MIXED SOURCE

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Abstract. We study competitive interaction between profit-maximizing firms that sell software and complementary goods or services. Firms choose between three business models: the proprietary model, the open source model, and the mixed source model. When a firm opens part of its software, users can access and improve the source code, which increases quality and value creation, but at the same time, it sets up a free open source competitor, which hampers the firm’s ability to capture value. We analyze three cases: monopoly, commercial firm vs non-profit open source project, and duopoly, and discuss the implications for optimal business strategy and equilibrium industry structure. We show that: (i) firms may become “more closed” in response to competition from an outside open source project; (ii) firms are more likely to open substitute, rather than complementary, modules to existing open source projects; (iii) when the products of two competing firms are similar in quality, firms differentiate through choosing different business models; and (iv) low-quality firms are generally more prone to opening some of their technologies than firms with high-quality products.

Keywords: Open Source, User innovation, Complementarity, Vertical Differentiation, Value Creation, Value Capture, Business models (JEL O31, L17, D43).

1. Introduction

As is well understood by now, commercial firms may benefit from participating in open source (OS) software development by selling complementary goods or services. For example, IBM sells consulting services and proprietary software that is complementary to the OS software that it develops, Red Hat sells subscription services, and Sun sells complementary hardware such as servers. In particular, the combination of OS software and proprietary extensions has grown into an important phenomenon. In fact, the expressions mixed source and hybrid source have been used to refer to a business model whereby a software firm releases an open source version of its software and derives revenue from selling proprietary complementary code. Examples include: JasperSoft (business intelligence software), Zimbra (server software for email and collaboration), SugarCRM (customer relationship management...
software), Hyperic (systems monitoring, server monitoring, and IT management software),
xTuple (enterprise resource planning software), Zenoss (enterprise IT management software),
Talend (data integration software), and Groundwork (IT management and network moni-
toring software).

Even fervent advocates for proprietary software have jumped on the bandwagon. Having
stated in 2001 that “open-source is an intellectual property destroyer [...] I can’t imagine
something that could be worse than this for the software business,” Microsoft has recently
switched course to embrace the notion of mixed source. Amongst other initiatives, it has
partnered with Novell to put some of Microsoft’s technologies on Linux and other open
platforms. The Mono project, for example, consists of porting the .Net framework onto
Linux and the Moonlight project is about providing an offer of Silverlight for Linux. And in
July 2009, Microsoft agreed to contribute some of its technology to Linux under a licensing
agreement that allows developers outside Microsoft to modify the code.

Open source has the potential to improve value creation because it benefits from the
efforts of a large community of developers. Proprietary software, on the other hand, results
in superior value capture because the intellectual property remains under the control of
the original developer. Industry observers, however, point out that strict open source and
proprietary approaches to software development “don’t work in a world where innovators
have to innovate, investors need a profit, employees need to eat, and customer needs must
be met.” The reasoning is that proprietary development leads to little innovation and open
source leads to little profits. According to Horacio Gutierrez (Microsoft’s Deputy General
Counsel for IP Licensing): “striking a balance between [embracing open-source software and
brandishing patents] is one of the key things every commercial technology company must do
in order to compete effectively.” Figure illustrates the “popular case” for mixed source.

Oracle’s April 2009, $7.4 million acquisition of Sun can be understood by reference to the
figure. Sun’s most important software products were the Java programming language and the
Solaris operating system. On November 13, 2006, Sun released Java as open source under the
GNU General Public License. Likewise, a large part of Solaris’s codebase was open sourced

1The quote is by James Allchin (Microsoft’s executive in charge of Windows 2000 rollout). See Ben Charny,
print/38795.
2Silverlight is a Web-based digital video technology by Microsoft. It is a plug-in for delivering media and
interactive applications for the Web.
3Microsoft announced the release of 20,000 lines of device driver code to the Linux community. See http://www.marketwatch.com/story/microsoft-defends-nuanced-open-source-approach.
in 2005. Industry observers claimed that Sun’s excessive openness prevented the company from capturing sufficient value. On the other hand, most of Oracle’s products, such as the Oracle Database or the Oracle Fusion Middleware, were proprietary. The combination of Sun and Oracle created a mixed source corporation aiming a better balance between the potential for value creation and the potential for value capture as shown in Figure 1. The choice of Sun was quite natural because the Fusion Middleware was built on Java and Solaris was the leading platform for the Oracle Database. Both firms’ main products were highly complementary.

While the straightforward rationale for mixed source presented in Figure 1 is appealing, the desirability (or lack thereof) of the mixed source business model is more nuanced. An implication of Figure 1 is that software firms always benefit from mixed source. However, we do not see every software firm opening (a portion of) its technologies. The question arises: under what circumstances should a profit-maximizing firm adopt a mixed source business model? And for firms considering the adoption of a mixed source business model, which technologies should be open and which should remain closed/proprietary? Should the core software be opened or is it better to open the extensions?

Likewise, the popular case for mixed source does not consider how business models of different industry players interact. While the adoption of mixed source may be optimal when the firm is alone in the market, competition may render this business model suboptimal. A question that has not yet been addressed in the literature is: how should firms respond to competitors’ adoption of mixed source business models?
Finally, the argument for mixed source of Figure 1 provides little advice on how firms that adopt that business model should price their products. What are the right pricing structures under mixed source compared to the proprietary business model? In this paper, we present a formal model that addresses these questions.

We set up a game-theoretical model where profit-maximizing firms that sell software and complementary goods or services (such as training or support services) must choose whether to open all or part of their technologies and the prices at which to sell their goods. The software that the firms offer is composed of two modules: a base (core) program and a set of extensions. The base program may be used without the extensions. The extensions, on the other hand, are valueless unless they are used in conjunction with a base program, i.e. the base program is an essential complement to the extensions (see Chen and Nalebuff 2006).

Firms may open the base, the extensions, or both.

Consistent with the terminology used in the software development community, we refer to the different degrees of openness as business models. Thus, we consider three business models. In the proprietary model, all the software offered by the firm is closed, and in the open source model, both modules are open. In the mixed source model, one module is open and the other one is closed. The mixed source model has two variants. In the open core model, the base program is open and the extensions closed, and in the open extensions model, the base program is closed and the extensions open.

Figure 2 shows examples of the different business models that we consider. Because the four business models are implemented in reality, it is important to provide a theoretical framework to guide the design of adequate business strategies.

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Figure 2. Examples of the different business models.

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6More generally, the extensions represent not only software modules, but all those technologies and protocols which have value only if used with the firm’s core software.
The trade-off in our model builds on Figure 1. When a module is opened, users can access and improve the source code, which increases quality and value creation. At the same time, however, the firm is setting up an open source competitor of its program which may cannibalize the revenues of the proprietary version. This hinders the firm’s ability to capture value. We study how this trade off is affected by: (i) the value of complementary services that the firm may offer, such as support for software implementation or tailoring the software to customer specifications; (ii) the strength of user innovation; and (iii) competitors’ responses, which include strategic reconfigurations in competitors’ business models and tactical price adjustments.

We present our model and analysis in three parts, each building on the previous one. In Section 3, we analyze the case of an incumbent firm that enjoys a monopoly in its market segment (i.e., there are no open or commercial substitutes for the firm’s software) and that sells its product (a combination of software and a complementary good or service) to a measure one of potential customers. We refer to the product sold by the monopolist as the commercial product. The incumbent monopolist chooses the business models through which it would like to operate. As long as the base module is kept proprietary, the firm remains a monopolist. The firm can benefit from user innovation by opening some (or all) of its software modules. However, when the base module is opened a free, open source product emerges. We refer to the open source product set up by the monopolist when opening the base module as the OS product. The willingness to pay for the OS product is lower than that for the commercial product because it does not come bundled with the complementary good or service.

We find that when the value of the complementary good ($z$) and the strength of user innovation ($\sigma$) are low, the firm will open the extensions only. For intermediate values of $z$ and $\sigma$, the firm prefers to open the base module instead of the extensions. For $z$ and $\sigma$ sufficiently large, the firm will open both modules. In the first two cases, the firm prefers to compete through a mixed source business model, and in the third case, the firm is using the pure open source business model. The intuition is simple. Large $z$ increases the vertical differentiation between the commercial product and the OS product. Therefore, the business stealing effect is mild in this case. Likewise, when $\sigma$ is large, the quality improvement that occurs when modules are opened, outweighs the negative competitive effect.

In Section 4, we consider the case in which the firm faces competition from an external open source project. This means that consumers may use a free open source product, even if the firm decides to compete through a proprietary business model. We allow the firm to replace any one of its modules with the available open source module/s if it desires to do so.

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7This effect is sometimes referred to as demand-side learning. See Casadesus-Masanell and Ghemawat (2006).
We find that the firm may become “more closed” in response to competition from an outside OS project: compared to the monopoly situation, the firm is more likely to use a proprietary business model. In particular, the firm will be wary of opening the set of extensions when the outside OS project has a base module. A combination of the outside base module and the firm’s extensions result in a stronger free competitor.

When considering which mixed source business model to employ, the firm will always prefer the one that adds less value to the outside OS project. Thus, the firm is more likely to open substitute, rather than complementary, modules to the outside OS project.

Finally, we find that the firm will prefer to adopt the modules developed externally when these are of higher quality than the firm’s own modules. When the outside OS project has modules of higher quality than those developed internally by the firm, the trade off between value creation and value capture of Figure 1 is broken: increased openness not only results in superior value creation but also in better value capture.

In Section 5 we study a profit-maximizing duopoly (firms $H$ and $L$) which, in addition to prices, choose the business models through which they would like to compete. We assume that firm $H$ has modules of higher quality than firm $L$. We analyze the following game: (i) in the first stage either $H$ or $L$ chooses business model, (ii) in the second stage, the other firm observes the first mover’s business model and chooses its own, (iii) finally, both $H$ and $L$ simultaneously determine the prices of their commercial packages. This section studies competition through business models: firms best responding to each other’s choices of business models, taking into account the equilibria of the subgames that different business models combinations set up.

We find that when the exogenous quality difference between $H$ and $L$ is low, one of the two firms competes through a proprietary business model and the other opens one module, generally the extensions. As the quality difference grows, cannibalization concerns lessen and both competitors elect to compete through the same mixed source business model. Likewise, when user innovation $\sigma$ is low, only one of the two firms competes through a mixed source business model. The other firm is proprietary. And when $\sigma$ is high, the equilibrium has both firms adopting a mixed source model.

We also find that both firms may prefer to compete through the proprietary business model when $H$ is the first mover. This business model profile, however, never occurs when $L$ is the leader. Thus, the low quality firm is more interested in competing through a mixed source business model than the high quality firm. Finally, we also show that quality leapfrogging may occur in equilibrium when $L$ moves first but that it never happens when $H$ leads.

Our theoretical model provides a formal base on which to build recommendations for management practice. In Section 6 we present simplified versions of our main results through
easily accessible matrixes. In each case, we discuss what drives our propositions and lemmas and present policy recommendations that can be directly applied to the design of optimal business strategies. We also discuss the implications of our findings for the strategy literature, particularly as they relate to the subfield of value-based business strategy.

The rest of the paper is organized as follows. We first position our contribution in relation to the literature. In Section 2 we introduce the basic elements of our model. Section 3 presents the monopoly benchmark. In Section 4 we analyze competitive interaction between a profit-maximizing firm and a non-strategic OS competitor. The more challenging case where the business models of two strategic, for-profit companies are endogenously determined is presented in Section 5. Section 6 discusses the managerial implications and relates our results to a number of real-world examples. Section 7 concludes. The proofs of all propositions and lemmas are in Appendixes A and B. For completeness, in Appendix C we present the analysis of a few additional cases that are less relevant in practice.

1.1. Related literature. Our paper contributes to the literature on the economics of open source. For the most part, early papers on OS were concerned with explaining why individual developers contributed to OS projects, allegedly for free (see Lerner and Tirole (2005) and von Krogh and von Hippel (2006) for excellent surveys). The most common explanations were: altruism, personal gratification, peer recognition, and career concerns. Bagozzi and Dholakia (2006), for example, demonstrate that participation in OS development is partly explained by social and psychological factors and Roberts, Hann, and Slaughter (2006) find that status and career concern motivations significantly influence developers’ levels of participation. Baldwin and Clark (2006) argue that the architecture of the code may affect the developers’ incentives to contribute. Specifically, they show that a modular codebase mitigates free riding in OS development.

While the contributions of individual developers have played a crucial role in the growth of open source software, the same is true of contributions by commercial firms. In fact, in a carefully executed empirical piece, Bonaccorsi, Rossi, and Giannangeli (2006) show that fully proprietary and fully open software firms are rare. Instead, firms typically adopt a hybrid business models. Shah (2006) investigates the effects of sponsorship of open source projects by commercial firms and finds that voluntary developers tend to contribute less, have different motivations for contributing, and take on fewer code maintenance tasks than in the absence of such sponsorship.

On the theory front, a handful of papers have introduced profit maximizing OS firms (Henkel 2004, Bessen 2006, Schmidtke 2006, Haruvy, Sethi, and Zhou 2008). In these papers, commercial firms may profit from OS by selling complementary goods and services. However,
these works consider OS firms in isolation, rather than in competition against proprietary firms. In fact, the interaction between open source and proprietary firms has been identified as an important topic for further research by Lerner and Tirole (2001, 2002, 2005).

The presence of complementarities has been documented by Fosfuri, Giarratana, and Luzzi (2008). The authors perform an econometric analysis and find that firms with a larger stock of hardware patents and trademarks are more likely to participate in OS.

The papers that study competition between the two development paradigms set up duopoly models of a profit-maximizing, proprietary firm and a community of not-for-profit/non-strategic OS developers that sell at zero price (Mustonen 2003, Bitzer 2004, Gaudeul 2005, Casadesus-Masanell and Ghemawat 2006, Economides and Katsamakas 2006, Lee and Mendelson 2008). In these papers, however, the business model choice by firms and the market structure are exogenously given. We endogeneize these constructs.

An important precedent for our paper is Llanes and de Elejalde (2009) which presents a model where profit-maximizing firms decide whether to be open source or proprietary. Firms profit from selling goods and services which are complementary to the software. The main trade-off is between the lower appropriability of the investment in R&D in OS and the duplication of effort in the proprietary model. These authors show that even when the appropriability of the investment in R&D is low, there are equilibria where both kinds of firms coexist in the same markets. These equilibria are characterized by an asymmetric market structure, with many small open source firms and a few large proprietary firms. Most importantly, this market structure arises endogenously, even though all firms are ex-ante symmetric.

Our paper builds upon this literature and presents a novel approach to the study of optimal business models and equilibrium market structure. Specifically, our modeling contributions are: (1) firms have multiple software modules and must decide which modules to open and which to keep proprietary, (2) firms choose not only between the two pure business models (open source and proprietary), but may also compete through a mixed source model, (3) we analyze the case of a monopolist, but also the cases in which a for-profit firm competes against a not-for-profit OS project, and against another for-profit firm. As a consequence of our modeling strategy, a variety of market structures may arise in equilibrium, combining proprietary, mixed source, and open source firms.

Our paper also contributes to an emerging literature in strategy that explores competitive interactions between organizations with different business models. While there are several formal models of asymmetric competition that exist in strategy (differences in costs, resource endowments, or information, mainly), the asymmetries that this literature wrestles with are
of a different nature: firms with fundamentally different objective functions, opposed approaches to competing, or different governance structures. The papers mentioned above on competition between open source and proprietary firms belong to this literature. In addition, Casadesus-Masanell and Yoffie (2007) study competitive interactions between two complementors, Microsoft and Intel, with asymmetries in their objectives functions stemming from technology—software vs. hardware. Casadesus-Masanell and Zhu (2009) study competitive interaction between a high-quality incumbent that faces a low-quality ad-sponsored competitor. Finally, Casadesus-Masanell and Hervas-Drane (2009) analyze competitive interactions between a free peer-to-peer file sharing network and a profit-maximizing firm that sells the same content at positive price and that distributes digital files through an efficient client-server architecture. For the most part, this literature has studied interactions between firms with exogenously given business models. We contribute by endogeneizing the choice of business model by two profit-maximizing firms that compete to attract customers.

