On the measurement of household welfare: Marginal willingness to pay and to accept

Abstract
The paper defines marginal measures of individual and household willingness to pay and to accept in a general flexible framework. It investigates and clarifies the relationship between the household measures and the individual (household members’) welfare measures. The results prove helpful for the theoretical analysis of household behavior, for the evaluation of nonmarket goods in environmental and public economics and for contingent valuation.

Keywords: Welfare measurement, household, marginal valuation, willingness to pay, willingness to accept, contingent valuation

JEL-codes: D11, D13, Q51
1. **Introduction**

This paper deals with the measurement of welfare in environmental economics. It considers changes in an environmental good and their implications for the welfare of households. A household is interpreted as a group of individuals who have decided to live together and to act jointly in accordance with a collective decision rule. Therefore a household forms a (new and) separate unit and can in general not be treated as a group of single individuals. As a consequence also an appropriate welfare analysis is required. In principle one can distinguish between discrete and marginal changes. The present paper examines marginal changes in welfare. They are relevant in (at least) two situations: Firstly, if the change in an environmental variable is (very) small, the corresponding welfare change can be measured by the marginal change. This case is relevant for contingent valuation (see Mitchell and Carson (1989)). Secondly, for the determination of an optimal policy the first-order conditions (of the problem) have to be considered. Here in general marginal welfare changes are involved.

An individual’s marginal welfare measure is the limit of a discrete welfare measure when the change in the environmental good tends to zero. This claim is true for an individual’s marginal willingness to pay and to accept measure which can be derived as the limit of one of the Hicksian measures. For a household things are more complicated since we at first have to determine appropriate measures of household welfare. Below we will base our examination on the measures proposed in Ebert (2008) who defines household willingness to pay (to accept) by means of an optimization problem. The household member maximize (minimize) the sum of the amounts of income they can do without (needs) jointly in order to be as well off as in the original (new) situation before (after) the change. This measure forms the basis of the analysis and allows us to derive a corresponding marginal household welfare measure as a limit. For simplicity we confine ourselves to a household consisting of two members. It turns out that the marginal household measures of willingness to pay and willingness to accept are in general different. Furthermore, the household measures are weighted sums of the individual measures. The sum of the weights normally differs from the number of household members. This property seems to be at variance with the principle of individualism usually imposed. It requires that individuals are taken into account and are counted equally. But the weighting is an implication of and consistent with the fact that the household members are not singles and have decided to behave according to a collective decision rule.

It seems that there is only one paper in the literature dealing with marginal household welfare measures. Strand (2007) considers the evaluation of marginal changes in a public good for
two-person households. The households decide on consumption by Nash bargaining. The author derives some results for a particular class of strongly separable (cardinal) utility functions and also examines the implications of various forms of altruism. The present paper investigates the problem of defining marginal measures of household welfare and of comparing individual and household measures in a general ordinal framework. A large class of collective decision rules is considered. Altruism is not discussed.

The paper is organized as follows: Section 2 defines the framework and discusses welfare measures for individuals. Section 3 introduces households and household measures of welfare. In section 4 an example is presented and investigated in some detail. It forms the background and motivation for section 5. Here the marginal measures of household welfare are examined and characterized for the general case and also for two more particular classes of household decision rules. Section 6 offers some conclusions.

2. Individual welfare measures
To begin with we describe the framework. There are three types of goods: \( X = (X_1, \ldots, X_k) \) denotes the vector of private goods, \( Y = (Y_1, \ldots, Y_l) \) the vector of household goods (which are public for all household members), and \( Q \) a nonmarket good. \( X \) and \( Y \) comprise market goods; \( Q \) can be a public good provided by the government or an environmental good. We assume that \( k \geq 1, l \geq 0, \) and \( k + l \geq 2, \) i.e., there are at least two market goods. The prices of the market goods are given by \( p = (p_{X_1}, \ldots, p_{X_k}, p_{Y_1}, \ldots, p_{Y_l}) \). In this section we consider one individual. It possesses a preference ordering defined over commodity bundles \( (X, Y, Q) \in \mathbb{R}_{+}^{k+l+1} \). The ordering is supposed to be continuous and strictly convex. Moreover, it is strictly increasing in \( X, Y, \) and \( Q \). Let \( U(X, Y, Q) \) be a once continuously differentiable (direct) utility function which is strictly concave and represents the preference ordering. The individual’s income is denoted by \( M \). Utility maximization implies the continuously differentiable indirect utility function \( IV(p, Q, M) \). Employing Roy’s identity and duality theory we are able to derive the individual’s marginal willingness to pay for \( Q \). It is denoted by

\[
\frac{\partial IV(p, Q, M)}{\partial Q} \frac{\partial Q}{\partial M}.
\]

