A dynamic minimax regret analysis of flood risk management strategies under climate change uncertainty with emerging information on peak flows

Abstract Investment decisions on flood protection are often guided by considerations of regret. The ‘minimax regret’ (MR) decision criterion is used to identify investments in flood protection which minimise worst-case regret. In this paper, we study the dynamic application of the MR decision criterion to analyse robustness of flood risk management (FRM) strategies under climate change uncertainty with emerging peak flow information. The approach supports identification of adaptive FRM strategies by including ‘learning scenarios’ about peak flow development. We implement the MR decision criterion dynamically to study optimal dike height and floodplain development in a conceptual FRM model. Outcomes of static and dynamic MR analysis are compared. It is shown that the dynamic model offers greater flexibility than the static model because it allows investments to be changed when new peak flow information emerges. We conclude that dynamic MR solutions are more robust than the solutions obtained from a static MR analysis of FRM investments due to ongoing changes in climate change impact projections.

Keywords minimax regret, dynamic regret, flood risk management, climate change adaptation, flexibility, robustness, learning
1. Introduction

Investment decisions on flood protection are often guided by considerations of regret, comparing an actual or a hypothetical outcome against a best achievable outcome. The ‘minimax regret’ (MR) decision criterion is used to support robust decision-making by identifying flood risk management (FRM) solutions with the least worst-case regret (cf. Hall and Solomatine 2008; Brekelmans et al. 2012). In its basic form, the MR objective is to minimise maximum regret for a defined set of scenarios (Niehans 1948; Savage 1951). In a one-shot or static MR application, this set is constant over time. However, recent literature advocates ‘flexible’ or ‘adaptive’ FRM strategies which can be adapted to new information at relatively low costs, motivated by the presence of ‘deep’ uncertainties, and the inherently limited capacity to predict the future and the possible emergence of new information (cf. Pahl-Wostl 2007; Kwadijk et al. 2010; Haasnoot et al. 2013). In this paper, we apply the MR decision criterion dynamically to FRM strategies under the possible emergence of new climate information.

Our starting point is an example of a static MR application to flood risk management to illustrate how climate change impact scenarios result in selection of flood protection measures using the MR criterion. Consider the choice between raising inland dikes and investment in a storm surge barrier.¹ Table 1(a) summarises total discounted costs, including investment and damage costs, associated with both investment options under a ‘low’ (s_L), ‘high’ (s_H) or ‘extreme’ (s_E) sea level rise scenario.

<table>
<thead>
<tr>
<th>Measure / Scenario</th>
<th>s_L</th>
<th>s_L</th>
<th>s_E</th>
</tr>
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<tbody>
<tr>
<td>Storm surge barrier</td>
<td>C₁</td>
<td>C₂</td>
<td>C₃</td>
</tr>
<tr>
<td>Inland dikes</td>
<td>C₄</td>
<td>C₅</td>
<td>C₆</td>
</tr>
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</table>

¹ Dutch policy makers were facing this dilemma in the 80s along the estuary of the “New Waterway” near Rotterdam. Raising the inland dikes would have required large-scale investments, which resulted in the decision to construct an innovative storm surge barrier (Bol 2005).
An optimal *ex-post* decision (*) minimises the costs of both flood protection measures for a given scenario. In this example, \( C_1 < C_4, C_2 < C_5 \) and \( C_6 < C_3 \). Corresponding *anticipated* regret values are displayed in Table 1(b).

<table>
<thead>
<tr>
<th>Measure / Scenario</th>
<th>( s_L )</th>
<th>( s_H )</th>
<th>( s_E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storm surge barrier</td>
<td>( C_4 - C_1 )</td>
<td>( C_5 - C_2 )</td>
<td>( C_3 - C_6 )</td>
</tr>
<tr>
<td>Inland dikes</td>
<td></td>
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</table>

The regret values in Table 1(b) are ‘absolute’ regret values obtained from subtracting the costs of an optimal *ex-post* alternative from the costs of the selected adaptation measure for the scenario.

Suppose that \( C_3 - C_6 > C_5 - C_2 > C_4 - C_1 \). MR of dike heightening is \( C_5 - C_2 \) and MR of investment in the storm surge barrier is \( C_3 - C_6 \). The optimal strategy, when applying the MR criterion, is to raise inland dikes if scenarios \( s_L, s_H \) and \( s_E \) are included in the MR analysis. Note that exclusion of extreme scenario \( s_E \) from the set of scenarios results in a decision to invest in the storm surge barrier rather than to raise the inland dikes. This implies that MR decisions are sensitive to the set of scenarios considered.

MR analysis of flood protection measures is challenged by climate scenario choices. Including scenarios that cover the complete uncertainty set, i.e. “the smallest closed set such that the probability of the data to take a value outside of this set is zero” (Ben-Tal et al. 2009), might include highly unlikely, but catastrophic climate scenarios and could lead to excessive investment costs or extreme outcomes, such as ‘abandoning land’ (cf. Clarke 2008; Lonsdale et al. 2008). Instead, ‘plausible high-end’ climate change impact scenarios can be used for the analysis, as has been suggested for the updating of FRM strategies (Katsman et al. 2011).

In addition, the set of ‘plausible’ climate change impact scenarios may be subject to change over time. Examples are recent reports of larger uncertainty ranges of sea level rise and extreme rainfall than previously reported (Wahl et al. 2013; KNMI 2014). In the long run, more extreme value observations and scientific progress may reduce or resolve climate uncertainties (cf. Baker 2005;
Khaliq et al. 2006). When new information emerges, anticipated regrets may change and investments can be adapted accordingly.

So far, few authors have discussed the dynamic application of the MR decision criterion with emerging information, and focus has mainly been on theoretical problems (Krähmer and Stone 2005; Hayashi 2011). To our best knowledge, no dynamic application of the MR decision criterion to FRM cases has been reported in the literature. This paper shows how the MR decision criterion can be applied dynamically to analyse practical FRM investment problems under climate change, and the important differences between static and dynamic MR analysis due to the possible emergence of new information. First, a procedure is developed to implement the MR decision criterion in a time-consistent manner building on the work of Hayashi (2011). Next, a conceptual FRM model is developed applying the dynamic MR procedure.

The structure of this paper is as follows. Section 2 provides a brief motivation for MR analysis. Section 3 introduces the MR decision criterion formally and develops a consistent procedure for its dynamic implementation. Section 4 describes a conceptual FRM model, and applies the dynamic MR procedure to this model. Section 5 presents results. Section 6 concludes and discusses implications of our findings.