2. The model

2.1. Preferences. Preferences are based on a variant of Gabszewicz and Thisse’s (1979) and Shaked and Sutton’s (1982) models of vertical product differentiation. There is a continuum of consumers of mass 1, who differ in their valuations of the available products. Consumers are indexed by \( \rho \), where \( \rho \sim U[0,1] \). Consumer \( \rho \)’s indirect utility from consuming good \( i \) is:

\[
    u_{\rho i} = \rho v_i - p_i,
\]

where \( v_i \) and \( p_i \) are the quality and price of product \( i \).

Given the list of qualities and prices for all products available, each consumer will choose the product that maximizes his indirect utility.

2.2. Technology. Consumers derive utility from consuming software modules and a complementary service \( z \). There are two software modules: a base module and a set of extensions, denoted by \( a \) and \( b \), respectively. With a slight abuse of notation, we use the same symbol to refer to a software module (or service) and to the value of that software module (or service). Let \( a > 1 \) and \( b > 1 \) be the value of modules \( a \) and \( b \), respectively, and \( z > 1 \) be the value of service \( z \). We assume zero marginal costs.

Software modules and service are complementary. The value of a package composed by software modules \( a \) and/or \( b \) and service \( z \) is the product of the value of its components. For example, if a package is composed by modules \( a \) and \( b \), its value is \( v = ab \), and if it also includes service, its value becomes \( v = abz \).

\(^8\)The exception is Casadesus-Masanell and Zhu (2009) who allow the incumbent (but not the entrant) to choose business model.
Module $a$ constitutes the base (core) of the program and module $b$ is a set of extensions that run over module $a$. This means that module $b$ has no value if it is not used with module $a$, so the value of a package which does not include module $a$ is zero. Similarly, support has value only when used in conjunction with software. Although we refer to $b$ as software extensions, $b$ represents more generally those technologies, protocols, documentation, and ideas that must be used in conjunction with the core software $a$. For instance, file format protocols are of little value if there is no core software with which to use them.

Modules may be opened or kept closed. The advantage of opening a module is that users can improve on it. We assume that when a module is opened, its value increases to $a_o = a (1 + \sigma q_o)$ or $b_o = b (1 + \sigma q_o)$ where $\sigma$ measures the extent of user innovation and $q_o$ is the measure of users of the open source module.

Opening a module increases the value of all packages containing that module. For example, if a package includes both modules and service, and the base module $a$ is opened, its value becomes $v = a_o b z$.

2.3. Expectations. When a module is opened, before users decide whether to adopt it or not, the firm and the users form expectations on the measure of users that will adopt it, $q_o^e$. Suppose, for example, that module $a$ is opened. Before users decide whether to adopt it or not, they form expectations on the value of that module. This value depends on how many users are expected to use the module in equilibrium: $a_o^e = a (1 + \sigma q_o^e)$.

What is the relation between $q_o^e$ and $q_o$? We follow Katz and Shapiro (1985) and assume fulfilled expectations. The expected number of users $q_o^e$ is taken as given by consumers and firms when they make their decisions. What Katz and Shapiro’s (1985) criterion requires is that those expectations be fulfilled in equilibrium: $q_o = q_o^e$. While the equilibrium $q_o$ depends on the expectation $q_o^e$, the advantage of this approach is that there is no need to specify how expectations are formed.

2.4. Business models. We consider four business models. Two pure models and two mixed models. The pure models are the proprietary and the open source business models. These are pure models because all modules are either closed or open:

- **Proprietary ($P$):** In the proprietary model both modules, $a$ and $b$, are closed. This model has no user innovation.
- **Open source ($O$):** In the open model both modules are open. User innovation is maximal in this case.

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9The user innovation term is similar to a network externality. In contrast to network externalities, however, if the firm decides not to open the module, user innovation is absent.
The two mixed models have one open module. The other module is closed. The two possibilities are:

**Mixed source with open a** ($M_a$): This business model has the base module open. When $M_a$ is adopted, an open source competitor that offers module $a$ for free will emerge.

**Mixed source with open b** ($M_b$): In this model, only the extensions $b$ are open. The implication is that no open source competitor emerges when this business model is adopted because the extensions cannot be used without a base module.

The most common definition of *business model* is “the logic of the firm, the way it operates to create and capture value for its stakeholders.” Thus, a firm’s *real* business model includes a broad range of organizational and competitive elements such as products & markets, sources of revenue, incentive systems, hiring policies, information technologies, and so on. Detailed descriptions of business models are often too complex to be amenable to mathematical treatment. We follow Casadesus-Masanell and Zhu (2009) and represent business models through profit functions. Thus, in our formal development the choice of business model is represented through the choice of profit function. Profit functions are reduced form representations of actual business models. As such, they do not capture the full richness of the entire business model. However, they allow the derivation of formal propositions, which is an important goal of our study. Representing business models through profit functions captures the idea that choosing a business model implies more than just determining the value of a particular variable or parameter: it entails the selection of “the logic of the firm, the way it operates to create and capture value for its stakeholders.”

2.5. **Cases.** We consider three alternative competitive scenarios. In Section 3 we analyze the case of a monopoly that must choose its business model and where to price its products. In Section 4 we examine a simple duopoly where a profit-maximizing firm faces an open source competitor that is nonstrategic. And in Section 5 we study the less tractable case of two for-profit firms that best-respond to each others’ pricing and business model choices.

While the specifics of each scenario differ (we explain them in detail in the corresponding section), the generic sequence of decisions that we study is as follows:

1. Firms decide which business model to adopt.
2. Expectations on $q_o$ are formed and firms decide where to price their products.
3. Given $q_o$, business models, and prices, consumers pick their preferred product.

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10. This business model is sometimes called *open core*. See [http://www.linuxinsider.com/story/66807.html](http://www.linuxinsider.com/story/66807.html) for example.

We solve for the subgame perfect equilibrium with fulfilled expectations.

2.6. **Strategy vs. tactics.** Building on Ghemawat (1991) and Casadesus-Masanell and Ricart (2009), we refer to the choice of business model as *strategy*: the business model is a set of committed choices that lays the ground for the tactical interactions that will occur down the line. Thus, business model choices are *strategic* whereas pricing choices are *tactical*. This terminology reflects the fact that while the choice of business model often implies commitments which may be difficult to reverse, pricing choices are easy to change.  

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2.7. **Modeling assumptions.** Before delving into the analysis, we discuss three important modeling assumptions.

*Base vs. extensions.* The main objective of this paper is to analyze the choice of business model. We begin by presenting a micro-foundation for technology and preferences, and then derive the profit functions that correspond to each business model. Technology is represented by a multiplicative specification: there are two software modules and the value of combining them is the product of the value of the components.

Our distinction between base and extensions reflects the fact that while some software modules can be used standalone, others require a base to function. This distinction captures important asymmetries between specific and general applications. Some software modules have general value, while others have been designed for highly specialized uses. The extent of module specificity affects its value to the open source community. In the case of general applications, a larger number of developers are likely to contribute and user creativity and innovation are likely to be enhanced.

In addition, and as we pointed out above, the extensions represent not only software modules, but also all those technologies, protocols, documentation, and ideas that require the core software to have value. Opening these has a different effects than opening technologies which can be easily applied elsewhere.

Finally, we have assumed that all consumers always prefer more modules. However, some consumers may prefer to use simpler software, with fewer features, instead more complex software. It would be interesting to see how introducing this trade-off between features and complexity affects the predictions of the model.

*User innovation.* For simplicity, we have taken a reduced form approach for describing user innovation: more users imply more user innovation and thus higher quality software. The strength of this relation is governed by the parameter \( \sigma \). This simple specification is very intuitive, and delivers powerful insights. To our knowledge, this is the first paper to  

12For example, once the a module has been opened, there is no way back to being proprietary because the code is in the public domain.
introduce user innovation in a game theoretical framework. An interesting direction for further research would be to “open the black box” of user innovation, by endogenizing users’ investment decisions. This approach would allow to study several interesting issues arising from user innovation.

First, having more developers working on the software may increase its complexity, as the need form communication increases. This is well summarized by Brooks’s Law: “adding more programmers to a late project makes it later”. Therefore, having more developers may imply a higher cost of coordination, and the firm may have to take charge of part of this cost.

Open source projects have responded to this challenge by becoming more modular, as the works of Baldwin and Clark (2006) and MacCormack, Rusnak, and Baldwin (2006) have demonstrated. This modularity may have a positive impact in the quality of the firm’s software and lower development costs. Moreover, open source allows for a higher speed of debugging, which increases software stability. In fact, open source proponents claim that open source effectively breaks Brooks’s law.

Second, in our model, all users value openness and user innovation. There may be some users who do not agree with the directions of development of the software. Open source provides some protection against this concern: given that the code is open, users may decide what to take and what to discard, and may choose to take different development paths. Also, some users may have security concerns: opening the code implies that everybody gets to see the code and learn how the program works. However, open source proponents argue that open source guarantees higher security, because users get to see the program they are running, and therefore may be able to detect security threats more easily.

Compatibility. In Sections 4 and 5 we introduce a competing product for the firm. In one case, the competing product is provided by a non-profit open source project. In the other case, the competing product is provided by another commercial firm. In both cases, we will assume full compatibility between the modules and service of the competing projects. In other words, when the firm opens some module, this module can be combined with the module of its competitor, and/or its support service. Likewise, if a competing module is open, the firm can include it in its commercial product, and it will be fully compatible with its other module and service.

We are focusing on the compatibility case because it maximizes the interaction between the firm and the open source project. If the competing modules were incompatible with those of the firm, then the competitor would not take advantage of the modules opened by the firm, and the firm will be more likely to become open (this case would be similar to the monopoly case analyzed in Section 3). Likewise, the firm will be less likely to consider adopting the
modules of a competitor if they are incompatible with the complementary good or service that the firm is providing. A thorough analysis of partial compatibility and incompatibility presents an interesting direction for future research, but is outside the scope of the present paper.

3. Monopoly

A monopolist facing a unit mass of consumers with utilities as described in Section 2.1 and a technology as given in Section 2.2 must decide to operate under one of four business models. Each business model gives rise to one product that the monopolist sells at positive price. We refer to the product sold by the monopolist as the commercial product. There is a one-to-one relationship between the four business models and the products offered by the monopolist. Letting $v_c$ be the quality of the commercial product, we have:

- $P$: $v_c = a b z$.
- $M_a$: $v_c = a_0 b z$.
- $M_b$: $v_c = a b_0 z$.
- $O$: $v_c = a_0 b_0 z$.

When the monopolist opens module $a$, an open source competitor emerges. The reason is that module $a$ can be used standalone and, thus, some users may prefer it to the monopolist’s commercial offering (which is of higher quality but sold at positive price). When $b$ is opened, however, no open source competitor appears because module $b$ cannot be used without the base $a$. Therefore, two of the available business models imply the emergence of competition ($O$ and $M_a$) while the other two ($P$ and $M_b$) ensure that the monopolist remains the only firm in the market.

Just as in the case of the commercial product, there is a one-to-one relationship between the four business models and the available open products. Letting $v_o$ be the quality of the open source product, we have:

- $P$: $v_o = 0$. No open source product available.
- $M_a$: $v_o = a_o$.
- $M_b$: $v_o = 0$. No open source product available.
- $O$: $v_o = a_o b_o$.

The sequence of decisions is as follows:

\[^{13}\text{The monopolist could consider selling several, horizontally differentiated, products. In Appendix 7 we allow the monopolist to produce and market as many products as she wishes, and we show that the monopolist will always prefer to offer only one product, which is the one that we are considering.}\]

\[^{14}\text{In this section, we will refer to the commercial firm as the monopolist regardless of the fact that, when } a \text{ is opened there is competition by an open source project.}\]
(1) The firm chooses a business model.
(2) Expectations are formed on the share of the population that adopt the available open modules. Given these expectations, the monopolist sets the price of its commercial product.
(3) Consumers choose between the commercial product, the open source product (if available), and staying out of the market.

We now solve the monopolist’s optimization problem. As usual, we proceed backwards. We first consider the consumers’ purchase decision (given that one or two products are offered) and derive the demand functions. Then, we find the profit-maximizing price under each business model. Finally, we let the monopolist choose a business model. We impose that expectations be fulfilled in equilibrium.

3.1. **Consumer demands.** Suppose first that there is no open source product available. This means that consumers have two options: they can either buy the commercial product and obtain utility \( u = \rho v_c - p \), or they can refrain from consuming and obtain \( u = 0 \). There is a cut-off point \( \rho_0 \) depending \( p \) such that only consumers with \( \rho > \rho_0 \) will buy the product of the firm at that price. The demand for the commercial product is:

\[
q_c = 1 - \frac{p}{v_c}.
\]

Suppose now that there is an open source alternative. Consumers can either buy the commercial product at price \( p \), consume the open source good for free, or stay out of the market. Since the open source product gives utility \( u = \rho v_o > 0 \), consumers never choose to stay out and the relevant comparison is between the commercial and the open source products. Figure 3 shows the utility schedules for the commercial and open source products.

![Figure 3. Consumer utility. Commercial and open source products.](image-url)
Demand for the commercial product and the open source alternative are, respectively:

\[ q_c = 1 - \frac{p}{v_c - v_o} \]
\[ q_o = \frac{p}{v_c - v_o} \]  

3.2. Pricing decision. In choosing prices, the firm is interested in maximizing profits:

\[ \pi = pq_c. \]

When the commercial product is the only one available, the optimal price is \( p = v_c/2 \), which implies an equilibrium quantity of \( 1/2 \) and profits of \( \pi = v_c/4 \). As is well-known, a monopolist that faces a linear demand function and has zero marginal cost of production trades off volume and markup optimally at a price equal half the choke price and ends up serving half of the market.

When there is an OS alternative, the inverse demand function that the monopolist faces is \( p = (v_c - v_o)(1 - q_c) \). This is a linear demand function. Thus, the optimal price is once again half the choke price \( v_c - v_o \), half of the market is served, and profits are \( \pi = (v_c - v_o)/4 \).

Finally, the optimal price and profits depend on the precise functional forms of \( v_c \) and \( v_o \) which, as seen above, depend on the business model chosen by the firm. Specifically, the profits that each business model generate are as follows:

- \( P \): \( \pi_p = abz/4 \).
- \( M_a \): \( \pi_a = a_o (bz - 1)/4 \).
- \( M_b \): \( \pi_b = ab_o z/4 \).
- \( O \): \( \pi_{ab} = a_o b_o (z - 1)/4 \).

3.3. Business model choice. The firm decides which module/s it should open, taking into account the expected number of users, its subsequent pricing decisions, the demand schedule, and that an open source competitor will emerge if module \( a \) is opened. The choice between business models depends on the comparison of profits which, in turn, depends on the strength of user innovation and the value of the modules and service (\( a, b, \) and \( z \)).

