j} \Pr(\theta_j) q^*_j(\theta_j)^2 \frac{\partial q^*_j(\theta_j)}{\partial \delta} + \\
+ \delta^2 \lim_{\delta \to 0} \sum_{\theta_j} \Pr(\theta_j) \left[ q^*_j(\theta_j)^2 \frac{\partial^2 q^*_j(\theta_j)}{\partial \delta^2} + \left( \frac{\partial q^*_j(\theta_j)}{\partial \delta} \right)^2 \right],
\]
and
\[
V^*_N \approx \kappa + \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) q^*(\theta_i)^2 + 2 \delta \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) q^*(\theta_i)^2 \frac{\partial q^*(\theta_i)}{\partial \delta} + \\
+ \delta^2 \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \left[ q^*(\theta_i) \frac{\partial^2 q^*(\theta_i)}{\partial \delta^2} + \left( \frac{\partial q^*(\theta_i)}{\partial \delta} \right)^2 \right].
\]

Using the equilibrium outputs from Lemma 1, we have
\[
\lim_{\delta \to 0} \sum_{\theta_j} \Pr(\theta_j) q^*_j(\theta_j)^2 = \lim_{\delta \to 0} \sum_{\theta_j} \Pr(\theta_j) q^*(\theta_j)^2,
\]
and
Proposition 1, for profits — i.e., $V^*$

Proof of Proposition 2.

Proof of Proposition 3. Equilibrium outputs are computed by solving the system of first-order conditions (1) and (2). Moreover, $\sum_{\theta_i} \Pr(\theta_i) q^*(\theta_i) = \frac{\beta - \bar{q}}{2 - \delta}$ and $q^*(\bar{q}) - q^*(\bar{q}) = \frac{\nu \Delta \bar{q}}{2\nu(1-\nu) - \alpha \delta}$. □
Proof of Lemma 2. Differentiating \( q^e(\theta) - q^e(\bar{\theta}) \) with respect to \( \alpha \),

\[
\text{sign} \frac{\partial [q^e(\theta) - q^e(\bar{\theta})]}{\partial \alpha} = \text{sign}\frac{\delta \nu}{(2 \nu (1 - \nu) - \alpha \delta)^2},
\]

\[\square\]

Proof of Proposition 4. Solving the system of first-order conditions (7) and (8) yields the equilibrium outputs with bilateral information sharing. Therefore,

\[
\sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^e(\theta_i, \theta_j) = \nu \left[ (\nu + \frac{\alpha}{\nu}) q^e(\bar{\theta}, \bar{\theta}) + (1 - \nu - \frac{\alpha}{\nu}) q^e(\bar{\theta}, \bar{\theta}) \right] + \\
+ (1 - \nu) \left[ (\nu - \frac{\alpha}{1 - \nu}) q^e(\bar{\theta}, \bar{\theta}) + (1 - \nu + \frac{\alpha}{1 - \nu}) q^e(\bar{\theta}, \bar{\theta}) \right] \\
= \frac{\beta - \bar{\theta}}{2 - \delta},
\]

and

\[
\sum_{\theta_i} \Pr(\theta_i) q^e(\theta_i) = \nu q^e(\theta) + (1 - \nu) q^e(\bar{\theta}) = \frac{\beta - \bar{\theta}}{2 - \delta}.
\]

Hence, \( \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^e(\theta_i, \theta_j) = \sum_{\theta_i} \Pr(\theta_i) q^e(\theta_i) \). Moreover,

\[
\sum_{\theta_i} \Pr(\theta_i) q^e(\theta_i) - \sum_{\theta_i} \Pr(\theta_i) q^e(\theta_i) = \frac{\beta - \bar{\theta}}{2 - \delta} - \frac{\beta - \bar{\theta} - (1 - \nu) \Delta \theta}{2 - \delta} = \frac{-\nu \Delta \theta}{2 - \delta} < 0.
\]

The rest of the proof is straightforward. \[\square\]

Proof of Proposition 5. Solving the system of first-order conditions (9), (10), (11) and (12) yields the equilibrium outputs with unilateral information sharing. Moreover, the expected output of both principals is

\[
\sum_{\theta_i} \Pr(\theta_i) q^e_1(\theta_i, \theta_j) = \sum_{\theta_j} \Pr(\theta_j) \sum_{\theta_i} \Pr(\theta_i|\theta_j) q^e_2(\theta_j, \theta_i) = \frac{\beta - \bar{\theta}}{2 - \delta},
\]

which is equal to the expected output when both principals share information. \[\square\]

Proof of Proposition 6. Suppose first that both \( \alpha \) and \( \delta \) are different from 0. A symmetric equilibrium where both principals share information exists if and only if \( V^e_i \geq V^e_{N,I} \). Using a second-order Taylor approximation around \( \delta = 0 \),

\[
V^e_i \approx \kappa + \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^e(\theta_i, \theta_j)^2 + 2 \delta \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^e(\theta_i, \theta_j) \frac{\partial q^e(\theta_i, \theta_j)}{\partial \delta},
\]

and

\[
V^e_{N,I} \approx \kappa + \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) q^e_1(\theta_i)^2 + 2 \delta \lim_{\delta \to 0} \sum_{\theta_i} \Pr(\theta_i) q^e_2(\theta_i) \frac{\partial q^e_2(\theta_i)}{\partial \delta}.
\]