2. Motivation

Management strategies are robust when they perform relatively well across a range of possible future states (Lempert et al. 2006). Robustness methods, including MR applications, are generally motivated by the presence of ‘deep’ uncertainties (Woodward and Bishop 1997; Dessai and Hulme 2007; Clarke 2008; Hine and Hall 2010; Green and Weatherhead 2014). Uncertainty is ‘deep’ when probability or outcome information is ambiguous or lacking. Decision-making under risk, in contrast, assumes known probabilities of possible outcomes (Hogarth and Kunreuther 1995). Climate change uncertainties have been classified as ‘deep’ (Kandlikar et al. 2005).
In this paper, robust solutions that minimise maximum regret are analysed (cf. Averbakh 2000). Regret is a context-dependent measure, because its value follows from outcome comparisons. Context-dependency implies that regret values may change under different scenarios or investment alternatives. As a result, no fixed relationship exists between a possible outcome and the objective variable assumed in standard economic theory (e.g. Yager 2004). Therefore, a decision-maker’s ability to anticipate regret induces axiom violations of expected utility theory (Loomes and Sugden 1982).

Minimisation of maximum regret implies an infinite aversion against worst-case regret for a given set of investment opportunities and scenarios. In flood risk management, decision-makers are often regret- or loss averse. The common use of safety margins reveals loss aversion. The ‘minimax’ criterion analyses maximum losses. However, ‘minimax’ solutions may be associated with large regret, which advocates MR analysis. Regret arises both from under- and overinvestment. For example, a ‘very high’ dike may be robust to virtually any climate change impact scenario with barely any flood risk remaining. Therefore, the largest dike investment possible minimises maximum losses. Yet, this is not a low-regret solution, as these investment costs will largely outweigh damage reduction under any scenario (cf. Brekelmans et al. 2012).

MR analysis has several practical advantages over other robustness and expected-value methods. Application of the MR criterion does not require probabilistic information. The use of subjective probability distributions in expected-value approaches is controversial, and might result in ‘bad’ adaptation decisions (Hall 2007). It is generally considered easier to formulate a range of scenarios, for example by setting parameter intervals, than to attach probability distributions to these intervals. Furthermore, the MR decision criterion does not rely on arbitrary scenario weights such as the Laplace decision criterion, which uses equal scenario probabilities, the Hurwicz decision criterion, which employs weights for pessimism and optimism, or combined decision criteria (e.g. Gaspars-Wieloch 2013). Moreover, MR analysis usually yields a unique solution. More recent robust decision-making methods, such as info-gap theory, give insights in the robustness of solutions under
different degrees of uncertainty (e.g. Hine and Hall 2010). Info-gap theory, however, does not prescribe which measures to implement.

3. Static versus dynamic regret

The MR decision criterion is mostly applied in static settings (Hayashi 2011). Let $C(s, z)$ be the Net Present Value (NPV) of the total costs of option $z$ under climate scenario $s \in \theta$. The maximum absolute regret ($R$) for mutually exclusive adaptation investment options $z, y \in Z$ is mathematically defined by (e.g. Kasperski 2008)

$$R(z) = \max_{s \in \theta} \{C(s, z) - \min_{y \in Z} C(s, y)\}. \quad (1)$$

The MR objective is given by

$$R^* = \min_{z \in Z} R(z). \quad (2)$$

Hence, the investment option that minimises maximum regret is

$$z^* = \arg\min_{z \in Z} \{R(z)\}. \quad (3)$$

Sequential repetition of MR analysis over time could lead to dynamically inconsistent decisions as discussed by Hayashi (2011). However, no procedure was provided to resolve this problem. Dynamic consistency “imposes that the sequence of choice dispositions of the decision-maker’s successive ‘selves’ has to be connected across date-events in a recursive manner” (Hayashi 2011). In other words, dynamic inconsistency arises when a decision-maker commits to an initial plan as his final, but would prefer to change it later on. Time-inconsistent decisions originate from context-dependency, as briefly explained in Section 2. Information arrival implies a change in scenarios. Therefore, regrets change on arrival of new information even if the scenario payoffs remain unchanged. This, in turn, may result in a change of the initial plan, which is dynamically inconsistent.

To address this problem, we suppose that a decision-maker is not only capable of anticipating regret, but also has the analytical capacity to anticipate future optimal decisions, i.e. future decisions
which minimise maximum regret for any hypothetical sequence of previous decisions and events. In contrast to the time-inconsistent implementation of Hayashi (2011), our method removes any future suboptimal decision by a backward induction procedure. We will first illustrate the procedure by means of an example and formalise it in Section 4.2.

Consider forward-looking regret in the two-period investment problem of Figure 1. At moment $t_0$, the decision maker either chooses to invest ($I_0$) or to postpone ($P_0$) investment. The set of climate scenarios at decision moment $t_0$ is $\theta_0 = \{s_{L,L}, s_{L,H}, s_{H,L}, s_{H,H}\}$. Under strategy $(I_0, P_1)$, i.e. invest at $t_0$ and postpone investment at $t_1$, total costs are assumed to be scenario-independent, which is displayed in the upper branch of the tree in Figure 1. Consider that at decision moment $t_1$ it will be known whether or not climate change is severe or less severe with scenarios $s_{H,L}, s_{H,H} \in \theta_1$ or $s_{L,L}, s_{L,H} \in \theta_1$, respectively. If the investment decision is postponed to moment $t_1$, one can either invest ($I_1$) at $t_1$ or postpone again ($P_1$).
Figure 1 Decision-event-outcome tree for a two-period investment problem. The sub-trees, marked (1) and (2), are solved to determine optimal decisions at decision moment $t_1$ given that the initial decision is to postpone the investment.

The anticipated regret at moment $t_1$ after decision $P_0$ and new information $s_{HL}, s_{HH} \in \theta_1$ does not depend on the costs (15) incurred up to moment $t_1$, but only on the consecutive outcome changes from the decision at $t_1$ (respectively, $+13, +10, +25, +18$). The reader can verify that the minimising maximum regret decision at moment $t_1$ is $I_1$ under climate information $s_{HL}, s_{HH} \in \theta_1$, and $P_1$ under information $s_{LL}, s_{LH} \in \theta_1$. This result is obtained from the static application of the MR decision criterion to sub-trees (1) and (2) in Figure 1 respectively.
The initial optimal decision is derived from deleting outcomes of sub-optimal decisions from the next period. In the example, outcomes of decision $P_1$ under scenarios $s_{HL}, s_{HH} \in \Theta_1$, and outcomes of decision $i_1$ under scenarios $s_{LL}, s_{LH} \in \Theta_1$ are removed. This results in the decision tree displayed in Figure 2. Applying MR Eqs. (1) and (2) to this problem leads to the optimal decision at $t_0$ to postpone investment. This is followed by investment at $t_1$ under climate information $s_{HL}, s_{HH} \in \Theta_1$, and no investment under climate information $s_{LL}, s_{LH} \in \Theta_1$. Note that this investment plan is dynamically consistent, because the decision-maker will stick to the initial plan throughout the time horizon under every course of events.