The first point to notice is that \( P \) is dominated by \( M_b \). \( M_b \) is unambiguously superior to \( P \) because it takes advantage of user innovation to produce a higher quality product but it does not set up an open source competitor.\[15\]

\[ \text{More generally, the firm can open all those technologies, protocols and ideas which have no value unless they are used with the base module, instead of remaining completely closed. For instance, in this monopoly case, if the firm’s software is the only way to access an specific format for saving data, or is the only way to compile code written in a given programming language, the firm may open the file format or language without hampering its ability to capture vale. This is exactly what happens in the cases of Microsoft’s .Net framework, Stata, and Mathematica.} \]
Given that $P$ is dominated by $M_b$, the three profit functions that must be compared are those for $M_a$, $M_b$, and $O$. The following thresholds are useful to state our first proposition which characterizes the monopolist’s choice of business model.

\[
\sigma_{a|b} := \frac{2}{bz - 2}, \quad \sigma_{a|ab} := \frac{b - 1}{b(z - 1)}, \quad \sigma_{b|ab} := \frac{1 + \sqrt{z(9z - 8)}}{4(z - 1)} - \frac{3}{4}.
\]

**Proposition 1.** Suppose that either $2 < b < 3$ and $z \leq \frac{2}{(3-b)b}$, or $b \geq 3$ hold. Then, if $\sigma \leq \sigma_{a|b}$, $M_b$ is the optimal business model; if $\sigma_{a|b} < \sigma \leq \sigma_{a|ab}$, $M_a$ is the optimal business model; and if $\sigma > \sigma_{a|ab}$, the optimal business model is $O$. Suppose, instead, that either $2 < b < 3$ and $z > \frac{2}{(3-b)b}$, or $1 \leq b \leq 2$ hold. Then, if $\sigma \leq \sigma_{b|ab}$, $M_b$ is the optimal business model; and if $\sigma > \sigma_{b|ab}$, $O$ is the optimal business model.

Figure 4 shows the regions on which each business model is optimal in the $(\sigma, z)$ space. When $\sigma$ and/or $z$ are large, the optimal business model is $O$, regardless of the values of all other parameters. Large $\sigma$ implies large user innovation when $a$ and $b$ are open. However, an open source competitor emerges, degrading the firm’s ability to capture value. Notwithstanding, when $\sigma$ is large the trade off is resolved in favor of opening the software to the maximal possible extent. When $z$ is large, the vertical differentiation between the commercial product and the open source product that business model $O$ sets up is large. As a consequence, the presence of the open source product does not harm much the ability of the firm to earn large profits. Thus, when $z$ is large, the firm prefers to open both modules.

When $\sigma$ is small the optimal business model is $M_b$ as there is little to gain from user innovation and there is no point of being too open. As $\sigma$ increases the preferred business model eventually becomes $M_a$. The difference between $M_b$ and $M_a$ is as follows. When $b$ is
opened, no open source competitor emerges. When $a$ is opened, a new competitor surfaces which forces prices down. But $M_b$ does not exploit user innovation to the same extent that $M_a$ does: when $M_b$ is chosen, $\frac{1}{2}$ of the market adopts the product and when $M_a$ is chosen, everybody adopts. The trade off is resolved in $M_a$’s favor when $\sigma$ is large.

3.4. **Value creation and value capture.** The profit-maximizing choice of business model depends on the resolution of a trade-off between value creation and value capture which are defined as follows:  

**Value creation ($V$):** Is the sum of the firm’s profits and consumer surplus:  

$$V = CS + \pi.$$  

**Value capture ($\theta$):** Is the proportion of the value created which is appropriated by the firm (in the form of profits):  

$$\theta = \frac{\pi}{CS + \pi}.$$  

Using the above definitions, firm profits can be expressed as $\pi = \theta V$. It is straightforward to compute $V$ and $\theta$:  

$$V = \frac{3}{8} v_c + \frac{1}{8} v_o,$$  

$$\theta = 2 \frac{(v_c - v_o)}{3 v_c + v_o},$$  

setting $v_o = 0$ when there is no open source alternative.

Figure 5a compares value creation and value capture for the different business models. As discussed above, value capture under $P$ coincides with that under $M_b$ but value creation is larger under $M_b$. Given that profits are $\pi = \theta V$, we can easily plot the isoprofit curves. Obviously, curves farther away from the origin correspond to higher profits and the profit-maximizing business model is the one reaching the highest isoprofit curve. Figure 5b shows the isoprofit curves and business models. In this example, the optimal business model is $M_b$.

The figure shows that there is a trade off between value creation and value capture: $M_b$ allows for maximal value capture but value creation is low, $M_a$ erodes value capture but

---

16Recall that the profit-maximizing price of the monopolist results in adoption by exactly half of the market.  
17When $2 < b < 3$ and $z \geq \frac{2}{(3-b)\sigma}$, whenever $M_a$ is superior to $M_b$, it turns out that $O$ is superior to $M_a$. This is why for $z \geq \frac{2}{(3-b)\sigma}$, $M_a$ is never chosen.

18The expressions **value creation** and **value capture** have been mainstream in strategy since the publication of Brandenburger and Stuart (1996), Brandenburger and Nalebuff (1997) and MacDonald and Ryall (2004). In Brandenburger and Stuart’s (1996), all transactions with positive value take place. Therefore, value created equals potential value. However, Brandenburger and Stuart also recognize that there may exist ‘frictions’ which may introduce a wedge between potential and realized value. In our model, for example, the monopolist cannot price discriminate. Hence, there are transactions with positive potential value that do not occur in equilibrium (i.e. there is a welfare loss). Our definition of value creation corresponds with actual value created, which is more relevant for studying the choice of an optimal business model. For a thorough discussion of value creation and value capture and endogenous business models, see Salas-Fumas (2009).

19The parameter values are: $a = 1$, $b = 1.5$, $z = 1.2$ and $\sigma = 0.1$. 

strengthens value creation, and $O$ deteriorates value capture further but value creation is at its maximum.

A glance at $V$ and $\theta$ reveals that regardless of the business model, value creation grows when $a$, $b$, $z$ or $\sigma$ increase. When $a$ or $\sigma$ increase, however, value capture remains constant. The reason is that value creation and profits grow at equal rates with $a$ or $\sigma$ regardless of whether there is an open source competitor or not. When $z$ increases, value capture remains constant for $P$ and $M_b$, and increases for $M_a$ and $O$. And when $b$ increases, value capture remains constant for $P$, $M_b$ and $O$, and increases for $M_a$.

Note that our definition of value is equivalent to welfare. This means that Figure 5a also serves to rank the different business models in terms of welfare. The socially optimal model (in a second best sense) is $O$. As we have seen, the socially optimal business model is not always chosen by the monopolist.

4. Competition from an outside OS project

In this section we analyze a situation where an incumbent firm (modeled like the monopolist of Section 3) faces a non-strategic competitor that enters the market with an independently developed open source product. Think, for example, of a non-profit open source project like Apache or Linux, competing against a for-profit firm like Microsoft or IBM. Because a substitute is available for the firm’s product even if it decides not to open any one of its modules, the open source project imposes competitive pressure on the incumbent.

Let $a \geq 1$ and $b \geq 1$ represent the external open source modules and their values. We will consider two cases: (1) the open source project has modules of lower quality than the firm, and (2) the open source project has modules of higher quality than the firm.\(^{20}\)

\(^{20}\)We are assuming that the open source project provides substitutes for both modules. In Appendix C we analyze alternative scenarios, in which the open source project provides only one of the two modules, or
As in Section 2.2, the value of a combination of modules and service is the product of their individual values, and a set of extensions (\(b\) or \(\hat{b}\)) has no value unless it is used with a base (\(a\) or \(\hat{a}\)). We assume full compatibility: either \(a\) or \(\hat{a}\) may be combined with \(b\) or \(\hat{b}\), and \(z\). Therefore, the incumbent may embed an outside open source module in its commercial software if this leads to higher profit. Likewise, the open source project may combine one of its modules with one of the modules opened by the incumbent.\(^{21}\)

The incumbent must choose the business model through which it would like to compete. The choice set is the same as in the previous section: \(P\), \(M_a\), \(M_b\), and \(O_{ab}\). The only difference is that the firm may adopt the module/s available from the open source competitor. Therefore, there are two flavors to the business model with open \(a\): \(M_a\) and \(M_{\hat{a}}\). Likewise for \(M_b\). Similarly, there are four flavors of business model \(O_{ab}\).

Each business model gives rise to one commercial product which is sold at a positive price, but in this duopolistic setting there is always an open source product available to consumers, regardless of the business model choice made by the incumbent.

In addition to the commercial product, consumers may have a choice over two open source products, the outside OS project and the open product set up by the firm (if it decides to compete through a business model that involves opening at least one module). A slight complication that arises in this case is that there may be multiple equilibria. The reason is that the expected value of open products (and, thus, demand for them) depends on how many users are expected to use them. In some circumstances, more than one way to coordinate purchases may satisfy Katz and Shapiro’s (1985) criterion discussed in Section 2.3. To sidestep this issue, we assume that consumers coordinate to choose the open source product with highest (exogenous) quality, before taking into account the effect of user innovation.

The firm must choose its business model and price. The optimization problem is similar to that of Section 3. Given price and qualities, consumers choose their preferred product. The sequence of decisions is:

1. The firm chooses business model and whether or not to adopt any of the available modules from the outside open source project.
2. Expectations are formed on \(q_o\) and the firm sets price.
3. Given \(q_{oe}\), the choice of business model and price, consumers choose between the open source product and the commercial product.

Given \(v_o\) and \(v_c\), the functional forms of demands and profits are the same as in the monopoly case (see Section 3.1). The only difference is that the open source products may provide one module with higher quality and another one with lower quality than those of the firm. The main conclusions of our analysis remain unaffected.

\(^{21}\)See our discussion of compatibility in Section 2.7.
now be based on the outside open source modules, the modules opened by the firm, or a combination of both.

4.1. The firm has modules of higher quality. We start by analyzing what happens when \( a > \hat{a} \) and \( b > \hat{b} \). As discussed above, the possibility of adoption of the modules provided by the outside OS project mean that there are multiple implementations of the mixed and open source business models. The possibilities are: \( M_a, M_{\hat{a}}, M_b, M_{\hat{b}}, O_{ab}, O_{\hat{a}\hat{b}}, O_{\hat{a}b}, \) and \( O_{\hat{a}\hat{b}} \). In addition, the firm may also choose \( P \).

We will show that: (1) the firm will never embed the external open source modules in its commercial offering, (2) when choosing between the different Mixed Source models, the firm will open the module that faces stronger competition from the open source project, and will keep closed the module which could potentially increase more the value of the open source product, and (3) the firm may choose to be \( P, M \) or \( O \), so all possible business models may arise as an equilibrium outcome.

We begin by showing that whenever the firm is considering a Mixed Source or Open Source business model, it will always choose to base its commercial product on its own modules, i.e. the firm will never adopt any module by the external open source project.

**Lemma 1.** When the modules of the firm have higher quality than the modules of non-strategic open source competitor, the firm will never choose to embed those modules in its commercial product, i.e. it will never choose the \( M_{\hat{a}}, M_{\hat{b}} \) or \( O_{\hat{a}\hat{b}} \) business models.

Lemma 1 is a consequence of our complementarity assumption. Suppose, for example, that the firm is considering a Mixed Source model with open base. The firm has to choose between \( M_a \) and \( M_{\hat{a}} \). The firm could be tempted to choose \( M_{\hat{a}} \) because opening \( a \) would imply a higher quality for the OS product. However, complementarity with \( b \) and \( z \) implies that the quality of the firm’s product will increase relatively more. Therefore, the firm will prefer \( M_a \) to \( M_{\hat{a}} \).

The above lemma implies that the firm will consider only \( P, M_a, M_b \) and \( O_{ab} \). The values of \( v_c \) and \( v_o \) and profits are:

- **\( P \):** \( v_c = abz \), \( v_o = \hat{a}_o \hat{b}_o \), and \( \pi_p = (abz - \hat{a}_o \hat{b}_o)/4 \).
- **\( M_a \):** \( v_c = a_o b z \), \( v_o = a_o \hat{b}_o \), and \( \pi_a = (a_o b z - a_o \hat{b}_o)/4 \).
- **\( M_b \):** \( v_c = a b_o z \), \( v_o = \hat{a}_o b_o \), and \( \pi_b = (a b_o z - \hat{a}_o b_o)/4 \).
- **\( O_{ab} \):** \( v_c = a_o b_o z \), \( v_o = a_o b_o \), and \( \pi_{ab} = a_o b_o (z - 1)/4 \).

Next, we show that, for given parameter values, either \( M_a \) dominates \( M_b \) or vice versa. When \( a/\hat{a} > b/\hat{b} \), the firm prefers to open \( b \). Likewise, when \( a/\hat{a} < b/\hat{b} \), the firm prefers to open \( a \). There are two ways to interpret this result. On one hand, the firm wants to open
the module which faces greater competition from the OS project. On the other hand, the firm wants to keep proprietary the module which is more complementary to the software of the OS competitor.

**Lemma 2.** $M_a$ is preferred to $M_b$ if and only if $a/\hat{a} < b/\hat{b}$, and vice versa.

The reason behind the previous result is that opening $a$ or $b$ produce the exact same benefit for the firm ($a_o b z = a b_o z$). The costs differ, however, because the quality of the open source competitor depends on which module is opened. The quality of the competitor is $a_o \hat{b}_o$ if $a$ is opened and $\hat{a}_o b_o$ if $b$ is opened. In comparing $M_a$ and $M_b$, the firm will prefer the business model that results in the lowest possible value for the outside OS project.

Finally, we proceed to characterize the optimal choice of business model for all possible values of $\sigma$ and $z$. The above lemmas imply that there are only three candidates for optimal business model: $P$, $O_{ab}$, and either $M_a$ or $M_b$.

Note that, contrary to the monopoly case, $M_b$ does not dominate $P$ for all parameter values. The reason is that when $b$ is opened, users of the open source project can combine $\hat{a}$ with $b$ to assemble a product of quality $\hat{a}_o b_o$. Having a stronger open source competitor may lead to decreased value capture by the for-profit firm.

The following thresholds are used in Proposition 2:

$$z_{alp} := \frac{\hat{b}(2 + \sigma)(2a(1 + \sigma) - \hat{a}(2 + \sigma))}{4a b \sigma} , \quad z_{b|p} := \frac{\hat{a}(2 + \sigma)(2b(1 + \sigma) - \hat{b}(2 + \sigma))}{4a b \sigma} ,$$

$$z_{ab|a} := \frac{2b(1 + \sigma) - \hat{b}(2 + \sigma)}{2b \sigma} , \quad \text{and} \quad z_{ab|b} := \frac{2a(1 + \sigma) - \hat{a}(2 + \sigma)}{2a \sigma} .$$

**Proposition 2** (Equilibrium regions). If $a/\hat{a} < b/\hat{b}$, then $P$ is optimal when $z < z_{alp}$, $M_a$ is optimal when $z_{alp} < z < z_{ab|a}$ and $O_{ab}$ is optimal when $z > z_{ab|a}$. If $a/\hat{a} > b/\hat{b}$, then $P$ is optimal when $z < z_{b|p}$, $M_b$ is optimal when $z_{b|p} < z < z_{ab|b}$ and $O_{ab}$ is optimal when $z > z_{ab|b}$.

Figure 6 illustrates the optimal strategy as a function of determined parameter values. For low values of $\sigma$ and $z$, the firm chooses the proprietary business model ($P$), for intermediate values, the firm chooses a mixed source model (either $M_a$ or $M_b$), and for high values of $\sigma$ and $z$, the firm becomes completely open ($O_{ab}$).