---

1 We use the abbreviation \( IV \) for the usual individual indirect utility function.
For the discussion below we want to demonstrate that marginal willingness to pay for \( Q \) is equal to the limit of the usual welfare measures for a nonmarginal (discrete) increase \( \Delta Q \) in \( Q \) if \( \Delta Q \) goes to zero. At first we introduce the Hicksian compensating variation \( CV \) for a discrete change in the nonmarket good from \( Q \) to \( Q_+ \) (we assume that \( Q < Q_+ \)). It is defined\(^2\) implicitly by

\[
IV(p, Q, M - CV(Q, Q_+)) = IV(p, Q, M)
\]

and represents the maximal amount of income the individual can do without after the increase in \( Q \) (from \( Q \) to \( Q_+ \)) if she is to be (at least) as well off as in the original situation, i.e., \( CV \) is the willingness to pay for \( \Delta Q \) (in the new situation). Here prices and income are kept fixed. The marginal change of the compensating variation is given by

\[
\frac{\partial CV(Q, Q_+)}{\partial Q_+} = \frac{\partial IV(p, Q, M - CV)/\partial Q_+}{\partial IV(p, Q, M - CV)/\partial M}
\]

(\text{use the implicit function theorem}). We therefore obtain the marginal willingness to pay for \( Q \) when we form the limit

\[
\lim_{Q_+ \to Q} \frac{\partial CV(Q, Q_+)}{\partial Q_+} = w(p, Q, M)
\]

since \( CV(Q, Q_+) \to 0 \) for \( Q_+ \to Q \).

Analogously we can consider the Hicksian equivalent variation \( EV \). It is defined implicitly by

\[
IV(p, Q, M + EV(Q, Q_+)) = IV(p, Q_+, M)
\]

and represents the minimal amount of income the individual has to receive in the old situation in order to be (at least) as well off as after the increase in \( Q \). The measure \( EV(Q, Q_+) \) can also be interpreted as the willingness to accept for \( \Delta Q_+ \) (if this change is not performed). We can determine the marginal change of \( EV \) by deriving

\[
\frac{\partial EV(Q, Q_+)}{\partial Q_+} = \frac{\partial IV(p, Q_+, M)/\partial Q_+}{\partial IV(p, Q, M + EV)/\partial M}
\]

and get the marginal willingness to accept as the limit

\(^2\) Since in the following the exogenous parameters \( p \) and \( M \) are not changed we do not mention them in the parameter list of \( CV \). See Ebert (1993) for the definition of individual measures of willingness to pay and to accept.
\[
\lim_{Q,\to Q} \frac{\partial EV(Q,Q_\ast)}{\partial Q_\ast} = w(p,Q,M)
\]

since \( EV(Q,Q_\ast) \to 0 \) for \( Q_\ast \to Q \).

In other words, \( w(p,Q,M) \) is equal to the marginal willingness to pay and to the marginal willingness to accept for \( Q \), if the individual faces the (exogenous) variables \((p,Q,M)\).

This discussion also demonstrates that willingness to pay and willingness to accept are in general not identical for a discrete change!

3. Household welfare measures

For the definition of household welfare measures we use the same framework as above and we follow Ebert (2008). For simplicity we assume that there are (only) two individuals \( i \) and \( j \) who form a household. (We also attach an index \( i \) and, respectively, \( j \) to the utility function etc. defined in section 2) In this case we have to introduce some assumptions about the household’s behavior (cf. e.g. Bergstrom (1997)). In order to keep things simple and to present a general analysis we do not go into any details. Instead we suppose that the two household members agree on a collective decision rule for the household. The household members have only their own income at their disposal. They contribute it to the household which spends the income in accordance with the decision rule. Therefore the decision rule and the members’ incomes determine the commodity bundles \((X_h,Y,Q)\), for \( h = i,j \), which individual \( i \) and \( j \) consume. For welfare measurement it is sufficient to know the utility levels attained. They are given by the collective indirect utility functions

\[
V_i(p,Q,M_i,M_j) := U_i(X_i,Y,Q) \quad \text{and} \quad V_j(p,Q,M_i,M_j) := U_j(X_j,Y,Q)
\]

and depend on the exogenous variables \( p \) and \( Q \), on all incomes, and on the collective decision rule. The decision process is not necessarily efficient. We assume that \( V_h \) is once continuously differentiable and strictly increasing in \( M_i \) and \( M_j \), for \( h = i,j \). Furthermore, we assume that \( V_h \) is concave in \((M_i,M_j)\). Concavity allows us to compare the utility levels achieved in particular situations. If two income distributions yield the same level of \( V_h \) then any mixture (i.e. convex combination) of these income distributions makes individual \( h \) better.