![Decision Tree](image)

**Figure 2** Decision-tree after removal of non-optimal branches of the second period
4. A flood risk management application

Section 4.1 describes a conceptual flood risk management (FRM) model for a stylised FRM investment problem. The dynamic MR procedure explained in the previous section is applied to this model in Sections 4.2-4.3.

4.1 A conceptual flood risk management model

Consider the problem of increasing river peak flows due to climate change and a river dike protecting agricultural land as well as urban area. The current river dike is expected to provide less flood protection in the future due to the impact of climate change on peak flows. A quick-scan of possible adaptation options suggests two alternative adaptation strategies: either raising the existing river dike, or creating detention compartments on agricultural land. In the latter case, agricultural land can be deliberately flooded in order to attenuate peak flows to prevent flooding of the downstream urban area. However, damages arise from yield losses when detention area is used. Figure 3 illustrates the problem setting studied in the remainder of this paper.

![Figure 3 Graphical representation of the problem setting for two detention compartments](image)

Figure 3 displays the length \( l_{\text{max}} \) and the width \( w_{\text{max}} \) of the agricultural land. The urban area is denoted by \( E_1 \), and the total area of agricultural land is denoted by \( E_2 \). Upstream of the urban area, the agricultural land can be used to create one or more rectangular detention compartments next to the existing primary dike. We assume that the existing dike can be used as one of the sides of the detention compartments. Detention compartment lengths are denoted by \( l_k \), with \( k = 1,2,\ldots,K \)
and \( l_k \leq l_{\text{max}} \), and compartment widths are denoted by \( w_k \) with \( w_k \leq w_{\text{max}} \). The compartments can be flooded in a cascading order such that the agricultural area that has to be flooded in order to prevent flooding of the urban area is minimised.

We study a two-period investment problem with an infinite time horizon divided into three parts: \([t_0, t_1)\), \([t_1, T)\), and \([T, \infty)\). At decision moments \( t_0 \) and \( t_1 \) investment can take place in either the primary dike or in detention storage. In the sequel, we denote these decisions moments by \( t_j \) (\( j = 0, 1 \)). At the initial decision moment \( (t_0) \), three different peak flow projections are available: a low \((s_L)\), a medium \((s_M)\) and a high \((s_H)\) peak flow scenario. We denote the set of peak flow scenarios at decision moment \( t_j \) by \( \theta_j \). At \( t_0 \) the set of peak flow scenarios consists of three scenarios, \( \theta_0 = \{s_L, s_M, s_H\} \). Every peak flow scenario describes the development of an annual peak flow distribution, which is shifting over time due to climate change. The annual maxima of peak flows \( Q \) are distributed according to a Gumbel distribution, a subtype of the Generalised Extreme Value (GEV) distribution without shape parameter, with cumulative distribution function (Gumbel 1941)

\[
F_t(Q,s) = \exp \left[ -\exp \left\{ -\frac{Q - \mu_t(s)}{\sigma_t(s)} \right\} \right],
\]

where \( \mu_t(s) \) is the location parameter, the mode, and \( \sigma_t(s) \) is the scale parameter for peak flow scenario \( s \) at year \( t \). Hence, a peak flow scenario \( s \) defines distribution parameters of the annual peak flow distribution for any year \( t \): \((\mu_0(s), \mu_1(s), \ldots; \sigma_0(s), \sigma_1(s), \ldots)\). Note that mean and variance follow from the location and scale parameters (e.g. Forbes et al. 2011).

It is not only uncertain how peak flows will develop, but it is also hard to predict whether or not peak flow uncertainty will be reduced in the future. The future range of peak flow projections depends on future peak flow observations and new insights from improved climate models. We model possible futures by three ‘learning scenarios’, each represented by one or more information sets at \( t_1 \):

- ‘no learning’ scenario: the set of peak flow scenarios remains the same as today, with no-learning information set \( \theta_1 = \{s_L, s_M, s_H\} \),
• ‘uncertainty reduction’ scenario: the set of peak flow scenarios becomes smaller, either $s_L$ or $s_H$ disappears from the original set, with uncertainty reduction set $\theta_1 = \{s_L, s_M\}$, or $\theta_1 = \{s_M, s_H\}$.

• ‘uncertainty resolution’ scenario: complete knowledge on the development of the annual peak flow distribution is obtained, with uncertainty resolution set $\theta_1 = \{s_L\}$, $\theta_1 = \{s_M\}$ or $\theta_1 = \{s_H\}$.

To capture the notion of information sets, consider information superset $\bar{\theta}_j$, which contains possible information sets at decision moments $t_j$. At decision moment $t_0$ the superset contains only one set as $\theta_0$ is given. At moment $t_1$, the information superset contains the information sets from the different learning scenarios, i.e. $\theta_1 = \{\{s_L, s_M, s_H\}, \{s_L, s_M\}, \{s_M, s_H\}, \{s_L\}, \{s_M\}, \{s_H\}\}$.

Our conceptual FRM model contains a stage-discharge relationship and a peak flow attenuation function (cf. Westphal et al. 1999; Vis et al. 2003). These functions characterise the risk of flooding under different investment decisions. The model also contains simple investment cost functions and damage cost functions. The model is described as follows:

**Indices**

- Decision moment index $j = 0, 1$
- Detention compartment index $k = 1, 2, \ldots, K$
- Year $t = 0, 1, 2, \ldots, \infty$

**Data**

- Set of peak flow scenarios at decision moment $t_j$ $\theta_j \subseteq \bar{\theta}_j$
- Peak flow scenario describing an annual peak flow distribution over time $s \in \theta_j$
- Location parameter of annual peak flow distribution in year $t$ under scenario $s$ $\mu_t(s)$
- Scale parameter of annual peak flow distribution in year $t$ under scenario $s$ $\sigma_t(s)$
- Stage-discharge function coefficients $\alpha_1, \alpha_2$
- Storage-attenuation function coefficients $\beta_1, \beta_2, \beta_3$
- Investment cost function coefficients of primary dike $c_1, c_2, \lambda$
Investment cost function coefficients of detention storage \( d_1, d_2, d_3 \)

Damage value per unit of flooded urban area per flooding event \( p_1 \)

Yield loss per unit of flooded agricultural land per flooding event \( p_2 \)

Length of agricultural land \( l_{max} \)

Maximum dike height of the primary dike \( h_{max} \)

Width of agricultural land \( w_{max} \)

Total urban area \( E_1 \)

Total agricultural area \( E_2 \)

**Stock variables**

Dike height of primary dike at decision moment \( t_j \) before heightening \( h_j \)

Storage volumes of existing compartments \( k = 1, ..., K_1 \) at moment \( t_j \) \( x_{jk} \)

**Decision variables**

Dike increment of primary dike at moment \( t_j \) \( u_j \)

Storage volume of new compartment \( k = K_1 + 1, ..., K \) at moment \( t_j \) \( v_{jk} \)

Detention length of compartment \( k \) at moment \( t_j \) \( l_{jk} \)

Detention width of compartment \( k \) at moment \( t_j \) \( w_{jk} \)

**Objective function at \( t_0 \)**

Minimisation of maximum regret at the initial decision moment \( (t_0) \) is defined by

\[
R^* = \min\{R_A, R_B\},
\]

with \( R_A \) maximum regret under optimal investment in the primary dike (option A) and \( R_B \) under optimal floodplain investment in detention compartments (option B). Sections 4.2-4.4 describe how \( R_A \) and \( R_B \) are obtained.