The intuition is simple. When $\sigma$ and $z$ are low, there is little benefit from opening modules as user innovation is weak. Moreover, low $z$ means that if the firm was to open any module,
the outside OS project would be very close in quality, so competitive pressure would be strong. Therefore, the firm is better off with a proprietary business model in this case. As $\sigma$ and $z$ increase, so does the value creation implications of opening modules and the vertical differentiation between the OS project and the firm’s product. When $\sigma$ and $z$ are sufficiently large, the additional value creation of opening one module outweighs the increased competitive pressure from the outside OS project. For the same reason, the firm is better off opening both modules when $\sigma$ and $z$ are very large.

An important implication of Proposition 2 is that the firm may prefer the proprietary (closed) business model when there is an outside OS competitor. In the monopoly case (Section 3), the firm never desired to be completely closed. This was because (i) opening the extensions was harmless if users did not have a base module with which to combine it and (ii) there was an innovation benefit when opening a module. When there is competition from an outside OS project, opening the extensions leads to increased competitive pressure, making $P$ optimal in some opportunities.

One final issue that we need to address is the possibility of leapfrogging. Leapfrogging refers to a situation where the outside open source project ends up with quality above that of the commercial firm’s product. This could happen if $\hat{a} \hat{b}$ was close to $ab$ and $z$ was close to 1. In this case leapfrogging could occur if the firm adopted a proprietary business model. It is easy to see, however, that leapfrogging never happens in equilibrium. If there was leapfrogging, the commercial firm would earn zero profits because the open source product would be of higher quality and given away for free. The commercial firm can easily prevent leapfrogging by opening $a$ and/or $b$.

Figure 6. Equilibrium business models.
4.2. **Open source competitor has modules of higher quality.** In the previous section, we have shown that if \( \hat{a} < a \) and \( \hat{b} < b \), the firm will *never* include \( \hat{a} \) nor \( \hat{b} \) in its commercial product. The following lemma shows that when \( \hat{a} > a \) and \( \hat{b} > b \), the firm will *always* embed both modules in its commercial product.

**Lemma 3.** *When the modules of the non-strategic open source competitor have higher quality than the modules of the firm, the firm will always choose to embed those modules in its commercial product, i.e. it will always choose the \( O_{\hat{a}\hat{b}} \) business model.*

The above result is somewhat surprising, and is once again due to the assumption of complementarity between modules and service. The firm could consider not adopting the open source modules, because doing so increases the quality of the open source product (the consumers of the firm engross the user base of the open source project, which increases its quality). However, complementarity implies that the quality of the firm’s product increases relatively more than that of the open source product: the firm shares now the same user base as the open source project, the firm’s product is based on higher quality modules, and its value is multiplied by \( z > 1 \)\(^{22}\).

4.3. **Value creation and value capture.** We now compare the different business models in terms of value creation \((V)\) and value capture \((\theta)\). The functional forms for \(V\) and \(\theta\) are the same as in Section 3.3. What changes are the values of \(v_o\) and \(v_c\) that enter in these expressions.

4.3.1. *The firm has modules of higher quality.* Next, we will analyze how the trade-off between value creation and value capture is affected by the presence of a non-strategic open source competitor with lower quality modules.

Figure 7 compares value creation and value capture for the different business models for \(a = 1.5, b = 1.5, z = 1.2\) and \(\sigma = 0.1\). Figure 7a shows a case in which \(a/\hat{a} > b/\hat{b}\) (\(\hat{a} = 1\) and \(\hat{b} = 1.2\)), and Figure 7b shows a case in which \(a/\hat{a} < b/\hat{b}\) (\(\hat{a} = 1.2\) and \(\hat{b} = 1\)).

These plots have some features in common with the monopoly case. For example, \(P\) is still the business model with lowest value creation and \(O_{ab}\) the model with highest value creation. Also, \(O_{ab}\) is the model with lowest value capture. The mixed models \((M_a\) and \(M_b\)) produce intermediate value creation and capture. However, there are important differences also. First, the line connecting \(P\) with \(M_b\) (or \(M_a\)) has negative slope indicating that the mixed models do not dominate \(P\). Second, \(M_a\) may be to the north-west of \(M_o\), as in Figure

\(^{22}\text{This result could be reversed if the open source modules were incompatible with the firm’s complementary good or if the different software products were horizontally differentiated.}\)
Clearly, the availability of outside OS modules affects the ranking of business models in terms of value creation and value capture.

4.3.2. Open source competitor has modules of higher quality. Figure 8 shows the ranking of business models when the outside OS project has modules of higher quality than those of the profit-maximizing firm, for $a = 2.5$, $\hat{a} = 3.5$, $b = 1.5$, $\hat{b} = 3$, $z = 4$, and $\sigma = 0.3$.

The plot shows clearly that the Open Source business model ($O_{\hat{a}\hat{b}}$) dominates all other business models. Thus, the tradeoff between value creation and value capture is completely broken when the OS competitor has modules of higher quality. Clearly, $O_{\hat{a}\hat{b}}$ reaches the highest isoprofit curve and therefore it is the best business model. In this case, private choice agrees with socially optimal behavior.
5. Competing through business models

In this section, we study competitive interaction between two firms that choose the business model through which they would like to compete. Specifically, we consider firms $H$ and $L$ endowed with modules $a_i$ and $b_i$, $i \in \{H, L\}$. We assume that the modules offered by $H$ are of higher quality than those offered by $L$: $a_H \geq a_L \geq 1$ and $b_H \geq b_L \geq 1$. The two-period model that we analyze is the natural generalization of that in Section 4. In the first period (strategic choices), firms choose business models. In the second period (tactical choices), firms choose the price at which their products are sold. We consider two alternative timings. In Subsection 5.1, $H$ chooses business model first and in Subsection 5.2, $L$ is the first mover.

As it will become apparent in what follows, the model with endogenous business models for both players is complex. To make progress, we simplify by assuming that $z = 1$ so that no additional willingness to pay is derived from service. This assumption reduces the attractiveness of opening modules. The reason is that the vertical differentiation between the commercial products and the open source competitors that are set up when base modules are opened is not as large as it would be if $z > 1$. Therefore, if we find that it is desirable to open modules when $z = 1$, it should be even more desirable if $z$ was greater than 1.

We are able to obtain analytical expressions of profits for each combination of business models of the two players. However, these expressions are long and intricate. This forces us to solve the first period of the game (the choice of business model) numerically. Analytical solutions are intractable. As we will see, several interesting results can be derived from the numerical analysis.

5.1. $H$ moves first. The extensive form game that we study has two periods. Strategic choices take place in the first period and tactical interaction occurs in the second period. The first period has two stages. In the first stage, $H$ chooses its business model and in the second stage $L$ chooses its business model.

As the first mover, $H$ has four business models at its disposal: $P, M_{a_H}, M_{b_H}$, and $O_{a_Hb_H}$. Because $L$ moves second, there are cases in which it has more than four business models to choose from. For example, if $H$ chose $M_{a_H}$, then in addition to $P, M_{a_L}, M_{b_L}$, and $O_{a_Lb_L}$, $L$ can also choose to compete through $M_{a_H}$ or $O_{a_Hb_L}$. However, whenever the $H$ and $L$ versions of a given module are open, all consumers will use the $H$ version (because it is free, compatible, and of higher quality). Therefore, if $H$ competes through model $M_{a_H}$ and $L$ competes through business model $M_{a_L}$, $L$’s effective model will be $M_{a_H}$ as even those consumers buying $L$’s product will combine $b_L$ with $a_H$.

23The *Mathematica* code is available from the authors.
Therefore, we will assume that whenever both firms open the same module, there is a \textit{de facto} adoption of the higher quality one by \(L\)'s customers.\(^{24}\) The implication is that, regardless of the business model choice made by \(H\) in the first stage, \(L\) effectively has four business models to choose from.\(^{25}\)

The timing of the game is:

1. \(H\) chooses business model.
2. \(L\) chooses business model.
3. Expectations are formed over the market shares of the different products. Given these expectations, \(H\) and \(L\) simultaneously choose prices, \(p_H\) and \(p_L\).\(^{26}\)
4. Consumers choose their preferred product.

Although we allow firms to be completely open, our assumption that \(z = 1\) implies that if a firm opened everything, then it would not be able to sell its commercial product at a positive price because the open source product would be as good as the commercial version. Therefore, opening both modules is a strictly dominated strategy for both firms and it can be ignored when solving for the subgame perfect equilibrium of the game.

Figure 9 shows a partial description of the game tree.\(^{27}\) Depending on \(H\) and \(L\)'s decisions on what modules to open, there may be two or three products available in the market. Three products are available when either firm opens module \(a\) because an open source competitor emerges. Otherwise, only two products are available.

Note that in this game \(H\) can never push \(L\) off the market by opening \(a\) and setting up a high-quality open source competitor. The reason is that \(L\) can always adopt the opened module and have quality at least as high as that of the free open source product. For the same reason, if \(H\) opens a module, \(L\) will never choose to open the complementary module. As a consequence, there are two branches of the game in Figure 9 that will never be part of an equilibrium: \(\{M_{a_H}, M_{b_L}\}\) and \(\{M_{b_H}, M_{a_L}\}\).

We solve for the subgame perfect equilibria recursively. We start by determining consumer choices and solving the pricing subgame given the business models, and later solve for the equilibrium business model choices.

\(^{24}\)Clearly, this is possible because we assume that the modules of different firms are compatible. Read Section 2.7 for more details.

\(^{25}\)More precisely, when \(H\) opens module \(i_H\) \((i \in \{a, b\})\), from the point of view of \(L\), opening \(i_L\) or adopting \(i_H\) are equivalent strategies. For ease of exposition, we collapse both strategies into one: adoption by \(L\).

\(^{26}\)As in previous sections, the market shares of the different products are taken as given by consumers and firms when they make their choices. Fulfilled expectations imply that in equilibrium, those expectations are correct.

\(^{27}\)The game tree is \textit{partial} because we have not written down explicitly the pricing subgame nor the payoff functions. Moreover, we have not included the strictly dominated strategy of being completely open.
5.1.1. Consumer demands and equilibrium prices with two products. If neither firm opens a base module, consumers choose between two commercial products. Let $v_i$ and $p_i$ be the quality and price of the product offered by $i \in \{H, L\}$. Demands are

$$q_H = 1 - \frac{p_H - p_L}{v_H - v_L} \quad \text{and} \quad q_L = \frac{p_H - p_L}{v_H - v_L} - \frac{p_L}{v_L}.$$  

Finding the equilibrium prices and substituting in, we obtain

$$q_H = \frac{2v_H}{4v_H - v_L} \quad \text{and} \quad q_L = \frac{v_H}{4v_H - v_L}.$$  

Equilibrium profits are

$$\pi_H = \frac{4v_H^2 (v_H - v_L)}{(4v_H - v_L)^2} \quad \text{and} \quad \pi_L = \frac{v_H v_L (v_H - v_L)}{(4v_H - v_L)^2}.$$
The particular profits in each subgame are obtained by substituting in the corresponding expressions for product quality, \( v_H \) and \( v_L \), given in Figure 9 and imposing that the firms’ and consumers’ expectations be fulfilled in equilibrium.

Consider, for example, the subgame where \( H \) has opened \( b_H \) and \( L \) has adopted \( b_L \). This is the subgame \( \{ M_{b_H}, M_{b_L} \} \). In this case, \( v_H = a_H b_H (1 + \sigma (q^e_H + q^e_L)) \) and \( v_L = a_L b_H (1 + \sigma (q^e_H + q^e_L)) \), where \( q^e_i (i \in \{ H, L \}) \) is the expected demand for \( i \). Substituting \( v_H \) and \( v_L \) in the equilibrium demands we have

\[
q_H = \frac{2a_H}{4a_H - a_L} \quad \text{and} \quad q_L = \frac{a_H}{4a_H - a_L}.
\]

Fulfilled expectations require \( q_H + q_L = q^e_H + q^e_L \). Hence,

\[
q^e_H + q^e_L = \frac{3a_H}{4a_H - a_L}.
\]

Substituting in the profit functions we obtain:

\[
\pi_H = \frac{4a_H^2 b_H (a_H - a_L)(a_H(4 + 3 \sigma) - a_L)}{(4a_H - a_L)^3},
\]

and

\[
\pi_L = \frac{a_H a_L b_H (a_H - a_L)(a_H(4 + 3 \sigma) - a_L)}{(4a_H - a_L)^3}.
\]

Other cases are solved similarly. The equilibrium expression for \( q^e_H + q^e_L \) is particularly simple in this example because \( q_H + q_L \) does not depend on \( q^e_H \) or \( q^e_L \). More generally, \( q_H + q_L \) will depend on \( q^e \) and the resulting expressions for \( q^e \) and profits are convoluted. This is why analytical solutions for the first period become intractable and we have to resort to a numerical analysis.

Regarding leapfrogging, the only subgame where it may occur is when \( H \) keeps both modules closed and \( L \) opens \( b_L \). Leapfrogging occurs in this subgame when \( \sigma \) is large. Let

\[
\sigma_{NLF} := \frac{3(a_H b_H - a_L b_L)}{a_L b_L} \quad \text{and} \quad \sigma_{LF} := \frac{3(a_H b_H - a_L b_L)}{2(a_L b_L)}.
\]

Clearly, \( 0 < \sigma_{LF} < \sigma_{NLF} \). It is not hard to see that when \( \sigma > \sigma_{LF} \) there is an equilibrium with leapfrogging. Likewise, when \( \sigma < \sigma_{NLF} \) there is an equilibrium without leapfrogging. Therefore, when \( \sigma_{LF} < \sigma < \sigma_{NLF} \) there are two equilibria. As in previous sections, we assume that whenever there are multiple equilibria, consumers coordinate to choose the equilibrium with higher (exogenous) quality. Therefore, when \( \sigma < \sigma_{NLF} \) the equilibrium that we consider has no leapfrogging.

Allowing for the possibility of leapfrogging is important because \( H \) has to determine what is \( L \)'s best response to each strategy it may take. In the game where \( H \) moves first, however,
leapfrogging does not occur in equilibrium because $H$ can always prevent it by opening $b_H$ and be at least as well off.

5.1.2. Consumer demands and equilibrium prices with three products. If at least one of the firms opens its base module $a$, there may be three products in equilibrium. Let $v_i$ and $p_i$, $i \in \{H, L, O\}$ be the quality and price of the different products, where $O$ refers to the open source product, and $p_O = 0$ (by definition of open source). Demands are

$$q_H = 1 - \frac{p_H - p_L}{v_H - v_L}, \quad q_L = \frac{p_H - p_L}{v_H - v_L} - \frac{p_L}{v_L - v_O}, \quad \text{and} \quad q_O = \frac{p_L}{v_L - v_O}.$$ 

After substituting in the equilibrium prices, demands become

$$q_H = \frac{2(v_H - v_O)}{4v_H - v_L - 3v_O}, \quad q_L = \frac{v_H - v_O}{4v_H - v_L - 3v_O}, \quad \text{and} \quad q_O = \frac{v_H - v_L}{4v_H - v_L - 3v_O}.$$ 

Equilibrium profits are

$$\pi_H = \frac{4(v_H - v_O)^2(v_H - v_L)}{(4v_H - v_L - 3v_O)^2}$$

and

$$\pi_L = \frac{(v_H - v_O)(v_L - v_O)(v_H - v_L)}{(4v_H - v_L - 3v_O)^2}.$$ 

The particular profits in each subgame are obtained by substituting in the corresponding expressions for product quality, $v_H$, $v_L$, and $v_O$, as given in Figure 9 and imposing that the firms’ and consumers’ expectations be fulfilled in equilibrium.

As in the previous case, there is one subgame where leapfrogging may occur: $\{P, M_{aL}\}$. As before, there are parameters for which the subgame has equilibria with and without leapfrogging, in which case we select the equilibrium without leapfrogging. While it is important to consider the possibility of leapfrogging to compute the equilibrium of the complete game, leapfrogging does not occur in the equilibrium.