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Hence,

\[ V^e_I - V^e_{N,I} \approx \lim_{\delta \to 0} \left\{ \sum_{\theta_i} \Pr (\theta_i) \left[ \sum_{\theta_j} \Pr (\theta_j | \theta_i) q^e (\theta_i, \theta_j)^2 - q^e_i (\theta_i)^2 \right] \right\} + \]

\[ + 2\delta \lim_{\delta \to 0} \left\{ \sum_{\theta_i} \Pr (\theta_i) \left[ \sum_{\theta_j} \Pr (\theta_j | \theta_i) q^e (\theta_i, \theta_j) \frac{\partial q^e (\theta_i, \theta_j)}{\partial \delta} - q^e_i (\theta_i) \frac{\partial q^e_i (\theta_i)}{\partial \delta} \right] \right\} . \]

And, using the outputs characterized in Propositions 3 and 5,

\[ V^e_I - V^e_{N,I} \approx -\frac{\alpha \delta \Delta \theta^2}{4(\alpha + (1 - \nu)^2)}. \]

(A8)

Therefore, there is a symmetric equilibrium where both principals share information if and only if \( \alpha \delta < 0 \).

A symmetric equilibrium where principals do not share information exists if and only if \( V^e_N \geq V^e_{I,N} \). Using a second-order Taylor approximation around \( \delta = 0 \),

\[ V^e_N \approx \kappa + \lim_{\delta \to 0} \sum_{\theta_i} \Pr (\theta_i) q^e (\theta_i)^2 + 2\delta \lim_{\delta \to 0} \sum_{\theta_i} \Pr (\theta_i) q^e (\theta_i) \frac{\partial q^e (\theta_i)}{\partial \delta}, \]

(A9)

and

\[ V^e_{I,N} \approx \kappa + \lim_{\delta \to 0} \sum_{\theta_i} \Pr (\theta_i) q^e_i (\theta_i)^2 + 2\delta \lim_{\delta \to 0} \sum_{\theta_i} \Pr (\theta_i) q^e_i (\theta_i) \frac{\partial q^e_i (\theta_i)}{\partial \delta}. \]

Hence,

\[ V^e_N - V^e_{I,N} \approx \lim_{\delta \to 0} \left[ \sum_{\theta_i} \Pr (\theta_i) q^e (\theta_i)^2 - \sum_{\theta_i} \Pr (\theta_i) q^e_i (\theta_i)^2 \right] + \]

\[ + 2\delta \lim_{\delta \to 0} \left[ \sum_{\theta_i} \Pr (\theta_i) q^e (\theta_i) \frac{\partial q^e (\theta_i)}{\partial \delta} - \sum_{\theta_i} \Pr (\theta_i) q^e_i (\theta_i) \frac{\partial q^e_i (\theta_i)}{\partial \delta} \right] . \]

And, using the outputs characterized in Propositions 3 and 5,

\[ V^e_N - V^e_{I,N} \approx \frac{\delta \alpha \Delta \theta^2}{4(1 - \nu)^2}. \]

(A10)

Therefore, there is a symmetric equilibrium where both principals do not share information if and only if \( \delta \alpha > 0 \).
Finally, it is easy to verify that \(\lim_{\alpha \to 0} (V_e^i - V_{N,i}) = \lim_{\alpha \to 0} (V_e^e - V_{I,N})\). While

\[
\lim_{\alpha \to 0} (V_e^i - V_{N,i}) = \frac{\nu (8 - \delta^2) \delta^2 \Delta \theta^2}{4(1 - \nu)(2 + \delta)^2(2 - \delta)} > 0
\]

and

\[
\lim_{\alpha \to 0} (V_e^e - V_{I,N}) = -\frac{\nu (8 - \delta^2) \delta^2 \Delta \theta^2}{4(1 - \nu)(2 + \delta)^2(2 - \delta)} < 0.
\]

Because the sign of (A8) is always opposite to the sign of (A10), the equilibria are in dominant strategies. ■

**Proof of Proposition 7.** Suppose that both \(\delta\) and \(\alpha\) are different from 0. Using (A7), (A9), and the equilibrium outputs in Propositions 3 and 4,

\[
V_e^i - V_e^e \approx \lim_{\delta \to 0} \left\{ \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^e(\theta_i, \theta_j)^2 - \sum_{\theta_i} \Pr(\theta_i) q^e(\theta_i)^2 \right\} +
\]

\[
+ 2\delta \lim_{\delta \to 0} \left\{ \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) q^e(\theta_i, \theta_j) \frac{\partial q^e(\theta_i, \theta_j)}{\partial \delta} - \sum_{\theta_i} \Pr(\theta_i) q^e(\theta_i) \frac{\partial q^e(\theta_i)}{\partial \delta} \right\}
\]

\[
\approx \frac{\Delta \theta^2}{4(1 - \nu)((1 - \nu)^2 + \alpha)} \left[ \frac{\alpha^2}{\nu(1 - \alpha)} - \frac{\alpha \delta(\alpha + 2(1 - \nu)^2)}{1 - \nu} \right],
\]

which is strictly positive for \(\delta\) small and different than 0. Moreover,

\[
\lim_{\alpha \to 0} (V_e^i - V_e^e) = \frac{(12 - \delta^2) \Delta \theta^2 \delta^2 \nu}{4(1 - \nu)(2 + \delta)^2(2 - \delta)^2} > 0.
\]