---

3 The incomes are ordered in the parameter list such that individual \( h \)’s income is the last parameter, for \( h = i,j \). Since the collective decision rule is kept fixed, it is not part of the list of arguments.
off. Such an income distribution is more balanced than the original ones, i.e. incomes are more equally distributed.

In analogy to the definition of $CV$ and $EV$ we can determine the individual willingness to pay and to accept $WTP_h$ and $WTA_h$ for an increase in $Q$ for $h = i, j$ by using the collective indirect utility functions. We define these measures for individual $i$ (and analogously for $j$) by

$$V_i(p, Q, M_j, M_j - WTP_i(Q, Q_j)) = V_i(p, Q, M_j, M_j)$$

and

$$V_i(p, Q, M_j, M_j + WTA_i(Q, Q_j)) = V_i(p, Q, M_j, M_j).$$

Their interpretation is obvious. Like $CV$ and $EV$ the measures $WTP_h$ and $WTA_h$ do in general not coincide. Furthermore, – given the household framework – we can derive $i$’s marginal willingness to pay and to accept by

$$MWTP_i(Q) = \lim_{\delta Q \to 0} \frac{\partial WTP_i(Q, Q_j)}{\partial Q} = \frac{\partial V_i(p, Q, M_j, M_j)}{\partial V_i(p, Q, M_j, M_j)} / \partial M_i$$

and

$$MWTA_i(Q) = \lim_{\delta Q \to 0} \frac{\partial WTA_i(Q, Q_j)}{\partial Q} = \frac{\partial V_i(p, Q, M_j, M_j)}{\partial V_i(p, Q, M_j, M_j)} / \partial M_i.$$  

Now $MWTP_h$ and $MWTA_h$ depend on the entire income distribution. The marginal measures are also identical (as in the case of a single individual), for $h = i, j$.

Finally we introduce the corresponding household measures$^4$ for a nonmarginal (discrete) increase $\Delta Q$ from $Q$ to $Q_j$ (cf. Ebert (2008)). Since we have to take into account the collective decision rule they also have to be based on the collective indirect utility function $V_h$. We define household willingness to pay by the solution to problem $HWTP$:

$$HWTP(Q, Q_j) := \max hwtp_i + hwtp_j$$

s.t. $V_i(p, Q_j, M_j - hwtp_j, M_j - hwtp_j) \geq V_i(p, Q, M_j, M_j)$, \hspace{1cm} (P_i)

and $hwtp_i \geq 0$, $hwtp_j \geq 0$.

$^4$ See Ebert (2008) for a definition of household measures in a more general framework. The approach used by Quiggin (1998) is different.
HWTP is the maximal sum of the amounts of income the household members can jointly do without in the new situation after the change in $Q$ if each household member is at least as well off as before the change. The underlying idea is the same as used above: maximal willingness to pay has to be constrained by the utility levels attained in the old situation. Furthermore, the amount each household member is willing to pay has to be nonnegative, i.e., any internal redistribution is excluded. The mutual interdependence between member $i$ and $j$ is taken into account. Problem HWTP can be solved as long as $\Delta Q$ is small enough. It is well defined since the constraints are concave.

Similarly we define household willingness to accept by the solution to problem HWTA:

$$HWTA(Q,Q,):=\min hwta_i + hwta_j$$

s.t. $V_i(p,Q,M_j + hwta_j,M_i + hwtp_i) \geq V_i(p,Q,M_j,M_i), \quad (A_i)$

$$V_j(p,Q,M_i + hwta_i,M_j + hwta_j) \geq V_j(p,Q,M_i,M_j) \quad (A_j)$$

and $hwta_i \geq 0, \ hwta_j \geq 0$.

HWTA is the minimal sum of the amounts the household members jointly have to receive in the original situation if they have to be at least as well off as after the change in $Q$.

In the definition of the household measures the constraints $(P_i)$-$\mathcal{(P)}_j$ and, respectively, $(A_i)$-$\mathcal{(A)}_j$ have to be satisfied simultaneously. It is clear that in the optimum at least one constraint is binding. It is possible that the other one is satisfied as a strict inequality, i.e., the corresponding household member may (still) be strictly better off. The nonnegativity constraints have also to be taken into account.