Having obtained the continuation profits for each of the nine possible subgames we now move to studying the first period. The first period has two stages. In the first stage, $H$ chooses business model. In the second stage, $L$ chooses business model. Thus, in choosing its business model, $H$ takes into account how $L$ will best respond through its choice of business models.

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28There are two subgames where even with one of the base modules open, there are only one or two active products in equilibrium. The subgames are $\{M_{aH}, M_{bL}\}$ and $\{M_{aL}, M_{bH}\}$. In these two subgames, $L$’s product is of equal quality than the open source product. Therefore, $L$ gets no demand if it sets a positive price. Moreover, depending on the parameter values, the quality of the open source product may be higher than that of $H$’s product. When this happens, there is only one product that obtains positive demand in equilibrium. As discussed above, these subgames are never played.
5.1.3. Business model choice. Figure 10 shows the equilibrium business model configurations for different parameter values. To produce the figure, we have set $a_L = b_L$, $a_H = \lambda a_L$, and $b_H = \lambda b_L$, where $\lambda > 1$. The larger is $\lambda$, the larger is the exogenous quality difference between the products of firms $H$ and $L$. The plots have $\lambda$ on the vertical axis, and the strength of user innovation $\sigma$ on the horizontal axis.

![Figure 10](image)

**Figure 10.** Equilibrium regions when $H$ moves first.

The labels inside the regions indicate the equilibrium business model configurations. The first element corresponds to the business model of the first mover ($H$ in this case), and the second element corresponds to the second mover. For example, $\{M_{b_H}, P\}$ means that $H$ has chosen to open $b_H$ (mixed source business model with open $b$) and $L$ has chosen to stay
closed (proprietary model). Likewise, \( \{M_{bH}, M_{bH} \} \) means that \( H \) has opened \( b_H \) and \( L \) has adopted \( b_H \).

Several features of these plots are worth highlighting:

1. When \( \lambda \) is low, \( H \) opens one of its modules and \( L \) does not adopt. The reason is that when \( \lambda \) is low, the modules of \( H \) and \( L \) are very similar in quality. Therefore, \( L \) prefers to increase its vertical differentiation with \( H \). Thus, \( H \) knows that if it opens a module, it will not be adopted by \( L \).

2. When \( \lambda \) is high but \( \sigma \) is low, \( H \) stays closed and \( L \) opens \( b_L \). \( H \) knows that if it opens a module, it will be adopted by \( L \). The negative competitive effect of \( L \) getting closer in quality is larger than the positive effect of winding up with higher quality due to increased user innovation. This happens because user innovation is weak and initial quality differences are large in that region.

3. There is a small region of intermediate values of \( \lambda \) and \( \sigma \) for which both firms choose \( P \), the closed business model. This is because the strength of user innovation and initial differences in quality are such that \( L \) would adopt any module opened by \( H \). Still, \( L \) prefers to remain closed when \( H \) chooses \( P \) because by opening a module in that case, \( L \) becomes too close in quality to \( H \).

4. For relatively large values of \( \lambda \) and \( \sigma \), both firms choose a mixed source business model. This is because user innovation is strong and \( H \) prefers to open a module even if \( L \) is expected to adopt, and \( L \) prefers to adopt because initial quality differences are sufficiently large.

5. When \( b_L \) is low, firms have more incentives to open \( b \) than to open \( a \). As \( b_L \) increases, it is more likely that firms will open \( a \). This is because if an open source product \( (O) \) was to arise when \( b_L \) is low, it would have quality close to that of \( L \)'s product. This, in turn, would imply low prices for \( L \), which would also affect \( H \). When \( b_L \) is large, the region corresponding to \( \{M_{bH}, M_{bH} \} \) is completely replaced by \( \{M_{aH}, M_{aH} \} \).

6. There are three business model configurations which never arise in equilibrium: \( \{M_{aH}, M_{bL} \} \), \( \{M_{bH}, M_{aL} \} \) and \( \{P, M_{aL} \} \).

5.1.4. **Value creation and value capture.** Strategic interaction affects the creation and capture of value associated to the different business models. The trade-off between value creation and capture depends on the equilibrium under analysis, and several configurations can be analyzed depending on parameter values. Nevertheless, there are regularities that can be

\[^{29}\text{This is a well known feature of Shaked and Sutton’s (1982) model of vertical product differentiation. If quality differences are small, price competition will be strong and will lead to low profits. In this case, both firms will benefit if the quality differential between their products increases. However, if initial differences in quality are large, \( L \) will benefit from getting (somewhat) closer to \( H \).}\]
easily described by studying a particular example. In this section, we focus on \( H \)'s actions, taking into account \( L \)'s best response for each possible business model choice.

Figure 11 shows value creation and value capture for both firms for the case of \( a_L = b_L = 5 \), \( \sigma = 0.5 \), and different values of \( \lambda \). For each case, value creation and capture for \( H \) is represented with small circles. \( L \) is represented with squares.

For all values of the parameters, \( L \) captures less value than \( H \). We can see that the nature of the trade-off for \( H \) changes as \( \lambda \) increases. For \( \lambda = 1.1 \), there is really no trade-off. \( M_{aH} \) provides higher capture and creation than \( M_{bH} \) and \( P \), and even though \( O \) gives more creation than \( M_{aH} \), value capture is 0 under \( O \).

![Figure 11. Value capture and value creation.](image)

As \( \lambda \) increases, the trade-off becomes more pronounced. For \( \lambda = 1.3 \), \( M_{bH} \) gives more capture than \( M_{aH} \), and for \( \lambda = 1.35 \) and \( \lambda = 1.6 \), \( P \) gives more capture than \( M_{aH} \) and \( M_{bH} \). This means that the trade-off becomes more noticeable as the quality differential between the firms increases.

5.2. \( L \) moves first. We now study the game where \( L \) chooses its business model first. The timing is as follows:
(1) $L$ chooses business model.
(2) $H$ chooses business model.
(3) Expectations are formed over the market shares of the different products. Given these expectations, $H$ and $L$ simultaneously choose prices, $p_H$ and $p_L$.
(4) Consumers choose their preferred product.

As in the previous case, opening both modules is a strictly dominated strategy, so we ignore that choice. In addition and as discussed above, if both firms open the same module, $L$’s consumers will replace the lower quality module by the higher quality one (because it is open and freely available) and there will be a de facto adoption of $H$’s module by $L$.

Figure 12 displays the partial representation of the game tree. Note that we have labeled the subgame where $L$ chooses $M_{a_L}$ and $H$ chooses $M_{a_H}$ as $\{M_{a_L}, M_{a_H}\}$. This reflects the fact that although $L$ has chosen to open $a_L$, those individuals that purchase $L$’s product, will combine $b_L$ with the best base module that is freely available. And that is $a_H$. Therefore, the business model profile $\{M_{a_L}, M_{a_H}\}$ is in fact $\{M_{a_H}, M_{a_H}\}$.

As in the model where $H$ chooses first, depending on $L$ and $H$’s business model choices, two or three products will compete to attract customers in the second period. In contrast to the game where $H$ chooses first, however, in this game even if three products are available in the second stage, in equilibrium it may be the case that only one or two of these products winds up receiving positive demand.

For example, consider the subgame where $L$ has opened $a_L$ and $H$ has opened $b_H$. In principle, there are three products available for customers to choose: the (commercial) products of firms $L$ and $H$ and the open source product that combines the opened $a_L$ and $b_H$. In this case, however, if $p_L > 0$, $L$’s product gets no demand because it has the same quality as the open product which is given away for free. Also, if $\sigma$ is large, then the open source product will be of higher quality than the product offered by $H$.

In Figure 12 we have enumerated the different subgames. Before solving for the equilibrium, we should point out the following features of the game:

(1) $O$ leapfrogs $L$: $L$ is pushed out of the market only when the quality of the open source product is higher than that of $L$’s product. In subgame 7, $O$ may leapfrog $L$.
(2) $O$ leapfrogs $H$: $H$ is pushed out of the market only when the quality of the open source product is higher than that of $H$’s product. In subgames 2, 3 and 4, $O$ may leapfrog $H$.

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30 The same applies to the game where $L$ chooses $M_{b_L}$ and $H$ chooses $M_{b_H}$.
31 In some subgames, there are parameter values for which there are multiple equilibria. In these cases, we have selected (a) the equilibrium with the largest number of products and (b) the equilibrium in which $H$ gets more demand.
(3) $L$ leapfrogs $H$: In subgames 2, 3, 4 and 6, $L$ may leapfrog $H$. When $L$ leapfrogs $H$, but $O$ does not, $H$ is not pushed out the market.

(4) No leapfrogging: Leapfrogging (either $L$ over $H$, $O$ over $L$, or $O$ over $H$) may occur in subgames 2, 3, 4, 6, and 7. In subgames 1, 5, 8, and 9 there is no leapfrogging.

As usual, we solve the game recursively. Because there are so many subgames with potential leapfrogging and so many possible equilibrium product configurations, this game is harder to solve than the one of Section 5.1.

First, we solve the second period. As we just discussed, the number of products in equilibrium depends on each business model configuration. There are four possibilities: (i) one open product, (ii) one open and one commercial product, (iii) two commercial products, and (iv) one open and two commercial products (three products). In case (i), both firms have zero-profits. We have analyzed case (ii) in Sections 3.1 and 3.2. Finally, cases (iii) and (iv) correspond to our analysis in Section 5.1.
Once we have the continuation profits for the nine subgames, we proceed to solve numerically for the first period outcomes (choice of business model). We present the equilibrium regions for different parameter values in the following section.

5.2.1. Business model choice. Figure 13 shows the equilibrium business model configurations for different parameter values. As in the previous section, we have set $a_L = b_L$, $a_H = \lambda a_L$, and $b_H = \lambda b_L$, where $\lambda > 1$.

The labels inside the regions indicate the equilibrium business model configuration. The first element in the label corresponds to $L$, the first mover. For example, $\{M_{bL}, P\}$ means that $L$ has chosen to open $b_L$, and $H$ has chosen to stay closed. Therefore, $\{M_{bL}, P\}$ when $L$ moves first is equivalent to $\{P, M_{bL}\}$ when $H$ moves first (given the same parameters, both of them generate the same equilibrium demands and payoffs).

Several features of these plots are worth underlining:

Figure 13. Equilibrium regions when $L$ moves first
(1) When $\lambda$ is low and/or $\sigma$ is low the equilibrium has one mixed source firm and one proprietary firm. When $\lambda$ is low, firms do not want to get too close in quality because of the harsh price competition that ensues when products are not too differentiated. This means that only one firm can take advantage of user innovation when $\lambda$ is low. The intuition why when $\sigma$ is low and $\lambda$ is large there is one mixed source firm and one proprietary firm is that if $H$ opened a module, it would be adopted by $L$ and the products would become insufficiently differentiated (from $H$’s viewpoint).

(2) There are two cases in which $L$ will be the unique mixed source firm: when $\lambda$ is large, but $\sigma$ is small, and when $\lambda$ is very small (close to 1). In the first case, $L$ opens $b_L$, but $H$ does not want to open $b_H$ because the effect of $L$ getting closer in quality is larger than the effect of higher quality due to user innovation. In the second case, $L$ opens some module, and leapfrogs $H$, but $H$ prefers to stay as the lower quality firm, with a larger quality differential, rather than overtaking $L$ and having very similar quality.

(3) When $\lambda$ and $\sigma$ are relatively high, both firms choose to be mixed source. The initial quality differential is very large, so $L$ wants to get closer in quality to $H$. On the other hand, user innovation is significant and both firms want to take advantage of the increase in quality from opening some module.

(4) When $a_L$ and $b_L$ are small, firms prefer to open $b$ rather than $a$. As $a_L$ and $b_L$ increase, it is more likely that firms will open $a$. The reason is that when firms open $a$ an open source product arises, and if $b_L$ is small this open source product would have a similar quality to $L$’s product. This, in turn, would imply low prices for $L$, which would also affect $H$.

5.2.2. Value creation and value capture. As in the previous case, the trade-off between value creation and value capture is affected by the strategic interaction between the two firms. In this section, we focus on the effects of $L$’s actions, taking into account $H$’s best response for each possible action.

Figure [14] shows value creation and value capture for both firms for the case of $a_L = b_L = 3$, $\sigma = 0.5$, and different values of $\lambda$. For each case, value capture for $H$ is represented with small circles, and value capture for $L$ is represented with squares.

A first result is that when $L$ plays first, it may capture more value than $H$. This will happen when $\lambda$ is very close to 1, so that both firms are very similar. In this case, if $L$

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32Having understood that when $\lambda$ is low the equilibrium has one mixed source firm and one proprietary firm regardless of the value of $\sigma$, we need only consider low $\sigma$ when $\lambda$ is high.
decides to open some module first, \( H \) will decide to remain closed, and as a result \( L \) will leapfrog \( H \).

An interesting result is that when \( \lambda \) is small, there is room for only one mixed source firm in the market. We can see that if \( L \) decides to remain closed, \( H \) will open \( a_H \), but if \( L \) opens some module, \( H \) will react by remaining completely closed.

We can also see that for small \( \lambda \), opening \( a_L \) gives higher value capture and value creation than opening \( b_L \), so there is no trade-off. For larger values of \( \lambda \), opening \( b_L \) gives higher value capture, so a trade-off between the two mixed source business models arises.

5.3. **Comparison of the two alternative timings.** The two games analyzed before share some common features, but also have important differences. The first difference is that leapfrogging may occur in equilibrium when \( L \) plays first. Second, when \( L \) plays first, the area in which both firms choose the proprietary business model \( P \) disappears. Therefore, when \( L \) plays first, it is more likely that we have at least one firm choosing Mixed Source. Finally, the area with open \( a \) is smaller than before. This is because \( L \) prefers a world with open \( b \) rather than open \( a \). Also, the equilibrium region where both firms open \( b \) is larger.
6. Managerial implications

We have presented a series of models in order to analyze the optimal choice of business model in industries where technological openness and user innovation are important. Our findings give a formal base to existing managerial practice, but also offer new insights... However, these results may be difficult to interpret because of the inherent complexity associated with the models.

In this section we present a simplified framework for understanding our results. In each case, we try to discuss the essence behind our propositions and lemmas, and to schematize the figures which illustrated those theorems. This has the advantage of yielding simple policy recommendations, which can be directly applied to the design of an optimal business strategy.

As in the real world, the choice of business model depends on the type of competition the firm faces, the relevance of user innovation, and how important is the complementary good or service the firm sells in addition to software.

Finally, we also discuss the implications of our findings for the strategy literature, particularly in the field of value-based business strategy.

6.1. Monopoly. Our analysis of the monopoly case shows that the firm will in general choose to be very open: when there are no competing products in the market, there is little harm in opening at least those technologies which have no standalone value.

This is why the firm will prefer to open the extensions to remaining completely closed, even when the value of service and user innovation are very small. Nevertheless, there are situations in which the firm may find it optimal to open the core or to become completely open. This will happen when user innovation and the value of the complementary good are high enough, in which case the firm will want to build a strong user base and profit from selling the complementary good.

The following figure is a stylized version of Figure 4. It shows the business policy recommendation for different degrees of the value of service and user innovation. The figure shows that the firm should choose OS when service and user innovation are high, and should choose a MS model when either service or user innovation, or both, are not very high.