**Constraints**

Detention compartment width and length:
\[ w_{jk} \leq w_{\text{max}} \quad \forall j, k \quad (6a) \]

\[ \sum_{k=1}^{K} l_{jk} \leq l \quad \forall j. \quad (6b) \]

Note that \( l_{jk} = 2w_{jk} \), which maximises the surface of a rectangular detention area for a given detention dike length, as one of the compartment sides is covered by the existing primary dike.

Detention volume is described by

\[ x_{jk} = 2w_{jk}^2g \quad \forall j, k \quad \Leftrightarrow \quad w_{jk} = \sqrt{\frac{x_{jk}}{2g}}, \quad (7) \]

where \( g \) is the ‘effective storage height’ capturing the distance between the critical height of the floodplain dikes and the datum, i.e. the reference level of the surface, which is assumed to be constant, and a fixed amount of storage in the subsurface per surface unit.

Dike height is characterised by

\[ h_{j+1} = h_j + u_j. \quad (8) \]

The case of mutually exclusive investment options is studied. At moment \( t_0 \) this is achieved by

\[ u_0 v_{0k} = 0 \quad \forall k, \quad (9a) \]

and at moment \( t_1 \) by

\[ h_1 v_{1k} = 0 \quad , \quad u_1 x_{1k} = 0 \quad \forall k. \quad (9b) \]
**Damage cost functions**

We assume that if river stage $S$ exceeds critical level ($\bar{S}$), the primary dike fails, otherwise it does not. In case of primary dike failure, flood losses are assumed to be constant and independent of inundation depth\(^2\)

$$D_1 = p_1 E_1 + p_2 E_2. \quad (10)$$

Annual use of detention compartments without primary dike failure gives damages from yield losses in the compartments approximated by:

$$D_2 \left( n(Q, x_{jk}), w_{jk}(x_{jk}) \right) = \sum_{k=1}^{n(Q,x_{jk})} 2w_{jk}^2 p_2,$$

where $n(Q, x_{jk})$ is the required number of detention compartments for a given peak flow event $Q$.

**Investment cost functions**

The investment cost functions of dike construction or heightening are represented by an exponential function described by Brekelmans et al. (2012). Dike increments are denoted by $u_j$, and the cost function of heightening the primary dike by

$$I_A(h_j, u_j) = \begin{cases} 
  c_1 + c_2 u_j e^{\lambda h_{j+1}} & \text{if } u_j > 0 \\
  0 & \text{if } u_j = 0 
\end{cases} \quad (12)$$

where $h_{j+1}$ is the relative dike height after heightening at $t_j$ with $h_0 = 0$, parameter $c_1$ the per kilometre heightening costs, $c_2$ the variable costs of heightening the primary dike and $\lambda$ the per centimetre incremental costs per kilometre of dike.

The storage cost function is represented by

\(^2\)In actual model applications more complex damage specifications can be included, e.g. damages depending on inundation depth and flood duration.
\[
I_B(v_{jk}) = \begin{cases} 
  d_1 + d_2 \sum_{k=K_1+1}^{K} v_{jk} + d_3 \sum_{k=K_1+1}^{K} \sqrt{v_{jk}} \quad \text{if } \sum_{k=K_1+1}^{K} v_{jk} > 0 \\
  0 \quad \text{if } \sum_{k=K_1+1}^{K} v_{jk} = 0 
\end{cases}
\]

where \( d_1 \) are the fixed costs of investment in detention storage, and \( d_2 \) is a linear cost parameter, which includes a cost estimate of average infrastructure protection and land purchase costs per unit of detention. Construction costs of the floodplain dikes surrounding the detention compartments are captured in \( d_3 \). This term reflects ‘economies of scale’, which implies that increasing the capital investment in detention results in a more than proportional increase in detention volume. Note that \( I_B(v_{jk}) \) does not depend on previously constructed detention compartments (indexed \( k = 1, \ldots, K_1 \)).

**Stage-discharge relationship**

Peak flow events result in increasing water levels, called surface water elevation, or stage. Stage can be studied in detail for peak flow events given the water bed form, roughness coefficients, and river bed elevation differences. An alternative is to identify a stage-discharge relationship, which can be directly applied to analyse FRM strategies (e.g. Hoekstra and De Kok 2008). Stage-discharge relationships are represented by a power function (e.g. Westphal et al. 1999)

\[
S = \alpha_1 Q^{\alpha_2} \quad \Leftrightarrow \quad Q = \left( \frac{S}{\alpha_1} \right)^{\frac{1}{\alpha_2}},
\]

where \( S \) is the surface water elevation, \( Q \) is the annual maximum peak flow discharge, and \( \alpha_1 \) and \( \alpha_2 \) are constants.

**Peak flow attenuation function**

In order to attenuate peak flows to a given peak flow base level \( Q_{\text{base}} \) required detention storage volume \( H(Q) \) is represented by a quadratic function of peak discharge \( Q \) (cf. Vis et al. 2003)

\[
H(Q) = \begin{cases} 
  \beta_1 + \beta_2 Q + \beta_3 Q^2 \quad \text{if } Q > Q_{\text{base}} \\
  0 \quad \text{if } Q \leq Q_{\text{base}} 
\end{cases}
\]

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4.2 The primary dike problem

We first derive optimal investment strategies for the primary dike. Consider the present value of the terminal costs $V_2$ from moment $T$ onwards. Define a cumulative distribution function $G_t(s, s)$, which is obtained by substituting Eq. (14) into Eq. (4). For the terminal condition we will assume that $\mu_t(s) = \bar{\mu}(s)$ and $\sigma_t(s) = \bar{\sigma}(s)$ from $T$ onwards. As a consequence, the annual flood probability $P_t(h, s)$ is constant on interval $[T, \infty)$ for a given dike height and peak flow scenario. Define this probability by $\bar{P}(h, s) = 1 - G_t(\bar{S}(h), s)$, where $\bar{S}(h)$ is the critical surface water elevation for a dike with height $h$. Terminal costs are given by

$$V_2(h, s, u = 0) = \sum_{t=T}^{\infty} \frac{\bar{P}(h, s)D_1}{(1 + \delta)^t} = \frac{\bar{P}(h, s)D_1}{\delta(1 + \delta)^{T-1}}$$

(16)