For a derivation of the marginal household measures we therefore introduce two conditions which make the following analysis easier:

**Property RCP [RCA]**

For all $Q$ there is an interval $I = (\underline{Q},\overline{Q})$ such that the set of constraints, which are binding in the solution to problem HWTP [HWTA], is the same for all $Q \in I$.

and$^5$

---

$^5$ The envelope theorem already implies differentiability in some, but not in all situations.
Differentiability DP [DA]

For all $Q$ there is an interval $I = (Q, Q')$ such that $HWTP(Q, Q') \ [HWTA(Q, Q')]$ is continuously differentiable in $Q$ for $Q \in I$.

Imposing these conditions we can define marginal household willingness to pay and to accept for $Q$ by:

$$MHWT_P(Q) = \lim_{\epsilon \to 0} \frac{\partial HWTP(Q, Q')}{\partial Q_\epsilon} \text{ and } MHWT_A(Q) = \lim_{\epsilon \to 0} \frac{\partial HWTA(Q, Q')}{\partial Q_\epsilon}.$$

Unlike $MWT_h$ and $MWTA_h$, the marginal household measures do not necessarily coincide as the example provided in the next section demonstrates.

4. Example

The computation of $MHWT$ [and $MHWT_A$] is complicated: In general one has to distinguish several cases which depend on the subset of the active constraints. Therefore it is helpful to consider an example as illustration and motivation:

We assume that there are two private goods $X_1$ and $X_2$ and set $p_{X_1} = p_{X_2} = 1$. Let household member $h$ possess the utility function

$$U_h(X_1, X_2, Q) = 2\delta_h X_1^{\gamma/2} X_2^{\gamma/2} + Q$$

for $\delta_h > 0$ and $h = i, j$.

We define a collective decision rule $D_\epsilon$ by supposing that the household members share their incomes: Each member receives the fraction $\epsilon$ of her own and $(1-\epsilon)$ of the partner’s income and then maximizes her utility (for $0 \leq \epsilon \leq 1$). Member $i$ then gets $M_i(\epsilon) = \epsilon M_i + (1-\epsilon) M_j$.

For $\epsilon = 1$ there is no redistribution. For $\epsilon = 0$ both incomes are interchanged. For $\epsilon = 1/2$ income is pooled.

Now we confine ourselves to $\delta_i < \delta_j$ and $0 < \epsilon < 1$, i.e. $j$’s marginal rate of substitution between good 1 (or good 2) and the environmental $Q$ is always higher than $i$’s rate.

Then we get the collective indirect utility function

$$V_h^*(p, Q, M_i, M_j) = \delta_h M_h(\epsilon) + Q \text{ for } h = i, j.$$
Since $WTP_h(Q, Q_r)$ is defined implicitly by

$$V^e_h(p, Q, M_i, M_h, WTP_h(Q, Q_r)) = V_h(p, Q, M_i, M_h)$$

we obtain

$$\delta_h \left( \varepsilon \left[M_h - WTP_h(Q, Q_r)\right] + (1 - \varepsilon)M_i \right) + Q_r = \delta_h \left( \varepsilon M_h + (1 - \varepsilon)M_i \right) + Q.$$ 

The definition yields

$$WTP(Q, Q_r) = (Q_r - Q)/(\delta_h \varepsilon) \text{ for } h = i, j.$$ 

Therefore, $WTP_j(Q, Q_r) < WTP_i(Q, Q_r)$ since $\delta_i < \delta_j$ by assumption.

In this example it is possible to derive $HWTP(Q, Q_r)$ explicitly. The solution of the optimization problem is presented in Table 1 where $\bar{\varepsilon} = \delta_j/(\delta_i + \delta_j)$ and $\bar{\varepsilon} = \delta_i/(\delta_i + \delta_j)$. It is easy to see that $HWTP$ is continuous in the parameter $\varepsilon$. We can distinguish four cases (A)-(D) and three transitional cases.

(A) If $\varepsilon$ is close to unity and greater than $\bar{\varepsilon}$, we have $HWTP > \max \{WTP_i, WTP_j\}$. Both constraints $(P_i)$ and $(P_j)$ are active and the contributions $hwtp_i$ and $hwtp_j$ are strictly positive (in the optimum of problem $HWTP$).

(A/B) For $\varepsilon = \bar{\varepsilon}$ we get a transitional case: $\frac{\varepsilon}{1 - \varepsilon} \cdot HWTP = WTP_j$ and $hwtp_j = 0$.