Our findings show the relevance of the Open Extensions model in the real world, which could be considered as an inexpensive way to become more open. Two good examples are Microsoft’s .Net and Stata. In the first case, Microsoft is committed to opening the languages which can be compiled with .Net (Visual Basic, C#, J#, etc.), even to the point of promoting open standards. However, users still need to use .Net to compile the programs developed with these languages. Likewise, StataCorp has opened hundreds of ado files,
which are econometric programs used to perform specific tasks (like performing an estimation of Maximum Likelihood for a particular econometric model). However, users still need to use Stata to compile those programs.

6.2. **Competition with a non-profit Open Source project.** The presence of an open source alternative affects the optimal decisions of the firm. The exact way in which it will affect them depends on whether the modules of the open source project have better or worse quality than the modules of the firm.

Our first finding is that the firm will always want to embed the modules of the open source project when they have higher quality. The result is due to our assumption of the complementarity between the modules and service. The firm may consider not adopting the open source modules because in doing so, it increases the demand for the open source project (the consumers of the firm engross the user base of the open source project, which increases its quality). However, the complementarity implies that the product of the firm increases relatively more than the open source product, which will always benefit the firm.  

A good example is IBM’s support of Linux. IBM had several competing operating systems (like Z/OS), but it began supporting Linux because it was a good operating system, with a larger user base, and it could profit from selling Linux related support and consultancy services. Currently, IBM provides support for over 500 software products running on Linux, and has more than 15,000 Linux-related customers worldwide.

The above results change when the firm has modules of higher quality. Figure XXX shows a stylized version of Figures 6a and 6b. As in the monopoly case, the firm will choose the Open Source model when user innovation and value of service are high. The firm will still choose a Mixed Source model when user innovation is intermediate or high. However,

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33 This result may not hold if the service of the firm is specific to its software, or if the product of the firm is horizontally differentiated, in addition to vertically differentiated.

in contrast with the previous case, the firm will choose the Proprietary model when user innovation is not very important.

Therefore, we have the surprising result that the firm may react to competition from a non-profit open source competitor by becoming more closed. This result defies the conventional wisdom that firms will always react to open source by becoming more open.

With respect to the module the firm should open when deciding to be Mixed Source, an interesting finding is that the firm should open the module which faces stronger competition, and keep closed the module that is more complementary to the software of the open source project.

6.3. Competition between for-profit firms. In this case, we analyzed the optimal decisions of two firms who differ in the quality of their modules: a high quality firm ($H$) and a low quality firm ($L$). For simplicity, we analyzed a case in which the complementary service does not play an important role, and therefore, firms will never choose the open source model. In this case, the relevant parameters are the initial quality difference between the two firms and the strength of user innovation.

The optimal strategy will depend on whether the firm is the quality leader or follower, and on whether the firm is the first to act, or it is reacting to the actions of other firms. Figures 17a and 17b show the optimal business model for both firms when $H$ and $L$ play first, respectively (stylized versions of Figures 10 and 13). In both figures, the first business model mentioned corresponds to the firm moving first, and the second to the firm moving second. For example, when $L$ moves first, user innovation is low and value of service is high, $L$ will choose to be Proprietary and $H$ will choose to be Mixed Source.

We can see in both figures that when the initial quality difference or user innovation are not high, the market will have one firm deciding to be Proprietary and another deciding to be Mixed Source. Therefore, there is a heterogeneity between the business models chosen by both firms, which arises endogenously in the model. When the quality difference and
user innovation are high, both firms choose the same business model, so homogeneity can be reached as a market outcome as well.

We can see that the stylized versions of both figures are similar (even though the original figures were somewhat different). However, there is an interesting difference which has to be remarked: if the firm with lower quality is the first to act, it may leapfrog the quality leader. This will happen when the initial quality difference is not very large, and the $L$ firm is the first to move. Both firms would prefer to move first, so the market outcome will depend on which firm is more agile to respond to the opportunity presented by open source. Second, when $L$ moves first, the equilibrium area in which both firms choose the proprietary business model dissappears. This means that the market will tend to be more open when firms with lower quality products are able to choose their strategy faster.

The duopoly case gives formal support to the observation that firms react to changes in their competitors business model by adapting their own business model. An interesting example is the case of IBM and JBoss. Both firms were competing in the middleware segment. Middleware is software which is used as a bridge to connect other programs, and is one of the most important market segments in the software industry. Initially, IBM was completely closed, but JBoss started gaining market share using an open source model and selling complementing proprietary software and support services. To respond, IBM bought a small firm called Gluecode, which was developing proprietary extensions over the open source middleware project Apache Geronimo, and completely opened such software. This move was aimed at reducing the user base of JBoss. Interestingly, IBM completely opened Gluecode’s software, but still maintained more advanced middleware programs proprietary. This example shows that there is a very active competition between for-profit firms, which are constantly redefining their business models.

---

6.4. **Business model heterogeneity.** Our previous analysis shows that under some circumstances, there is heterogeneity in the choice of business model. On one side, we obtain the result that the more similar are the firms’ products, the more heterogeneous are their business models. Thus, our model suggests that when product differentiation is low, firms will differentiate through business models. On the other side, firms will also choose to compete with different business models when user innovation is low.

Therefore, the recommendation for managers is: “compete differently than the first mover when your product is similar to that of your competitor,” but also, “compete differently than the first mover when user innovation is mild.”

6.5. **Value creation vs. value capture.** Our analysis shows that the key trade-off when choosing the optimal business model is that between value creation and value capture, that is, the conflict between increasing total value generated (consumer surplus plus producer surplus), and increasing the fraction of that value which is obtained by the firm.

There are several points worth discussing. First, as Salas-Fumas (2009) indicates, the choice of business model varies depending on whether firms act to maximize value creation or profits.\(^{36}\)

In our base model, maximum value creation occurs when the business model chosen is OS, but this may not be the model that maximizes profits or value capture: OS will be optimal from a profit-maximizing viewpoint only if the value of the complementary good is sufficiently large or there is an open competitor that is of higher quality.

Second, the trade-off between value creation and value capture is affected by the presence of competitors. There are some cases in which less openness does not necessarily mean more value capture. This is because how much value is captured and how much is created depends on the business models of different industry players. When choosing a business model, one needs to foresee what will be the reaction (in terms of business model choice) of other firms in the industry.

Third, the usual recommendation that firms should ”maximize the creation and capture of value” does not apply generally. For example, [provide example of definition of strategy that is related to the creation and capture of value] ... Figures XX, XX, and XX, reveal that increased value creation generally goes hand-in-hand with lower, rather than higher, value capture. Therefore, value creation and value capture are not monotone transformations of one another. Thus, it is often the case that as value creation increases, value capture plummets.

\(^{36}\)Salas-Fumas (2009) defines value creation in the same way that we do, but he reserves the term value capture for the money amount of value grabbed by the firm (i.e. profits), whereas we use it to indicate the proportion of value grabbed by the firm.
While the popular case for mixed source of Figure 1 already recognizes that improved value creation does not necessarily imply better value capture, our analysis qualifies the recommendations from Figure 1. We see that MS is not always the optimal business model. Sometimes P or OS may deliver higher profits.

7. Conclusion

This paper is a contribution to the literature analyzing open source software and user innovation. We develop a model in which firms have multiple software modules and must decide which modules to open and which to keep proprietary. These firms must choose not only between the two “pure” business models (open source and proprietary), but may also compete through a mixed source model. We analyzed the monopoly case, the case in which a for-profit firm competes against a non-profit OS project, the case in which it competes against another for-profit firm.

Our model presents several interesting findings, which we have presented in a simplified framework in the previous section. This framework can be used in a direct way in the design of an optimal business strategy.

We think that open source is a very interesting and current topic, which deserves further study. In particular, it would be interesting to study how the results of this paper would change if products can be made compatible or incompatible, or if firms could engage in horizontal differentiation.

In addition, our paper contributes to the recent strategy literature analyzing the competitive interactions between organizations with different business models. This line of research has become increasingly interesting, as new technologies, regulatory changes, and new customer demands have allowed firms to implement new approaches to competing in a wide range of industries spanning from airlines (e.g., Ryanair) to furniture (e.g., IKEA) and from the circus (e.g., Cirque du Soleil) to betting (e.g., Betfair). In fact, many of the fastest-growing firms in the recent past appear to have taken advantage of opportunities sparked by globalization, deregulation, or technological change to “compete differently” and to innovate in their business models (see Kim and Mauborgne (2005) and Markides (2008) for additional examples).

Where do the business models that we observe come from? Our contention is that, to a large extent, the configuration of business models in an industry is the equilibrium outcome of a search process for higher profits. We have proposed and illustrated a methodology for the study of endogenous business models; a two-period game where in the first period (the strategy period) business models are chosen and in the second period (the tactics period) firms interact by making tactical choices as allowed by their models. By choosing particular
business models, firms are committing to the tactical subgames that will ultimately determine the profits that they will earn. To the best of our knowledge, ours is the first paper in the strategy literature where firms interacting in the market place, in addition to best responding to each other through tactical choices (such as prices), compete by reconfiguring their business models.\textsuperscript{37}

Our approach to modeling competition through business models has similarities with the biform games introduced by Brandenburger and Stuart (2007). A biform game is a two-period game in which players first make strategic choices that determine the second period cooperative game that ensues. The first period is the strategy period where the bounds of the tactical subgame are set. The difference between our approach and biform games is that our second period is a noncooperative game. Therefore, the size of the final “pie of value” in our setting depends on the first period business model choices and on the equilibrium tactical interaction. In the biform game formulation, the size of the pie is determined by the first period choices and it remains fixed in the second period.

One significant implication of our approach to modeling the second period is the following. In Brandenburger and Stuart (2007), the larger the value created in the strategy period, the better off all firms end up because every unit of value created winds up being shared between all industry participants in the second period (in exogenously given proportions). In our formulation, however, larger value created in the strategy period does not imply improved value capture for all participants because (i) the equilibrium of the tactical game is generally not efficient; and (ii) the bargaining power of the different firms in the second period depends on the business models chosen. As a consequence, and as pointed out by Salas-Fumas (2009), the advice “choose your business model to create and capture as much value as possible” does not apply generally. As our results show, value creation and value capture need not be positively correlated. Perhaps better advice for profit-maximizing firms competing through business models is: “choose your business model to capture as much value as possible, taking into account the likely strategic and tactical reactions of other firms with which you will interact.”

The generic two-period game that we have presented can be applied to all sorts of competitive settings such as strategies to fight ad-sponsored rivals, strategies to fight low cost entrants, strategies to fight platform players, and the like. We hope that our analysis of mixed source is helpful to researchers and practitioners willing to consider competition beyond tactics and to have provided a solid first step towards a general framework for the study of competition through business models.

\textsuperscript{37}Casadesus-Masanell and Zhu (2009) consider business model reconfigurations by one firm only. In the present paper, both competitors business models are endogenous.
REFERENCES


Appendix A: Proofs of theorems in text

Proposition 1. Suppose that either $2 < b < 3$ and $z \leq \frac{2}{(3-b)b}$, or $b \geq 3$ hold. Then, if $\sigma \leq \sigma_{a|b}$, $M_b$ is the optimal business model; if $\sigma_{a|b} < \sigma \leq \sigma_{a|ab}$, $M_a$ is the optimal business model; and if $\sigma > \sigma_{a|ab}$, the optimal business model is $O$. Suppose, instead, that either $2 < b < 3$ and $z > \frac{2}{(3-b)b}$, or $1 \leq b \leq 2$ hold. Then, if $\sigma \leq \sigma_{b|ab}$, $M_b$ is the optimal business model; and if $\sigma > \sigma_{b|ab}$, $O$ is the optimal business model.

Proof. The thresholds
$$\sigma_{a|b} := \frac{2}{b z - 2}, \quad \sigma_{a|ab} := \frac{b - 1}{b(z - 1)}, \quad \sigma_{b|ab} := 1 + \sqrt{\frac{z(z - 8)}{4(z - 1)}} - \frac{3}{4}$$
are such that:

$$\pi_a > \pi_b \quad \text{if and only if} \quad \sigma > \sigma_{a|b},$$
$$\pi_a > \pi_{ab} \quad \text{if and only if} \quad \sigma < \sigma_{a|ab},$$
$$\pi_b > \pi_{ab} \quad \text{if and only if} \quad \sigma < \sigma_{b|ab}.$$

We study the six possible orders between the thresholds, taking into account that $\sigma > 0$, $b > 1$, and $z > 1$.

A little algebra shows that the only two possibilities are: $\sigma_{a|ab} < \sigma_{b|ab} < \sigma_{a|b}$ and $\sigma_{a|b} < \sigma_{b|ab} < \sigma_{a|ab}$. The other four rankings are impossible, given our assumptions on the values that the model parameters may take. Let’s consider the two possible orders separately.

Case 1: $\sigma_{a|ab} < \sigma_{b|ab} < \sigma_{a|b}$. This happens when $1 < b \leq 2$ and $z > \frac{2}{b}$ or $2 < b < 3$ and $z > \frac{2}{3b-2}$. Figure 18 illustrates this situation.

![Figure 18. $\sigma_{a|ab} < \sigma_{b|ab} < \sigma_{a|b}$](image)

Case 2: $\sigma_{a|b} < \sigma_{b|ab} < \sigma_{a|ab}$. This happens when $2 < b < 3$ and $1 < z < \frac{2}{3b-2}$ or when $b \geq 3$. Figure 19 illustrates this situation.
Lemma 1. When the modules of the firm have higher quality than the modules of non-strategic open source competitor, the firm will never choose to embed those modules in its commercial product, i.e. it will never choose the $M_a$, $M_b$ or $O_{ab}$ business models.

Proof. We have to prove that $M_a$, $M_b$ and $O_{ab}$ are always dominated by other model. It will suffice to compare $M_a$ with $M_b$, $M_b$ with $M_b$, and $O_{ab}$ with $O_{ab}$.

We will now prove that $M_a$ is dominated by $M_b$ for all parameter values. In any of the two business models, in equilibrium $1/2$ of the users consume the commercial product, and $1/2$ of the users choose the open source product. With the first business model, the firm’s product has quality $v_c = \hat{a}(1+\sigma)bz$ and the open source product has quality $v_o = \hat{a}(1+\sigma)\hat{b}(1+\sigma/2)$. Profits are $\hat{a}(1+\sigma)(bz-\hat{b}(1+\sigma/2))$. With the second business model, the firm’s product has quality $v_c = a(1+\sigma)bz$ and the open source product has quality $v_o = a(1+\sigma)\hat{b}(1+\sigma/2)$. Profits become $a(1+\sigma)(bz-\hat{b}(1+\sigma/2))$. Clearly profits are higher under $M_a$ because $a > \hat{a}$. Even though the open source product increases its quality when the firm open its product, in comparison with the case in which it adopts the inferior OS module, the quality of the commercial product increases even more, because of its complementarity with $b$ and $z$.

The proofs for $M_b$ and $O_{ab}$ follow the same steps, appropriately redefining the firm profits in each case.

Lemma 2. $M_a$ is preferred to $M_b$ if and only if $a/\hat{a} < b/\hat{b}$, and vice versa.

Proof. To see this, note that profits under $M_a$ and $M_b$ are, respectively, $\pi_a = (a_o b z - a_o \hat{b}_o)/4$ and $\pi_b = (a b_o z - \hat{a}_o b_o)/4$. The result follows directly from observing that $a_o b z := a b_o z$. Hence, the comparison is between $a_o \hat{b}_o$ and $\hat{a}_o b_o$ or, more simply, between $a \hat{b}$ and $\hat{a} b$.