For a given dike height $h$, peak flow development scenario $s$ and dike increment decision $u$, the present value of the expected costs under scenario $s$ from decision moment $t_1$ onwards is

$$V_1(h, s, u) = \frac{1}{(1 + \delta)^{t_1}} I_a(h, u) + \sum_{t=t_1}^{T} \frac{P_t(h_2, s)D_1}{(1 + \delta)^t} + V_2(h_2, s, 0),$$

(17)

where $h_2 = h + u$. Given dike height $h$ and information set $\theta \in \Theta_1$, the MR decision at moment $t_1$ is

$$u^*(h, \theta) = \arg\min_{u \in [0, h_{\max} - h]} \max_{s \in \Theta} \left( V_1(h, s, u) - \min_{u \in [0, h_{\max} - h]} V_1(h, s, u) \right).$$

(18)

For the perfect learning case, the MR computation (Eq. 18) is trivial, as $\Theta_1$ contains only one scenario for this case ($s_L$, or $s_M$, or $s_H$). As a result, maximum regret is zero under the optimal strategy, which coincides with a deterministic cost minimising investment.

Once all regret minimising decisions $u^*(h, \theta)$ for decision moment $t_1$ have been identified, $V_1(h, s, u)$ is replaced by $V_1^*(h, \theta(s)) = V_1(h, \theta(s), u^*(h, \theta))$, i.e. any $u(h, \theta) \neq u^*(h, \theta)$ will not be implemented at decision moment $t_1$. If the primary dike is raised at the first decision moment ($t_0$) the optimal investment in the primary dike at this moment is
\[ u^* = \arg \min_{u \in [0,h_{\max}]} \max_{\theta_1 \in \Theta_1} \left( I_A(h = 0, u) + \sum_{t=0}^{t_1} P_t(u,s) D_1 \left( 1 + \delta \right)^t + V_1^*(h_1 = u, s(\theta_1)) \right) \]

\[ - \min_{u \in [0,h_{\max}]} \left( I_A(h = 0, u) + \sum_{t=0}^{t_1} P_t(u,s) D_1 \left( 1 + \delta \right)^t + V_1^*(h_1 = u, s(\theta_1)) \right). \] (19)

### 4.3 The floodplain problem

Next, we derive optimal investment strategies for the floodplain. Recall that \( n(Q, x_{jk}) \) is the number of required detention compartments for a given peak flow \( Q > Q_{base} \). The number of required detention compartments is determined by

\[
\sum_{k=1}^{n-1} x_{jk} < H(Q > Q_{base}) \leq \sum_{k=1}^{n} x_{jk},
\] (20)

where \( H(Q) \) follows from inserting \( Q \) in Eq. (15). Hence, the maximum peak flow \( Q \) for which the number of required detention compartments is equal to \( n \) is defined by

\[
Q_{up}(n) = \frac{1}{2\beta_3} \left( -\beta_2 \pm \sqrt{\beta_2^2 - 4\beta_3 (\beta_1 - \sum_{k=1}^{n} x_{jk})} \right). \] (21a)

The minimum peak flow for which \( n \) detention compartments are required is

\[
Q_{low}(n) = \frac{1}{2\beta_3} \left( -\beta_2 \pm \sqrt{\beta_2^2 - 4\beta_3 (\beta_1 - \sum_{k=1}^{n-1} x_{jk})} \right) + dQ. \] (21b)

The probability \( P_{nt} \) that the number of used detention compartments is equal to \( n \) under peak flow scenario \( s \) in year \( t \) is

\[
P_{nt}(s) = \Phi \left( Q_{up}(n), s \right) - \Phi \left( Q_{low}(n), s \right), \] (22)
with $P_{nt}(s) = \bar{P}_n(s)$ on interval $[T, \infty)$. Define $Q_{\text{max}}(K_1) = Q_{\text{up}}(n = K_1)$, which is the maximum peak flow whose water can be stored in the existing $K_1$ detention compartments without causing flooding of the urban area. The present value of the total expected damage costs under scenario $s$ on interval $[T, \infty)$ is

$$V_2(x_k, s, v_k = 0) = \sum_{t=T}^{\infty} \sum_{n=1}^{K_1} \left( \frac{P_n(s)D_2(n, w_k(x_k))}{(1 + \delta)^t} \right) + \sum_{t=T}^{\infty} \left( \frac{1 - F(Q_{\text{max}}(K_1))}{(1 + \delta)^t} \right) D_1,$$

(23)

where the first term of the right-hand side consists of expected damages due to the use of detention compartments to prevent flooding, and the second term contains expected damages due to failure of the primary dike.

The remainder of the floodplain problem follows the same lines as the primary dike problem. Given compartment volumes $x_k$ and information set $\theta \in \Theta_1$, the MR decision at moment $t_1$ is

$$v_k^*(x_k, \theta) = \arg \min_{v_k} \max_{s \in B} \left( V_1(x_k, v_k, s(\theta)) - \min_{u_k} V_1(x_k, u_k, s(\theta)) \right).$$

(24)

Again, $V_1(x_k, s, v_k)$ is replaced by $V_1^*(x_k, s(\theta)) = V_1(x_k, s(\theta), v_k^*(x_k, \theta))$ and is substituted in the MR equation for moment $t_0$. This results in the optimal initial investment in the floodplain.

4.4 Investment selection and threshold-to-switch

The choice for either investment in the primary dike or in floodplain development follows from the cost differences between the options given the optimal investment strategies under the different learning scenarios. Between-option regrets are calculated as follows:

$$R_A = \max_{\theta_1 \in \Theta_1} \max_{s \in \Theta_1} \{V_A(\theta_1, s) - V^*(\theta_1, s)\}$$

(25a)

$$R_B = \max_{\theta_1 \in \Theta_1} \max_{s \in \Theta_1} \{V_B(\theta_1, s) - V^*(\theta_1, s)\}$$

(25b)

where $V_A(\theta_1, s)$ and $V_B(\theta_1, s)$ are the total discounted costs under scenario $s$ and information set $\theta_1 \in \Theta_1$ for investment in the primary dike, and investment in detention compartments, respectively.
The maximum regret minimising investment option is obtained by substituting $R_A$ and $R_B$ from Eq. (25a) and Eq. (25b) in Eq. (5).