(B) When $\varepsilon$ decreases further, it seems to be possible that both constraints remain active. But then $j$’s contribution $hwtp_j$ becomes negative, i.e. $j$ would have to receive a transfer from $i$. Transfers within the household are a priori excluded by the definition of $HWTP$. Therefore we now obtain $hwtp_j = 0$; i.e., in this case $j$’s constraint $(P_j)$ is active, $i$’s constraint $(P_i)$ becomes inactive. Furthermore, $HWTP = \frac{\varepsilon}{1 - \varepsilon} \cdot WTP_j$. In this situation the factor $\varepsilon/(1 - \varepsilon)$ depends on the parameters $\delta_i$ and $\delta_j$. For $\varepsilon = \bar{\varepsilon}$ it is equal to $\delta_j/\delta_i > 1$ and can in principle be arbitrarily large.

(B/C) $\varepsilon = 1/2$ is also a transitional case: the household pools its total income. $HWTP = WTP_j = hwtp_i + hwtp_j$. There is no unique solution as far as the contributions $hwtp_i$ and $hwtp_j$ are concerned.
(C) If $\varepsilon$ is further decreased we get a kind of corner solution: $HWTP = WTP_j$ and $hwtp_i = 0$. Again both constraints $(P_i)$ and $(P_j)$ could bind, but then some redistribution of income would be required (which is again excluded by assumption).

(C/D) Finally, we obtain the transitional case $\varepsilon = \frac{1}{\varepsilon}$. Here i’s constraint also becomes active.

(D) If $\varepsilon$ is smaller than $\varepsilon$, both individuals’ constraints are binding and we have

$$HWTP < \min\{WTP_i, WTP_j\} = WTP_j.$$

This discussion illustrates the various cases which can occur in the computation of $HWTP$ for the collective decision rule $D_\varepsilon$ where $0 < \varepsilon < 1$. It turns out that $WTP_h$ is only part of the resulting expression for $HWTP$ if individual h’s constraint h is active. Collecting the results we observe that

$$HWTP = \begin{cases} 
\bar{\alpha}_i WTP_i + \bar{\alpha}_j WTP_j & \text{both constraints } (P_i) \text{ and } (P_j) \text{ active} \\
\bar{\alpha}_j WTP_j & \text{only constraint } (P_j) \text{ active}
\end{cases}$$

where $\bar{\alpha}_i = \bar{\alpha}_j = \varepsilon$, $0 < \bar{\alpha}_i + \bar{\alpha}_j < 2$, and $\bar{\alpha}_j \geq 1$.

Thus $HWTP$ is in general a weighted sum of the individual measures $WTP_i$ and $WTP_j$. The sum of the weights is not necessarily equal to the number of household members. Furthermore, when only one constraint is binding the weight (weakly) exceeds unity and may be large.

Now we turn to the marginal household measure: In our case the assumptions DP and RCP are satisfied. Therefore the marginal household measure is well defined, i.e., marginal household willingness to pay is a weighted sum of the marginal individual measures. Again, $MWTP_h$ is taken into account in the household measure explicitly only if constraint h is active.

The next section demonstrates that we obtain an analogous result for $MHWTP$ in the general framework.

For later use we also derive $HWTA$ for our Example. It can be explained in the same way as $HWTP$: 
\[ HWT(A(Q, Q_v), Q_v) = \begin{cases} 
\varepsilon (WTA_i + WTA_j) & \text{for } 1 > \varepsilon \geq \varepsilon

WTA_i & \text{for } \varepsilon \geq \varepsilon \geq \frac{1}{2}

\frac{\varepsilon}{1 - \varepsilon} WTA_i & \text{for } \frac{1}{2} \geq \varepsilon \geq \varepsilon

\varepsilon (WTA_i + WTA_j) & \text{for } \varepsilon \geq \varepsilon > 0
\end{cases} \]

\( HWTA \) allows us to determine the marginal household willingness to accept.

5. Marginal measures of household welfare

Household willingness to pay and to accept can be calculated explicitly for the example presented in the last section since the underlying optimization problems are linear. One cannot expect to derive an explicit solution for arbitrary preference orderings. Nevertheless it is possible to describe the structure of \( MHWTP \) and \( MHWTA \) precisely in the general case.

At first we concentrate on household willingness to pay and therefore we now consider the solution of problem \( HWTP \) in more detail. Assuming that \( \lambda_h \) denotes the Lagrangean multipliers associated with the constraints \( (P_h) \) for \( h = i, j \) we can derive the derivative of \( HWTP(Q, Q_v) \) by means of the envelope theorem. It yields

\[
\frac{\partial HWTP(Q, Q_v)}{\partial Q_v} = \lambda_i \frac{\partial V_i}{\partial Q_v} + \lambda_j \frac{\partial V_j}{\partial Q_v}
\]

\[
= \lambda_i \frac{\partial V_i}{\partial M_i} \left[ \frac{\partial V_i}{\partial Q_v} \right] + \lambda_j \frac{\partial V_j}{\partial M_j} \left[ \frac{\partial V_j}{\partial Q_v} \right] + \lambda_h \frac{\partial V_h}{\partial M_h}
\]

\[
= : \omega_i MWTP_i(Q_v) + \omega_j MWTP_j(Q_v)
\]

where \( \omega_h = \lambda_h \frac{\partial V_h}{\partial M_h} \) can be interpreted as a weight, for \( h = i, j \).