Proposition 2 (Equilibrium regions). If $a/\hat{a} < b/\hat{b}$, then $P$ is optimal when $z < z_{a/p}$, $M_a$ is optimal when $z_{a/p} < z < z_{ab/a}$ and $O_{ab}$ is optimal when $z > z_{ab/a}$. If $a/\hat{a} > b/\hat{b}$, then $P$ is optimal when $z < z_{b/p}$, $M_b$ is optimal when $z_{b/p} < z < z_{ab/b}$ and $O_{ab}$ is optimal when $z > z_{ab/b}$. 

\[ \pi_a = (a_o b z - a_o \hat{b}_o)/4 \text{ and } \pi_b = (a b_o z - \hat{a}_o b_o)/4. \]

The result follows directly from observing that $a_o b z := a b_o z$. Hence, the comparison is between $a_o \hat{b}_o$ and $\hat{a}_o b_o$ or, more simply, between $a \hat{b}$ and $\hat{a} b$. 

Proposition 2 (Equilibrium regions). If $a/\hat{a} < b/\hat{b}$, then $P$ is optimal when $z < z_{a/p}$, $M_a$ is optimal when $z_{a/p} < z < z_{ab/a}$ and $O_{ab}$ is optimal when $z > z_{ab/a}$. If $a/\hat{a} > b/\hat{b}$, then $P$ is optimal when $z < z_{b/p}$, $M_b$ is optimal when $z_{b/p} < z < z_{ab/b}$ and $O_{ab}$ is optimal when $z > z_{ab/b}$.
Proof. It is easy to see that:

\[ \pi_a > \pi_p \quad \text{if and only if} \quad z > \frac{\hat{b}(2 + \sigma)(2a(1 + \sigma) - \hat{a}(2 + \sigma))}{4a b \sigma} := z_{a|p}, \]

\[ \pi_b > \pi_p \quad \text{if and only if} \quad z > \frac{\hat{a}(2 + \sigma)(2b(1 + \sigma) - \hat{b}(2 + \sigma))}{4a b \sigma} := z_{b|p}, \]

\[ \pi_{ab} > \pi_a \quad \text{if and only if} \quad z > \frac{2b(1 + \sigma) - \hat{b}(2 + \sigma)}{2 b \sigma} := z_{ab|a}, \]

\[ \pi_{ab} > \pi_b \quad \text{if and only if} \quad z > \frac{2a(1 + \sigma) - \hat{a}(2 + \sigma)}{2 a \sigma} := z_{ab|b}. \]

Given this, the proposition follows from Lemma 2 and a straightforward comparison of the thresholds presented above.

Lemma 3. When the modules of the non-strategic open source competitor have higher quality than the modules of the firm, the firm will always choose to embed those modules in its commercial product, i.e. it will always choose the \(O_{\hat{a}\hat{b}}\) business model.

Proof. With any business model, if the commercial product ends up with lower quality than the open source product in equilibrium, it has zero demand and thus firm’s profit become zero. The firm will then adopt some or all of the OS modules. Therefore, we can concentrate on the case in which \(z\) is large enough, so that the commercial product would have higher quality than the open source product in equilibrium if the firm were to choose a business model other than \(O_{\hat{a}\hat{b}}\).

In equilibrium, then, the commercial product and the open source product have each \(1/2\) of the market. This means that if the firm adopts an OS module, the user innovation term of any module will be \((1 + \sigma)\), whereas if the firm does not adopt the OS module the user innovation term is \((1 + \sigma/2)\). Therefore, when the firm adopts the module, there are two effects: (1) the commercial product increases quality because it changes modules from \(a\) or \(b\) to \(\hat{a}\) or \(\hat{b}\), which have higher quality, (2) the commercial product and the open source product increase quality because the user innovation term for each module goes from \((1 + \sigma/2)\) to \((1 + \sigma)\). However, the second effect is higher for the firm, because it gets multiplied by \(z > 1\). Clearly, the firm will benefit from adopting both OS modules.
APPENDIX B: MULTIPRODUCT MONOPOLIST

Not for publication.

In our analysis of the monopolist, we have imposed that the firm offers one commercial product only. Moreover, we have assumed that the product that the firm commercializes is composed of the base module, the extensions, and service. Because the independent modules and service are available to the monopolist for separate commercialization, one question that arises is whether the monopolist prefers to offer more than one commercial product.

The answer turns out to be negative: even if we allowed the monopolist to sell more than one product, she will choose to offer only the product that we have considered: a base with extensions and service.

Lemma 4. If we let the monopolist choose how many commercial products to sell, she will choose to offer one product only. Moreover, the chosen product is the one that we have considered in Section 3.

Proof. We consider all possible product combinations available to the monopolist.

Case 1: Monopoly with four products. The monopoly offers the base module at price $p_1$, the base module together with the set of extensions at price $p_2$, the base module with service at price $p_3$, and the complete package (base module, extensions, and service) at price $p_4$.

Let the qualities of the three products be $v_1$, $v_2$, $v_3$, and $v_4$. Clearly, $0 < v_1 < v_2 < v_3 < v_4$. The demand functions for each of the products are:

\[
\begin{align*}
q_1 &= \frac{p_2 - p_1}{v_2 - v_1} - \frac{p_1}{v_1} \\
q_2 &= \frac{p_3 - p_2}{v_3 - v_2} - \frac{p_2 - p_1}{v_2 - v_1} \\
q_3 &= \frac{p_4 - p_3}{v_4 - v_3} - \frac{p_3 - p_2}{v_3 - v_2} \\
q_4 &= 1 - \frac{p_4 - p_3}{v_4 - v_3}
\end{align*}
\]

Profits are $\pi_{1,2,3,4} = q_1 p_1 + q_2 p_2 + q_3 p_3 + q_4 p_4$. Solving the system of three first-order conditions yields:

\[
p_1 = \frac{v_1}{2}, \quad p_2 = \frac{v_2}{2}, \quad p_3 = \frac{v_3}{2}, \quad \text{and} \quad p_4 = \frac{v_4}{2}.
\]

Substituting in the demand functions, we obtain: $q_1 = 0$, $q_2 = 0$, $q_3 = 0$, and $q_4 = \frac{1}{2}$. Therefore, the profit-maximizing monopolist prefers to offer the complete product only.

Case 2: Monopoly with three products. The monopoly offers the base module at price $p_1$, the base module together with the set of extensions or the base module with service at price $p_2$, and the complete package (base module, extensions, and service) at price $p_3$.
Let the qualities of the three products be $v_1$, $v_2$, and $v_3$. Clearly, $0 < v_1 < v_2 < v_3$. The demand functions for each of the products are:

\[
q_1 = \frac{p_2 - p_1}{v_2 - v_1} - \frac{p_1}{v_1} \\
q_2 = \frac{p_3 - p_2}{v_3 - v_2} - \frac{p_2 - p_1}{v_2 - v_1} \\
q_3 = 1 - \frac{p_3 - p_2}{v_3 - v_2}
\]

Profits are $\pi_{1,2,3} = q_1 p_1 + q_2 p_2 + q_3 p_3$. Solving the system of three first-order conditions yields:

\[
p_1 = \frac{v_1}{2}, \quad p_2 = \frac{v_2}{2}, \quad \text{and} \quad p_3 = \frac{v_3}{2}.
\]

Substituting in the demand functions, we obtain: $q_1 = 0$, $q_2 = 0$, and $q_3 = \frac{1}{2}$. Therefore, the profit-maximizing monopolist prefers to offer the complete product only.

**Case 3: Monopoly with two products.** There are several combinations of two products that the monopolist may offer. In the analysis that we are now conducting, they are all equivalent. For example, the firm may offer the base module at price $p_1$ and the base module together with the set of extensions at price $p_2$. Or it may offer the base module at price $p_1$ and the base module together with the set of extensions and service at price $p_2$.

Let the qualities of the two products be $v_1$ and $v_2$. Clearly, $0 < v_1 < v_2$. The demand functions for each of the products are:

\[
q_1 = \frac{p_2 - p_1}{v_2 - v_1} - \frac{p_1}{v_1} \\
q_2 = 1 - \frac{p_2 - p_1}{v_2 - v_1}
\]

Profits are $\pi_{1,2} = q_1 p_1 + q_2 p_2$. The profit-maximizing solution has: $q_1 = 0$ and $q_2 = \frac{1}{2}$. Once again, the profit-maximizing monopolist prefers to offer the complete product only.

**Case 4: Monopoly with two products when there is an open source alternative.** This case corresponds to the monopolist opening module $a$, which sets up an OS competitor, and fighting the open source alternative with two, rather than one commercial product. The two products are: the base module with the set of extensions sold at $p_1$, and the complete product (base module, extensions, and service) sold at $p_2$.

Let the quality of the open source product be $v_0$ and those of the commercial products be $v_1$ and $v_2$. Clearly, $0 < v_o < v_1 < v_2$. The demand functions for each of the commercial products are:

\[
q_1 = \frac{p_2 - p_1}{v_2 - v_1} - \frac{p_1}{v_1 - v_0} \\
q_2 = 1 - \frac{p_2 - p_1}{v_2 - v_1}
\]
Profits are $\pi_{1,2} = q_1 p_1 + q_2 p_2$. The profit-maximizing solution has: $q_1 = 0$ and $q_2 = \frac{1}{2}$. Once again, the profit-maximizing monopolist prefers to offer the complete product only. The free open source product obtains a market share of $\frac{1}{2}$.

Case 5: Monopoly with three products when there is an open source alternative. This last case corresponds to the monopolist opening module $a$, which sets up an OS competitor, and fighting the open source alternative with three commercial products. The three products are: the base module with the set of extensions sold at $p_1$, the base module with service sold at $p_2$, and the complete product (base module, extensions, and service) sold at $p_3$. Following the same steps as in the above cases, it is easy to see that the profit-maximizing monopolist prefers to offer the complete product only.

APPENDIX C: ALTERNATIVE COMPETITIVE SCENARIOS. 

Not for publication.

In this section, we analyze alternative scenarios when there is a non-profit open source competitor for the products of the firm. We will see that the main conclusions of our preceding analysis still hold.

7.1. Relevant business models. The possibility of adoption of the modules provided by outside OS project means that there are multiple implementations of the mixed and open source business models. The possibilities are: $M_a$, $M_\hat{a}$, $M_b$, $M_\hat{b}$, $O_{ab}$, $O_{\hat{a}b}$, $O_a \hat{b}$, and $O_{\hat{a}\hat{b}}$. The following lemma says that given the values of $a$, $\hat{a}$, $b$, and $\hat{b}$, only one implementation of each business model is relevant.

Lemma 5. When choosing between $M_i$ and $O_{ij}$ ($i, j \in \{a, \hat{a}, b, \hat{b}\}$, $i \neq j$) business models, the profit-maximizing firm will never choose models embedding modules of lower quality. If $\hat{a} > a$ ($\hat{a} \leq a$), the firm will never choose an $M_i$ or $O_{ij}$ business model that uses $a$ ($\hat{a}$). Likewise, if $\hat{b} > b$ ($\hat{b} \leq b$), the firm will never choose an $M_i$ or $O_{ij}$ business model that uses $b$ ($\hat{b}$).

Proof. There are many cases depending on which modules are provided by the OS project. The continuation profit for the incumbent is always $(v_c - v_o)/4$, but the specific values of $v_c$ and $v_o$ depend on the case under analysis.

For ease of exposition, we will decompose quality into two parts: $v_c = m_c k_c$ and $v_o = m_o k_o$. $m_c$ and $m_o$ depend on whether the OS module has been adopted or not. $k_c > 1$ and $k_o \geq 1$ are constants that do not change regardless of adoption of the OS module. For example, suppose that $\hat{a} > a$ and that the firm is opening $b$, with $b \leq \hat{b}$. We are interested in comparing the payoffs of adopting $\hat{a}$ with those of opening $a$. If the firm opens $a$, then $v_c = a(1 + \sigma/2)b(1 + \sigma)z$, and $v_o = \hat{a}(1 + \sigma/2)b(1 + \sigma)$. If the firm adopts $\hat{a}$, then $v_c = \hat{a}(1 + \sigma)b(1 + \sigma)z$, and $v_o = \hat{a}(1 + \sigma)b(1 + \sigma)$. In this case, $k_c = b(1 + \sigma)z$ and $k_o = b(1 + \sigma)$.
Recall that the user innovation term depends on the expectations of the measure of consumers using an open module. When the commercial product \( C \) and the outside open source product \( OS \) are based on different open modules, the user innovation term is \((1 + \sigma/2)\) for both products. When \( C \) and \( OS \) share the same open module, the user innovation term becomes \((1 + \sigma)\).

The decomposition \( v_c = m_c k_c \) and \( v_o = m_o k_o \) allows us to classify every possible case into one of three possibilities:

1. \( \hat{a} > a \) or \( \hat{a} \leq a \), for any value of \( b \) and \( \hat{b} \).
2. \( \hat{b} > b \) or \( \hat{b} \leq b \), when \( \hat{a} \) exists or \( a \) is open.
3. \( \hat{b} > b \) or \( \hat{b} \leq b \), when \( \hat{a} \) does not exist and \( a \) is not open.

We analyze each case separately.

**Case 1:** \( \hat{a} > a \) or \( \hat{a} \leq a \), for any value of \( b \) and \( \hat{b} \).

Suppose \( \hat{a} > a \). We do not impose any restrictions on \( b \) and \( \hat{b} \): \( b \) may be open or closed, there may be a \( \hat{b} \) or not, and \( \hat{b} \) may be larger or smaller than \( b \) if the former exists. If the firm opens \( a \), \( v_c = a(1 + \sigma/2)k_c \) and \( v_o = \hat{a}(1 + \sigma/2)k_o \). If the firm adopts \( \hat{a} \), \( v_c = \hat{a}(1 + \sigma)k_c \) and \( v_o = \hat{a}(1 + \sigma)k_o \). Clearly, the firm will always adopt \( \hat{a} \). Suppose now that \( \hat{a} \leq a \). If the firm opens \( a \), \( v_c = a(1 + \sigma)k_c \) and \( v_o = a(1 + \sigma)k_o \). If the firm adopts \( \hat{a} \), \( v_c = \hat{a}(1 + \sigma)k_c \) and \( v_o = \hat{a}(1 + \sigma)k_o \). In this case, the firm will open \( a \). Therefore, if the firm is considering a \( MS \) or \( OS \) model with open \( a \), it will include the higher quality module in its commercial version.

**Case 2:** \( \hat{b} > b \) or \( \hat{b} \leq b \), when \( \hat{a} \) exists or \( a \) is open.

The proof is essentially the same as that of case 1.

**Case 3:** \( \hat{b} > b \) or \( \hat{b} \leq b \), when \( \hat{a} \) does not exist and \( a \) is not open.

In this case, if the firm opens \( b \), then \( v_c = b(1 + \sigma/2)k_c \) and \( v_o = 0 \). If the firm adopts \( \hat{b} \), then \( v_c = \hat{b}(1 + \sigma)k_c \) and \( v_o = 0 \). Therefore, if the firm is considering a \( MS \) or \( OS \) model with either \( b \) or \( \hat{b} \) open, it will choose the model with \( \hat{b} \). The opposite result is obtained when \( \hat{b} < b \). This concludes the proof.

**Lemma 5** offers a powerful simplification: the firm will only consider four business models. For example, if \( \hat{a} > a \) and \( \hat{b} \leq b \) or \( \hat{b} \) does not exist, the firm will only consider \( P \), \( M_{\hat{a}} \), \( M_b \) and \( O_{\hat{a}b} \); if \( \hat{b} > b \) and \( \hat{a} \leq a \) or \( \hat{a} \) does not exist, the firm will only consider \( P \), \( M_a \), \( M_{\hat{b}} \) and \( O_{ab} \); and so on.

Interestingly, **Lemma 1** is a particular case of **Lemma 5**. The intuition is the same as before, complementarity implies that the firm always benefits more than the open source project from adopting superior modules.