To examine differences in flexibility between the two investment options, we will also study the threshold-to-switch from investment in the primary dike, to investment in the detention compartments based on adaptation capital accumulation. For comparison, the average of the total discounted capital investments under the specified learning scenarios is used as a negative measure of flexibility, i.e.

$$\bar{K}_A = I_A(u_0^*) + \frac{1}{L} \sum_{i=1}^{L} \frac{I_A(h_1 = u_0^*, u_1^*(\theta_1(I)))}{(1 + \delta)^{t_1}}$$ (26a)

$$\bar{K}_B = I_B(v_{j=0,k}^*) + \frac{1}{L} \sum_{i=1}^{L} \frac{I_B(v_{j=1,k}^*(\theta_1(I)))}{(1 + \delta)^{t_1}}.$$ (26b)

where $L = |\theta_1|$, which is the number of learning scenarios. Next, the average total discounted capital is minimised under an acceptable maximum regret, where maximum regret is allowed to be higher than $R^*$. Thus,

$$\text{min}(\bar{K}_A, \bar{K}_B) \quad s.t. \quad R_A \leq (1 + \alpha)R^* \quad , \quad R_B \leq R^*(1 + \alpha).$$ (27)

Switching can occur if $\bar{K}_A < \bar{K}_B$ and $R_A > R_B$, or if $\bar{K}_B < \bar{K}_A$ and $R_B > R_A$. The switching threshold $\tilde{\alpha}$ is:

$$\tilde{\alpha} = \frac{|R_A - R_B|}{R^*}.$$ (28)

Thus, $\tilde{\alpha}$ reflects the amount of additional maximum regret as a fraction of $R^*$ that has to be allowed in order to reduce adaptation capital accumulation by switching between the investment options.

5. Implementation and results
In this section, results are presented of a numerical implementation of the conceptual FRM model. The results illustrate the effects of emerging information on optimal initial investment and on the optimal decisions after information arrival.\(^3\) Outcomes of static and dynamic MR analysis are compared.

5.1 Implementation

The conceptual model of Section 4 is calibrated with peak flow information and information on dike and detention investment options from the lower Rhine River for which data was readily available from the literature (cf. Vis et al. 2003; Hoekstra and De Kok 2008; Hurkmans et al. 2010). We use our own assumptions where appropriate. Baseline annual peak flow distribution of the river Rhine at gauging station Lobith are reported by Hoekstra and De Kok (2008). They describe an ‘extreme’ peak flow scenario in which the design discharge with a return period of 1250 years increases from 16000 m\(^3\)/sec. to 20000 m\(^3\)/sec. by the year 2100. We adopt this scenario as an upper scenario for 2100 and define two other peak flow scenarios for 2050 and 2100. We consider that the design peak flow of 16000 m\(^3\)/sec. today changes to 17000, 18500 and 19000 m\(^3\)/sec. in 2050 under scenarios \(s_L\), \(s_M\) and \(s_H\), respectively, and to 16700, 18000 and 20000 m\(^3\)/sec. in 2100. Note that the frequency of occurrence of peak flow extremes in the second half of this century decreases under scenarios \(s_L\) and \(s_M\) as compared to 2050. This is in line with the general findings reported in Hurkmans et al. (2010). Peak flow distribution parameter estimates are obtained for intermediate years by interpolation of the original and the scenario parameters (Kharin and Zwiers 2005).

Table 2 summarises the other parameters for the numerical case study. The investment cost function is taken from a Dutch dike ring at the Lobith-Westervoort-Doetinchem area (den Hertog and Roos 2008). Land use is assumed to be predominantly agricultural. Illustrative dimensions of the rural and urban areas are considered to describe the surface area of the case study. The effective distance

\(^3\) Note that optimal decisions after information arrival have been called ‘optimal recourse’ decisions in the operations research literature.
between the datum and the water table is assumed to be small, which implies the retention capacity in the subsurface to be limited. The dimensions of the agricultural land ($l_{\text{max}}$ and $w_{\text{max}}$) are calibrated such that the demand for storage capacity is not restricted by the area dimensions. In practice, this is not always the case at downstream river locations, for example, at the lower river Rhine.

Floodplain construction is usually relatively expensive in comparison with raising an existing dike. This is mainly due to the purchase of land and the costs of protection of infrastructure in the area (cf. Vis et al. 2003; Brouwer and van Ek 2004). We consider a Dutch average agricultural land price of 3.5 €/m$^2$, and consider that 5% of the floodplain area has to be converted at this price with no alternative use. Note that farmers are compensated for inundation damages, which enters the model through the damage function. A fraction of 0.7 of the land acquisition costs is used to represent the average infrastructure protection costs per m$^2$, for example to protect roads and bridges within a detention area.\footnote{These costs depend on local conditions. For example, in Brouwer and van Ek (2004) a fraction of 2445/1790=1.4 is reported.} This results in an estimate of 0.3 €/m$^2$, which is divided by the effective storage height ($g$), here assumed to be equal to 1 meter to obtain the value of parameter $d_2$. We consider that floodplain dikes are relatively inexpensive as compared to primary dikes, and assume 1.4 million €/km to calibrate parameter $d_3$. Fixed costs of detention are assumed to be lower than of the primary dike at 10 million €. A primary dike might involve more planning costs, because it stretches out over an entire dike ring (cf. den Hertog and Roos 2008). For flooding of residential and agricultural land constant value losses per square meter per flooding event are used, respectively, derived or taken from the literature (Vis et al. 2003; de Moel and Aerts 2011).
Table 2 Default calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Based on:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>$5.170 \cdot 10^3$</td>
<td>$m^3 s^{-1}$</td>
<td>Hoekstra and De Kok (2008): Rhine at Lobith</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>$1.519 \cdot 10^3$</td>
<td>$m^3 s^{-1}$</td>
<td>Hoekstra and De Kok (2008): Rhine at Lobith</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.7953</td>
<td>$m^{1-3\alpha_2}$</td>
<td>Hoekstra and De Kok (2008)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.3229</td>
<td>-</td>
<td>Hoekstra and De Kok (2008)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>$1.093 \cdot 10^{10}$</td>
<td>$m^3$</td>
<td>Vis et al. (2003): quadratic regression</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>$-1.53 \cdot 10^6$</td>
<td>$s$</td>
<td>Vis et al. (2003): quadratic regression</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>53.43</td>
<td>$m^{-3} s^2$</td>
<td>Vis et al. (2003): quadratic regression</td>
</tr>
<tr>
<td>$c_1$</td>
<td>$3.5625 \cdot 10^7$</td>
<td>€</td>
<td>den Hertog and Roos (2008): ring 48</td>
</tr>
<tr>
<td>$c_2$</td>
<td>$1.425 \cdot 10^6$</td>
<td>€ cm$^{-1}$</td>
<td>den Hertog and Roos (2008): ring 48</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.0063</td>
<td>cm$^{-1}$</td>
<td>den Hertog and Roos (2008): ring 48</td>
</tr>
<tr>
<td>$d_1$</td>
<td>$1.0 \cdot 10^7$</td>
<td>€</td>
<td>own assumption: see in-text explanation</td>
</tr>
<tr>
<td>$d_2$</td>
<td>0.3</td>
<td>€ m$^{-3}$</td>
<td>own assumption: see in-text explanation</td>
</tr>
<tr>
<td>$d_3$</td>
<td>4.0</td>
<td>€ m$^{-3}$</td>
<td>own assumption: see in-text explanation</td>
</tr>
<tr>
<td>$l_{max}$</td>
<td>$4.0 \cdot 10^4$</td>
<td>m</td>
<td>own assumption: see in-text explanation</td>
</tr>
<tr>
<td>$w_{max}$</td>
<td>$2.0 \cdot 10^4$</td>
<td>m</td>
<td>own assumption: see in-text explanation</td>
</tr>
<tr>
<td>$E_1$</td>
<td>$8.0 \cdot 10^7$</td>
<td>$m^2$</td>
<td>own assumption: see in-text explanation</td>
</tr>
<tr>
<td>$p_1$</td>
<td>75.60</td>
<td>€ m$^{-2}$</td>
<td>de Moel and Aerts (2011): residential value *damage factor (0.3)</td>
</tr>
<tr>
<td>$p_2$</td>
<td>0.11</td>
<td>€ m$^{-2}$</td>
<td>Vis et al. (2003)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.04</td>
<td>-</td>
<td>den Hertog and Roos (2008): risk-free rate + risk premium</td>
</tr>
<tr>
<td>$g$</td>
<td>1</td>
<td>m</td>
<td>own assumption: see in-text explanation</td>
</tr>
</tbody>
</table>