Here the functions have to be evaluated at the optimal solution \( (hwt_p(Q_v), hwt_p(Q_v)) \).

Assuming that the limit exists we then obtain

\[
MHWTP(Q) = \omega_i MWTP_i(Q) + \omega_j MWTP_j(Q),
\]

(1)

\[ ^6 \] As long as \( \lambda_i > 0 \) and \( \lambda_j > 0 \) the derivative is continuous in \( Q_v \).
i.e., marginal household willingness to pay for \( Q \) is related to the individual marginal measures. It can be represented as a weighted sum of the individual measures of marginal willingness to pay. At this point we do not know a lot about the weights: A weight \( \omega_h \) is strictly positive if and only if the Lagrangean multiplier \( \lambda_h \) is strictly positive, i.e. if and only if the constraint \((P_h)\) is binding. This outcome is consistent with our finding for the above example.

Fortunately the weights can be characterized more precisely when the underlying optimization problem \( HWTP \) is investigated. We obtain

**Proposition 1**

Assume that RCP and DP are fulfilled for \( Q < Q_+ \). Then the weights in (1) satisfy

(i) \( 0 \leq \omega_h \leq 1 \) for \( h = i, j \) if both constraints of \( HWTP(Q, Q_+) \), \((P)\) and \((P_j)\), are active for \( Q_+ \in (Q, \bar{Q}) \).

(ii) \( \omega_i \geq 1 \) and \( \omega_j = 0 \) \( [\omega_i = 0 \text{ and } \omega_j \geq 1] \) if only the constraint \((P_i)\) of \( HWTP(Q, Q_+) \) is active for \( Q_+ \in (Q, \bar{Q}) \).

Thus again the constraints for the utility levels in problem \( HWTP \) play an important role for the structure of \( MHWTP \). If both constraints are active the sum of the weights is bounded, \( 0 < \omega_i + \omega_j < 2 \). If only constraint \((P_i)\) is active the weight \( \omega_i \) is only bounded from below. Here the nonnegativity constraints do not have to be discussed explicitly. The Example in section 4 demonstrates that the sum of the weights can be equal to any value from the interval \((0,2)\). Furthermore \( \omega_i [\omega_j] \) can in principle be arbitrarily large if only \((P_i)\) \((P_j)\) is binding.

An explanation for these outcomes is given in the proof. Formally the weight \( \omega_h \) is related to the Lagrangian multiplier of constraint \((P_h)\). The proof therefore demonstrates that individual \( h \)'s willingness to pay is only relevant for the household welfare measure if a further increase of \( h \)'s contribution \( hwt \) to \( HWTP \) would violate \((P_h)\), for \( h = i, j \).

At first sight it is surprising that the weights are not equal to unity and their sum is not equal to the number of household members. In our example we get \( \omega_i = \omega_j = 1 \) as an extreme case if \( \varepsilon \) tends to unity. But for \( \varepsilon = 1 \) each individual gets \( M_h(\varepsilon) = M_h \), i.e. there is no longer any connection between both household members. Then each individual acts independently.
Given this result we conclude: The fact that the sum of the weights is strictly smaller than the number of household members is implied by the household’s collective action. The household members act jointly and form a separate (new) unit which is not simply the union of two individuals.

This discussion also indicates that the household members’ preferences and the collective decision rule have an impact on the magnitude of the weights. In the following we introduce two properties of the collective indirect utility functions which have implications for the weights. At first we consider

**Incentive Compatibility IC**

\[
\frac{\partial V_i}{\partial M_i}(p,Q,M_j,M_i) \geq \frac{\partial V_j}{\partial M_j}(p,Q,M_j,M_i)
\]

for all \((p,Q,M_j,M_i)\) and for \(i, j = 1,2\) and \(i \neq j\).

We call a decision process incentive compatible if a member’s utility gain is maximal when his own income is increased by one Euro. Although incomes are exogenous, the name of the principle seems to be justified. This property of the decision rule is relevant in the stage when the individuals agree on the collective household decision rule: If it is violated, a selfish individual will not join the household. Though we consider marginal utilities, the condition is (perfectly) ordinal. Furthermore, no interpersonal comparisons of utility are performed.