Consider now agents’ expected rents and let \(U^e(s_i)\) be \(A_i\)’s equilibrium utility when the information upon which \(P_i\) conditions her contract is \(s_i, s_i \in \{\theta_i, (\theta_1, \theta_2)\}\). Without information sharing,

\[
\sum_{\theta_i} \Pr(\theta_i) U^e(\theta_i) = \nu \Delta \theta q^e(\bar{\theta}). \tag{A11}
\]

When instead both principals share information,

\[
\sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) U^e(\theta_i, \theta_j) = \nu \Delta \theta \sum_{\theta_j} \Pr(\theta_j|\theta) q^e(\bar{\theta}, \theta_j). \tag{A12}
\]

Taking the difference between (A11) and (A12),

\[
\sum_{\theta_i} \Pr(\theta_i) \left[ \sum_{\theta_j} \Pr(\theta_j|\theta_i) U^e(\theta_i, \theta_j) - U^e(\theta_i) \right] = \nu \Delta \theta \left[ \sum_{\theta_j} \Pr(\theta_j|\theta) q^e(\bar{\theta}, \theta_j) - q^e(\bar{\theta}) \right].
\]
First, notice that

\[
\nu \Delta \theta \lim_{\alpha \to 0} \left[ \sum_{\theta_j} \Pr(\theta_j|\theta) q^e(\bar{\theta}, \theta_j) - q^e(\bar{\theta}) \right] = -\frac{\nu^2 \delta^2 \Delta \theta^2}{2(1 - \nu)(2 - \delta)(2 + \delta)} < 0.
\]

Suppose now that \( \alpha \neq 0 \) and that \( \delta \) is small. Using a first-order Taylor approximation,

\[
\nu \Delta \theta \left[ \sum_{\theta_j} \Pr(\theta_j|\theta) q^e(\bar{\theta}, \theta_j) - q^e(\bar{\theta}) \right] \approx -\frac{\Delta \theta^2}{2(1 - \nu)(\alpha + (1 - \nu)^2)} \times \\
\times \left[ \frac{\alpha^2}{\nu (1 - \nu) - \alpha} - \frac{\delta \alpha (\alpha \nu + (1 - \nu)^2(1 + \nu))}{2(1 - \nu)} \right],
\]

which immediately implies the result. ■

**Proof of Proposition 9.** By equation (5), the maximal transfer in state \((\bar{\theta}, \bar{\theta})\) that is compatible with the incentive compatibility constraint (5) is such that

\[
e^c(\bar{\theta}, \bar{\theta}) = \theta q^e(\bar{\theta}, \bar{\theta}) - \frac{\Pr(\theta|\bar{\theta})}{\Pr(\bar{\theta}|\theta)} (e^c(\bar{\theta}, \bar{\theta}) - \theta q^e(\bar{\theta}, \bar{\theta})) + \Delta \theta \frac{\Pr(\theta|\bar{\theta})}{\Pr(\bar{\theta}|\theta)} q^e(\bar{\theta}, \bar{\theta}) + \Delta \theta q^e(\bar{\theta}, \bar{\theta}).
\]

\[
\Rightarrow \hat{e}^c(\bar{\theta}, \bar{\theta}) = \max_{e^c(\bar{\theta}, \bar{\theta})} \left\{ e^c(\bar{\theta}, \bar{\theta}) : e^c(\bar{\theta}, \bar{\theta}) - \theta q^e(\bar{\theta}, \bar{\theta}) \geq 0 \right\} = \theta q^e(\bar{\theta}, \bar{\theta}) + \Delta \theta \frac{\Pr(\theta|\bar{\theta})}{\Pr(\bar{\theta}|\theta)} q^e(\bar{\theta}, \bar{\theta}) + \Delta \theta q^e(\bar{\theta}, \bar{\theta}).
\]

The transfer \(\hat{e}^c(\bar{\theta}, \bar{\theta})\) satisfies the agent’s limited liability constraint in state \((\bar{\theta}, \bar{\theta})\). Moreover, substituting this transfer into condition (14) yields

\[
\frac{\Pr(\theta|\bar{\theta})}{\Pr(\bar{\theta}|\theta)} q^e(\bar{\theta}, \theta) > 0.
\]

Hence, agents have no incentive to collude when they receive transfer \(\hat{e}^c(\bar{\theta}, \bar{\theta})\). ■
References