The decision space of dike height is discretised to steps of $\Delta u = 1$ cm with a maximum dike height of $h_{max}$. Dike increments of the primary dike ($u_t$), therefore, take values $0, \Delta u, \ldots, h_{max} - h_t$.

The detention volumes are discretised in steps of $\Delta v_k = 50 \ (10^6 m^3)$ and the construction of a maximum of two detention compartments per period (2x2) are allowed. We implemented and solved the problem in Matlab R2013a. Running times are modest (seconds to minutes) due to the specified number of decision moments and the coarse grid. The code is available upon request.

5.2 Results

Figure 4 displays the optimal dike height strategy for the uncertainty base case without learning, and optimal strategies for the specified learning scenarios. The optimal increment at $t_0$ is 55 cm for the uncertainty base case without learning, followed by an increment of 33 cm at $t_1$. This strategy is optimal under the static minimisation of maximum regret, in which it is assumed that the information set remains unchanged over time ($\theta_0 = \theta_1 = \{s_L, s_M, s_H\}$). Interestingly, the initial
optimal increment is 58 cm under the dynamic minimisation of maximum regret for the learning scenarios contained in $\Theta_1 = \{\{s_L, s_M, s_H\},\{s_L, s_M\},\{s_M, s_H\},\{s_L\},\{s_M\},\{s_H\}\}$. This result of an increase of the initial investment due to future learning is counter-intuitive at first sight. However, it is a consequence of both the timing of the second decision, which is in the year 2050, as well as the optimal decisions that would follow at this decision moment after information arrival, which are zero (i.e.: no heightening) if peak flow increase turns out to be low ($\Theta_1 = \{s_L\}$), or low or moderate ($\Theta_1 = \{s_L, s_M\}$). Under the higher peak flow scenarios $\{s_M, s_H\}$, $\{s_M\}$ and $\{s_H\}$, optimal increments are 48 cm, 37 cm, and 58 cm at $t_1$, respectively. As a result, total discounted costs and regret decrease under worst-case scenarios by increasing the initial investment as compared to the base case strategy without learning.

Figure 4 Optimal dike height strategy for the uncertainty base case without learning (static regret) and optimal strategies under a number of learning scenarios (dynamic regret)
The optimal floodplain development is to first invest in one detention compartment with a storage capacity of 250 million m$^3$ for the default calibration with economies of scale ($d_3 = 4.0$). Recall that parameter $d_3$ represents the economies of scale component of the investment cost function (Eq. 13). For this case, the reduction in damages from yield losses by the creation of a second detention compartment does not outweigh the investment costs to create it. The optimal decisions after information arrival are displayed in Table 3. The investment pattern is similar to the one found for the dike height problem. Under scenarios $\{s_L, s_M\}$, $\{s_L\}$ and $\{s_M\}$ and $d_3 = 4.0$ no second investment would be required. For this case, information that peak flow increase is moderate or high ($\{s_L, s_M\}$), or just high ($\{s_M\}$) results in a second investment of 250 million m$^3$ and 400 million m$^3$, respectively. When uncertainty is not reduced at $t_1$, an additional 150 million m$^3$ is needed. Without economies of scale ($d_3 = 0.0$) two unequally sized detention compartments would be constructed at the initial decision moment ($t_0$) with volumes of 100 million m$^3$ and 200 million m$^3$.

For any case, the total storage volume remains below the 2000 million m$^3$ that would be required at the end of the century to accommodate a peak flow event with a return period of 1250 years under peak flow scenario $s_H$. This implies that a higher flood probability will be accepted with time, which is caused by the relatively high investment costs. This effect is smaller when investment in the floodplain would be less expensive (for example, if $d_2 = 0$, or $d_3 = 0$), which results in larger optimal detention storage volumes; a total of 800 million m$^3$ of storage would be optimal under learning scenario $\theta_1 = \{s_H\}$ and $d_3 = 0$.

<table>
<thead>
<tr>
<th>$\theta_1$</th>
<th>$v_{12}^*$ ($x_k = 0, \theta_1$) (million m$^3$)</th>
<th>$v_{13}^*$ ($x_k = 0, \theta_1$) (million m$^3$)</th>
<th>$v_{13}^*$ ($x_k = 0, \theta_1$) (million m$^3$)</th>
<th>$v_{14}^*$ ($x_k = 0, \theta_1$) (million m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>${s_L, s_M, s_H}$</td>
<td>0</td>
<td>150</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>${s_L, s_M}$</td>
<td>0</td>
<td>0</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>${s_M, s_H}$</td>
<td>0</td>
<td>250</td>
<td>100</td>
<td>300</td>
</tr>
<tr>
<td>${s_L}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${s_M}$</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>${s_H}$</td>
<td>0</td>
<td>400</td>
<td>150</td>
<td>350</td>
</tr>
</tbody>
</table>
Table 4 displays a comparison of the NPVs of the total costs under different learning and peak flow scenarios associated with both investment options. Corresponding regret values are displayed as well. Based on the dynamic application of the MR criterion investment in the primary dike is the preferred option. This conclusion follows from the application of Eq. (5), i.e.: \( \min \{7.3; 11.0\} = 7.3 \).