This property restricts the class of admissible collective indirect utility functions. We are able to establish:

**Proposition 2**

Assume that IC, RCP and DP are fulfilled. Then the weights in (1) satisfy \(1 \leq \omega_i + \omega_j < 2\).

In this case it turns out that the sum of the weights is bounded from below by unity. An investigation of the problem HWTP demonstrates that – given IC – the measure HWTP is always greater than the minimal individual willingness to pay of those members whose constraint is active. In other words, one can always determine \(WTP_i\) or \(WTP_j\) as a lower bound for HWTP. This property drives the above result.

Income pooling describes another particular way of household behavior. Then the distribution of income among the household members does not play any role; i.e., the level of the collective indirect utility function depends (only) on total household income. We introduce
Income Pooling IP

There is a function $\tilde{V}_i$ such that $V_i(p, Q, M_j, M_i) = \tilde{V}_i(p, Q, M_i + M_j)$ for all $(p, Q, M_i, M_j)$ and $i, j = 1, 2$ and $i \neq j$. This assumption postulates that the result of the decision process is independent of the income distribution. It does not necessarily require that the household members explicitly decide to pool their incomes. Furthermore, this definition does not imply that the household decision process can be represented by the unitary model of household behavior in which a household welfare function\(^7\) is maximized. It is well known that the collective indirect utility functions can locally depend on total household income in the noncooperative model of household behavior, when there is a household good (cf. Warr (1982), Bergstrom, Blume, and Varian (1986) or Itaya, deMeza, and Myles (1997)). Then, as long as each household member contributes to the provision of this good, the result is invariant with respect to any redistribution of income within the household.

In this case $HWTP$ is always equal to the minimal individual willingness to pay for $Q$ (cf. Ebert (2008) and Ebert’s (2010) comment on Munro (2005)). Therefore it is easy to see that we get

**Proposition 3**

Assume that IP, RCP, and DP are fulfilled. Then the weights in (1) satisfy: $\omega_i + \omega_j = 1$.

If the household pools income the sum of the weights is equal to unity. The household appears to behave like one individual. Then of course $MHWTP$ is equal to $MWTP_h$ for either $h = i$ or $h = j$; i.e. since in this case in general only one household member’s constraint is binding, only his marginal willingness to pay is relevant. As one has to expect, the results derived in section 4 are consistent with the general results: The collective decision process $D_\varepsilon$ is incentive compatible for $\varepsilon \geq 1/2$ and the household pools income for $\varepsilon = 1/2$.

Analogously to (1) we can determine the household’s marginal willingness to accept: Assuming that the limit exists we obtain

$$MHWTA(Q) = \omega_i MWTA_i(Q) + \omega_j MWTA_j(Q).$$

\(^7\) Browning, Chiappori, and Lechene (2006) and Apps and Rees (2009) also discuss this topic.
and

**Proposition 4**

Assume that RCA and DA are fulfilled. Then the weights in (2) satisfy

(i) \( 0 \leq \omega_h \leq 1 \) for \( h \neq i, j \) if both constraints of HWTA\((Q, Q_+), (A_i) \) and \((A_j)\), are active for \( Q_+ \in (Q, \bar{Q}] \).

(ii) \( \omega_i \geq 0 \) and \( \omega_j \geq 0 \) \([\omega_i = 0 \) and \( \omega_j \geq 1] \) if only constraint \((A_i)[(A_j)]\) of HWTA\((Q, Q_+)\) is active.

(iii) \( 1 \leq \omega_i + \omega_j \leq 2 \) if (IC) is satisfied.

(iv) \( \omega_i + \omega_j = 1 \) if (IP) is satisfied – as far as the weights are concerned.

The results are essentially analogous. The explicit solution of problem HWTA in section 4 is, of course, consistent with proposition 4.

The analysis in section 2 has shown that – when the marginal (individual) measures are concerned – marginal willingness to pay and to accept coincide. This is no longer the case for the household measures. Our example demonstrates that for \( \bar{\varepsilon} > \varepsilon > 1/2 \) we obtain

\[
MHWTP(Q) = \frac{\varepsilon}{(1-\varepsilon)} MWTP, (Q) = \frac{1}{\varepsilon} \left[(1-\varepsilon)\delta_j\right]
\]

and

\[
MHWTA(Q) = MWTA (Q) = \frac{1}{\varepsilon} [\varepsilon \delta_j] \]

and therefore also \( MHWTP(Q) < MHWTA(Q) \). Thus the question arises under what circumstances the marginal household measures are identical. Calling \( MHWTP \) [MHWTA] regular if in the solution of HWTP\((Q, Q_+)\) \([HWTA(Q, Q_+)]\) both constraints \((P_i)\) and \((P_j)\) \([(A_i) \) and \( (A_j)\)] are active, we are able to establish

**Proposition 5**

Assume that (RCP and DP) and (RCA and DA) are satisfied for \( Q < Q_+ \). Then

\[
MHWTP(Q) = MHWTA(Q)
\] if these measures are regular.