**Table 4** NPV of total costs under different learning and actual scenarios and corresponding regret values in million €

<table>
<thead>
<tr>
<th>( \theta_1 )</th>
<th>Scenario</th>
<th>NPV dikes</th>
<th>NPV floodplain</th>
<th>( R_A^* )</th>
<th>( R_B^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( {s_L, s_M, s_H} )</td>
<td>( s_L )</td>
<td>288.7</td>
<td>285.1</td>
<td>3.6</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>( s_M )</td>
<td>360.1</td>
<td>361.4</td>
<td>0.0</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>( s_H )</td>
<td>420.2</td>
<td>428.2</td>
<td>0.0</td>
<td>8.0</td>
</tr>
<tr>
<td>( {s_L, s_M} )</td>
<td>( s_L )</td>
<td>278.2</td>
<td>271.2</td>
<td>6.9</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>( s_M )</td>
<td>366.5</td>
<td>359.2</td>
<td>7.3</td>
<td>0.0</td>
</tr>
<tr>
<td>( {s_M, s_H} )</td>
<td>( s_M )</td>
<td>360.7</td>
<td>362.2</td>
<td>0.0</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>( s_H )</td>
<td>411.2</td>
<td>422.2</td>
<td>0.0</td>
<td>11.0</td>
</tr>
<tr>
<td>( {s_L} )</td>
<td>( s_L )</td>
<td>278.2</td>
<td>271.2</td>
<td>6.9</td>
<td>0.0</td>
</tr>
<tr>
<td>( {s_M} )</td>
<td>( s_M )</td>
<td>359.5</td>
<td>359.2</td>
<td>0.3</td>
<td>0.0</td>
</tr>
<tr>
<td>( {s_H} )</td>
<td>( s_H )</td>
<td>409.9</td>
<td>419.0</td>
<td>0.0</td>
<td>9.1</td>
</tr>
</tbody>
</table>

Despite that the primary dike is the maximum regret minimising investment option, more adaptation capital accumulates over time when the primary dike is raised as compared to the option to invest in detention storage. Table 5 reports the Present Value (PV) of dike and detention investment costs under the different learning scenarios. Due to the postponement of investment, as well as due to the relatively high unit costs of detention and the resulting reduction in investment, total discounted investment costs of floodplain investment are relatively low.

**Table 5** Comparison of total discounted investment costs of the investment options

<table>
<thead>
<tr>
<th>( \theta_1 )</th>
<th>PV dike investments</th>
<th>PV floodplain investments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( {s_L, s_M, s_H} )</td>
<td>180.8</td>
<td>172.9</td>
</tr>
<tr>
<td>( {s_L, s_M} )</td>
<td>154.7</td>
<td>148.2</td>
</tr>
<tr>
<td>( {s_M, s_H} )</td>
<td>194.8</td>
<td>183.4</td>
</tr>
<tr>
<td>( {s_L} )</td>
<td>154.7</td>
<td>148.2</td>
</tr>
<tr>
<td>( {s_M} )</td>
<td>185.9</td>
<td>148.2</td>
</tr>
<tr>
<td>( {s_H} )</td>
<td>203.8</td>
<td>198.0</td>
</tr>
<tr>
<td>Average</td>
<td>179.1</td>
<td>166.5</td>
</tr>
</tbody>
</table>
Recall that the average of total discounted investment costs under the specified learning scenarios was defined as a negative measure of flexibility. The average PV of floodplain investments is lower ($\min\{179.1; 166.5\} = 166.5$). If a decision-maker would be willing to accept an additional maximum regret of $11.0 - 7.3 = 3.7$ million Euros, switching to investment in the floodplain would be optimal (Eq. 27). The threshold-to-switch is $100\% \times \frac{|7.3 - 11.0|}{7.3} = 51\%$ of $R^*$ (Eq. 28).

6. Conclusions and discussion

This paper presents a dynamic ‘minimax regret’ (MR) modelling approach for the analysis of flood risk management (FRM) investment problems under climate change. The approach supports the identification of adaptive FRM strategies by the inclusion of ‘learning scenarios’ about climate change impacts. We show how the MR decision criterion can be applied dynamically in order to analyse investments in flood protection. The important differences between static and dynamic MR analysis of FRM investments due to the possible emergence of new climate information are highlighted. The key message of this paper is that dynamic MR solutions are more robust than the solutions obtained from a static MR analysis of FRM investments due to ongoing changes in climate change impact projections.

In recent work the importance of the emergence of new information has been stressed for the successful adaptation to climate change, for example with the development of methods related to ‘adaptation tipping points’ and ‘adaptive pathways’ (cf. Kwadijk et al. 2010; Haasnoot et al. 2013). Unlike these methods, a dynamic MR analysis provides detailed economic advice on optimal management strategies.

Robustness concepts are normative in nature. In this paper, a ‘narrow’ definition of robustness is employed, i.e.: minimisation of maximum regret. However, other robustness approaches, such as info-gap theory and analytic robustness methods, may give complementary insights in FRM strategies that perform relatively well across a wide range of scenarios (cf. Lempert et al. 2006; Hine and Hall 2010).
So far, applications of the MR decision criterion have mostly been restricted to static settings (Hayashi 2011). Whereas static MR analysis provides insights in the ability of a flood protection measure or a system to remain functioning under scenarios of future disturbances, it is implicitly assumed that this set of scenarios does not change over time. Dynamic MR analysis, in contrast, incorporates future information, which improves the robustness of decisions over time (cf. Mens et al. 2011).

The FRM problem solved in this paper could also be addressed from the perspective of a social planner with an expected-value based cost-benefit optimisation procedure (van der Pol et al. 2014). However, this approach assumes risk-neutrality and requires information on the probabilities of future states and events. In Europe, cost-benefit analysis is predominantly applied for economic appraisal of flood protection and other adaptation measures, although its use is controversial (Turner 2007; Watkiss et al. 2014).

We have shown that the implementation of the dynamic MR decision is complex, and that a backward induction procedure is required to ensure dynamic consistency. This procedure, however, is computationally intensive. The computation time is determined by the number of learning scenarios and the number of decision moments. Even if the number of learning scenarios is constant over time, computation time is exponential in the number of decision moments. However, the presented case illustrates that this is no obstacle to the application of the dynamic MR procedure to FRM problems as long as the number of decision moments is small. In this paper, the setting was restricted to two decision moments, and mutually exclusive investment options. For further research it would be interesting to study a multi-period case with complementary investment options.

The inspection of average adaptation capital accumulation might be a useful extension to obtain insights in the overall flexibility of dynamic MR solutions. We argue that adaptation capital accumulation is risky, as invested capital can lose some or all of its value under new information on climate change impacts. The concept of ‘value-at-risk’ originates from finance (Linsmeier and
The value-at-risk, however, cannot be quantified in the absence of information on the likelihoods of value losses.

The conceptual FRM model presented in this paper is stylised regarding the cost functions and the risk of flooding. For example, fixed damages per flooding event were assumed independent of dike height, flood scenario and flood duration. Detention areas were assumed to be rectangular, and the infiltration potential was assumed to be constant independent of previous weather. For the dynamic MR approach to be applicable for decision support of real-world FRM investment decisions, the dynamic MR approach can be combined with a rainfall-runoff-inundation model.

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