Thus for regular measures we do not have to distinguish between a household’s willingness to pay and willingness to accept. Both marginal household measures are identical.
6. Conclusion

The above analysis demonstrates that the marginal household measures are related to the marginal individual measures. But it also shows that marginal household willingness to pay for a nonmarket good $Q$ is in general not equal to, but less than the unweighted sum of the individual measures. This result is a consequence of the interdependence between both household members. The details depend on the collective decision rule.

This outcome is surprising at first sight. Since $Q$ is by assumption a public or environmental good which can be consumed by every individual without any rivalry one would expect that the household’s marginal willingness to pay for $Q$ is equal to the unweighted sum of the individuals’ marginal willingness to pay. This idea is also based on the postulate of individualism used in theoretical and applied economics. It postulates that for the measurement of welfare in society solely its members’ preferences count (cf. e.g., Harberger (1971) and Just, Hueth and Schmitz (2004)). Of course, the household measure is in principle based on the individual measures. But our analysis demonstrates that all household members are not necessarily taken into account and that a household member’s willingness to pay may be taken into consideration only partially. Therefore our results seem to be at variance with the principle of individualism which assumes that the individuals concerned act independently. This assumption is, however, violated by the households considered in this paper. Here it is always supposed that there is a common, collective decision process. The household members depend on one another and do no longer act independently. This kind of interdependence is a fundamental property of our model. It seems to be justified when several individuals are living in one household and behave on the basis of a common household decision rule. Thus the principle of individualism is no longer appropriate for welfare measurement when individuals form a household and the household unit acts in accordance with a collective decision rule.
References


Ebert, U. (2008), The relationship between individual and household welfare of WTP and WTA, Wirtschaftswissenschaftliche Diskussionspapiere V-310-08, University of Oldenburg, Oldenburg.


Munro, A. (2005), Household willingness to pay equals individual willingness to pay if and only if the household income pools, Economics Letters 88, 27-230.


<table>
<thead>
<tr>
<th>case</th>
<th>( \epsilon )</th>
<th>((P_i)) active</th>
<th>((P_j)) active</th>
<th>(hwtp_i)</th>
<th>(hwtp_j)</th>
<th>HWTP</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(1 &gt; \epsilon &gt; \overline{\epsilon})</td>
<td>X</td>
<td>X</td>
<td>&gt;0</td>
<td>&gt;0</td>
<td>(\epsilon(WTP_i+WTP_j))</td>
</tr>
<tr>
<td>A/B</td>
<td>(\epsilon = \overline{\epsilon})</td>
<td>X</td>
<td>X</td>
<td>&gt;0</td>
<td>=0</td>
<td>(\epsilon(WTP_i+WTP_j) = \frac{\epsilon}{1-\epsilon}WTP_j)</td>
</tr>
<tr>
<td>B</td>
<td>(\bar{\epsilon} &gt; \epsilon &gt; \frac{1}{2})</td>
<td>X</td>
<td>&gt;0</td>
<td>=0</td>
<td>(\frac{\epsilon}{1-\epsilon}WTP_j)</td>
<td></td>
</tr>
<tr>
<td>B/C</td>
<td>(\epsilon = \frac{1}{2})</td>
<td>X</td>
<td>(\geq 0)</td>
<td>(\geq 0)</td>
<td>(\frac{\epsilon}{1-\epsilon}WTP_j = WTP_j)</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>(\frac{1}{2} &gt; \epsilon &gt; \epsilon)</td>
<td>X</td>
<td>=0</td>
<td>&gt;0</td>
<td>WTP_j</td>
<td></td>
</tr>
<tr>
<td>C/D</td>
<td>(\epsilon = \epsilon)</td>
<td>X</td>
<td>X</td>
<td>=0</td>
<td>&gt;0</td>
<td>WTP_j = (\epsilon(WTP_i+HWTP_j))</td>
</tr>
<tr>
<td>D</td>
<td>(\epsilon &gt; \epsilon &gt; 0)</td>
<td>X</td>
<td>X</td>
<td>&gt;0</td>
<td>&gt;0</td>
<td>(\epsilon(WTP_i+WTP_j))</td>
</tr>
</tbody>
</table>

Table 1: Possible patterns of the solution for HWTP