Having established this helpful result, we move on to analyzing some of the cases that may arise. We first study the case of competition by a low-quality outside OS project. We consider two cases: (1) the entrant competes with a base module only, and (2) the entrant competes with the set of extensions only. Remember that we have analyzed the case in which the entrant competes with both modules in the main text.
7.2. **Outside OS project competes with base module** \( \hat{a} \leq a \). We begin studying the case where the outside OS project consists only of a base module \( \hat{a} \) which is of lower quality than the firm’s corresponding module \( a \). Regardless of the business model chosen by the firm, consumers have the option of consuming an OS product with value \( v_0 > 0 \). However, the firm affects \( v_0 \) through its choice of business model. For example, if the firm decides not to open any modules, then \( v_0 = \hat{a}_0 \). If the firm opens module \( a \), then \( v_0 = a_0 \) because \( \hat{a} \leq a \) and all consumers prefer the firm’s open module. If the firm opens module \( b \) only, then \( v_0 = \hat{a}_0 b_0 \) because full compatibility implies that consumers may combine modules from different developers. Finally, if the firm opens both \( a \) and \( b \), then \( v_0 = a_0 b_0 \).

The firm’s profits are \((v_c - v_o)/4\), where \( v_c \) is the value of the commercial product. Lemma 5 states that the firm will never adopt \( \hat{a} \). Therefore the four business models that the firm will consider, the corresponding values \( v_c \) and \( v_o \), and profits are:

- **\( P \):** \( v_c = a b z \), \( v_o = \hat{a}_0 \), and \( \pi_p = (a b z - \hat{a}_0)/4 \).
- **\( M_a \):** \( v_c = a_0 b z \), \( v_o = a_0 \), and \( \pi_a = a_0 (b z - 1)/4 \).
- **\( M_b \):** \( v_c = a_0 b_0 z \), \( v_o = \hat{a}_0 b_0 \), and \( \pi_b = (a_0 b_0 z - \hat{a}_0 b_0)/4 \).
- **\( O_{ab} \):** \( v_c = a_0 b_0 z \), \( v_o = a_0 b_0 \), and \( \pi_{ab} = a_0 b_0 (z - 1)/4 \).

Contrary to the monopoly case, \( M_b \) does not dominate \( P \). The reason is that when \( b \) is opened, users of the open source project can combine \( \hat{a} \) with \( b \) to assemble a product of quality \( \hat{a}_0 b_0 \). Having a stronger open source competitor may lead to decreased value capture by the for-profit firm. We begin by comparing \( M_a \) and \( M_b \). The following lemma states that there is a threshold \( \sigma_{a|b} \) independent of \( z \) such that \( M_a \) is preferred to \( M_b \) if and only if \( \sigma > \sigma_{a|b} \).

**Lemma 6.** Let \( \sigma_{a|b} := 2 \left( \frac{a}{ab} - 1 \right) \). When \( \sigma > \sigma_{a|b} \), then \( M_a \) is preferred to \( M_b \), and if the opposite happens, \( M_b \) is preferred to \( M_a \). In particular, if \( b > a/\hat{a} \), then \( M_b \) is never optimal.

**Proof.** \( M_a \) is preferred to \( M_b \) when \( \pi_a > \pi_b \), that is, when \( a_0 (b z - 1)/4 > (a_0 b_0 z - \hat{a}_0 b_0)/4 \). Rearranging this inequality we obtain the desired result. The opposite result is obtained similarly. \( \square \)

The intuition behind this result is similar to that behind Lemma 2. The firm prefers to open the module which adds less value to the outside OS product. An implication of Lemma 6 is that when \( \sigma > \sigma_{a|b} \), the firm considers (at most) \( P \), \( M_a \), and \( O_{ab} \) as candidates to an optimal business model. And when \( \sigma < \sigma_{a|b} \), it considers (at most) \( P \), \( M_b \), and \( O_{ab} \).

We now fully characterize the firm’s choice of business model. The following are helpful for Proposition 3:

\[
\begin{align*}
\varepsilon_{a|p} &:= \frac{2 a (1 + \sigma) - \hat{a} (2 + \sigma)}{2 a b \sigma}, \\
\varepsilon_{b|p} &:= \frac{\hat{a} (2 + \sigma) (b (1 + \sigma) - 1)}{2 a b \sigma}, \\
\varepsilon_{a|b} &:= 1 + \frac{b - 1}{b \sigma}, \quad \text{and} \quad \varepsilon_{a|b} := \frac{2 a (1 + \sigma) - \hat{a} (2 + \sigma)}{2 a \sigma}.
\end{align*}
\]
Proposition 3 (Equilibrium regions). If \( \sigma > \sigma_{a|b} \), then \( P \) is optimal when \( z < z_{a|p} \), \( M_a \) is optimal when \( z_{a|p} < z < z_{a|b} \), and \( O_{a|b} \) is optimal when \( z > z_{a|b} \). If \( \sigma < \sigma_{a|b} \), \( P \) is optimal when \( z < z_{b|p} \), \( M_b \) is optimal when \( z_{b|p} < z < z_{a|b} \), and \( O \) is optimal when \( z > z_{a|b} \).

Proof. Noticing that

\[
\pi_a > \pi_p \quad \text{if and only if} \quad z > \frac{2a(1 + \sigma) - \hat{a}(2 + \sigma)}{2ab} := z_{a|p},
\]

\[
\pi_b > \pi_p \quad \text{if and only if} \quad z > \frac{\hat{a}(2 + \sigma)(b(1 + \sigma) - 1)}{2ab} := z_{b|p},
\]

\[
\pi_{a|b} > \pi_a \quad \text{if and only if} \quad z > 1 + \frac{b - 1}{b\sigma} := z_{a|b},
\]

\[
\pi_{a|b} > \pi_b \quad \text{if and only if} \quad z > \frac{2a(1 + \sigma) - \hat{a}(2 + \sigma)}{2a\sigma} := \sigma_{a|b}.
\]

this Proposition follows from Lemma 6 and a straightforward profit comparison.

Proposition 3 describes how the choice of business model varies with \( \sigma \) and \( z \). For low values of \( \sigma \) and \( z \), the firm chooses the proprietary business model (\( P \)), for intermediate values, the firm chooses a mixed source model (either \( M_a \) or \( M_b \)), and for high values of \( \sigma \) and \( z \), the firm becomes completely open (\( O_{a|b} \)). Figure 20 shows the regions for different parameter values.

![Figure 20. Equilibrium business models.](image)

It is interesting to note the similarity of this case with the case in which the firm competes with both modules. What is really important behind these two cases is that there is an outside base module which can be combined with the extensions of the firm, if it decides to open them.

As a final remark, note that leapfrogging by the open source project may not happen in equilibrium in this case. The commercial firm can easily prevent leapfrogging by opening \( a \).
7.3. Outside OS project competes with add-on module $\hat{b} \leq b$. We now consider the case in which the outside OS project consists only of extensions which are of lower quality than those of the profit-maximizing firm.

Because the outside open source project consists of $\hat{b}$ only, it cannot be used alone; it needs a base module with which to combine. The implication is that this case is similar to the monopoly of Section 3 in that if the firm decides not to open the base module, the competitor open source product has no value and no effect in the market.

The values of $v_c$ and $v_o$ and profits under each business model in this case are:

- **P**: $v_c = abz$, $v_o = 0$, and $\pi_p = abz/4$.
- **M_a**: $v_c = abz$, $v_o = a_o \hat{b}_o$, and $\pi_a = (a_o b z - a_o \hat{b}_o)/4$.
- **M_b**: $v_c = a b_o z$, $v_o = 0$, and $\pi_b = (a b_o z)/4$.
- **O_{ab}**: $v_c = a_o b_o z$, $v_o = a_o b_o$, and $\pi_{ab} = a_o b_o (z - 1)/4$.

As in the monopoly case (and for the same reason), $P$ is dominated by $M_b$. This, together with Lemma 5, implies that the firm needs only consider $M_a$, $M_b$, $O_{ab}$. The following are helpful to characterize the firm’s choice of business model:

$$z_{a|b} := \frac{\hat{b}(1 + \sigma)(2 + \sigma)}{b \sigma}, \quad z_{ab|a} := \frac{2b(1 + \sigma) - \hat{b}(2 + \sigma)}{2b \sigma}, \quad \text{and} \quad z_{ab|b} := \frac{2(1 + \sigma)^2}{\sigma(3 + 2\sigma)}.$$

**Proposition 4.** If $\frac{b}{\hat{b}} > \frac{(2 + \sigma)(3 + 2\sigma)}{2(1 + \sigma)}$, then $M_b$ is optimal when $z < z_{a|b}$, $M_a$ is optimal when $z_{a|b} < z < z_{ab|a}$, and $O_{ab}$ is optimal when $z > z_{ab|a}$. If $\frac{b}{\hat{b}} < \frac{(2 + \sigma)(3 + 2\sigma)}{2(1 + \sigma)}$, then $M_b$ is optimal when $z < z_{b|ab}$ and $O_{ab}$ is optimal when $z > z_{ab|b}$. $M_a$ is never optimal when $\frac{b}{\hat{b}} < \frac{(2 + \sigma)(3 + 2\sigma)}{2(1 + \sigma)}$. Notice that a sufficient condition is $b < 3\hat{b}$.

**Proof.** As in the monopoly case (and for the same reason), $P$ is dominated by $M_b$. This, together with Lemma 5, implies that the firm needs only consider $M_a$, $M_b$, $O_{ab}$. Thus, only three thresholds are relevant, $z_{a|b}$, $z_{ab|a}$, and $z_{ab|b}$:

$$\pi_a > \pi_b \quad \text{if and only if} \quad z > \frac{\hat{b}(1 + \sigma)(2 + \sigma)}{b \sigma} := z_{a|b},$$

$$\pi_{ab} > \pi_a \quad \text{if and only if} \quad z > \frac{2b(1 + \sigma) - \hat{b}(2 + \sigma)}{2b \sigma} := z_{ab|a},$$

$$\pi_{ab} > \pi_b \quad \text{if and only if} \quad z > \frac{2(1 + \sigma)^2}{\sigma(3 + 2\sigma)} := z_{ab|b}.$$

If $z_{ab|b} > z_{ab|a}$, $M_a$ is always dominated by $M_b$ or OS. Rearranging this inequality, we get $\frac{b}{\hat{b}} < \frac{(2 + \sigma)(3 + 2\sigma)}{2(1 + \sigma)}$. Therefore, if $\frac{b}{\hat{b}} < \frac{(2 + \sigma)(3 + 2\sigma)}{2(1 + \sigma)}$, then $M_a$ is never optimal, and the firm will choose between $M_b$ and $O_{ab}$. When $\frac{b}{\hat{b}} > \frac{(2 + \sigma)(3 + 2\sigma)}{2(1 + \sigma)}$, the firm will choose between $M_a$, $M_b$ and $O_{ab}$. The optimal choice is found by comparing the different thresholds.
The proposition states that depending on the values of $\sigma$ and $z$, there may be two or three optimal business models in equilibrium. For low values of $\sigma$ and $z$, the firm opens $b$. For high values of $\sigma$ and $z$, the firm opens both modules. And for intermediate values of $\sigma$, the firm may consider opening $a$. Opening $a$ happens only when $b$ is large (more precisely, only when $b/\hat{b}$ is large). In this case, although opening $a$ creates a more formidable outside open source competitor, there is a large increase in the value of the commercial package because of the complementarity between $a$ and $b$. If $b$ is low, on the other hand, the firm will prefer to keep $a$ closed to prevent having to remain a monopoly. Figure 21 illustrates the two situations that may emerge.

**Figure 21. Equilibrium business models.**

7.4. **Outside OS project competes with either a base module $\hat{a} > a$ or an add-on module $\hat{b} > b$.** In this subsection we consider the simple case in which one of the modules available through the outside OS project has higher quality than that of the firm.\(^{38}\) Remember that we have already analyzed the case in which the open source project provides both modules, and both modules have higher quality than those of the firm in the main text.

The following lemma says that when $\hat{a} > a$ or $\hat{b} > b$, only two business models may arise: one of the two mixed models ($M_\hat{a}$ or $M_\hat{b}$) and the open source model ($O_{\hat{a}\hat{b}}$).

**Lemma 7.** If $\hat{a} > a$ and $\hat{b} \leq b$ or $\hat{b}$ does not exist, the firm will choose $M_\hat{a}$ or $O_{\hat{a}\hat{b}}$. If $\hat{b} > b$ and $\hat{a} \leq a$ or $\hat{a}$ does not exist, the firm will choose $M_\hat{b}$ or $O_{a\hat{b}}$.

**Proof.** There are many cases, depending on the modules offered by the OS project. Remember that profits are always $(v_c - v_o)/4$, but the exact form of $v_c$ and $v_o$ change with the case under analysis.\(^{38}\) The other module may not be offered by the outside OS project or it may be of lower quality.
Suppose first that \( \hat{a} > a \). We assume nothing regarding \( b \): \( b \) may be open or closed, \( \hat{b} \) may exist or not, and if it exists \( \hat{b} \) may be larger than \( b \) or not. If the firm does not adopt \( \hat{a} \), \( v_c = a k_c \) and \( v_o = \hat{a}(1 + \sigma/2)k_o \), where \( k_c > 1 \) and \( k_o \geq 1 \) are constants which do not depend on the decision to adopt \( \hat{a} \) (for example, if \( \hat{b} < b \) and \( b \) is not open, \( k_c = b z \) and \( k_o = \hat{b}(1 + \sigma/2) \)). If the firm adopts \( \hat{a} \), then \( v_c = \hat{a}(1 + \sigma)k_c \) and \( v_o = \hat{a}(1 + \sigma)k_o \). Clearly, the firm will always adopt \( \hat{a} \) whenever \( \hat{a} > a \).

Suppose now that \( \hat{b} > b \), and that \( \hat{a} \) does not exist and \( a \) is not open. If the firm does not adopt \( \hat{b} \), then \( v_c = b k_c \) and \( v_o = 0 \). If the firm adopts \( \hat{b} \), \( v_c = \hat{b}(1 + \sigma/2)k_c \) and \( v_o = 0 \). Therefore, the firm will adopt \( \hat{b} \).

Finally, suppose that \( \hat{b} > b \), and that \( \hat{a} \) exists or \( a \) is open. \( \hat{a} \) may be larger than \( a \) or not. If the firm does not adopt \( \hat{b} \), then \( v_c = b k_c \) and \( v_o = \hat{b}(1 + \sigma/2)k_o \). If the firm adopts \( \hat{b} \), \( v_c = \hat{b}(1 + \sigma)k_c \) and \( v_o = \hat{b}(1 + \sigma)k_o \). Therefore, the firm will always adopt \( \hat{b} \) if \( \hat{b} > b \).

Lemma 7 extends the results of Lemma 8. The intuition is the same as before. The firm compares \( a b z \) to \( \hat{a}_o b z, \hat{a}_b o z, \) or \( \hat{a}_o b o z \). Incorporating the available open modules has an obvious positive effect: because \( \hat{a} > a \) and \( \hat{b} > b \) the resulting product will be of higher quality than \( a b z \). Moreover, because \( \hat{a} \) and \( \hat{b} \) are open modules, there is also a benefit from user innovation. The drawback of adopting \( \hat{a} \) and/or \( \hat{b} \) is that the quality of the outside OS project also increases (because more users end up adopting the module/s and there is more user innovation). As a result, competitive pressure intensifies. But because the software modules and service \( z \) are complementary, the increase in quality in the commercial product is always larger than that of the outside OS product and, thus, the positive effect always dominates the negative effect.

The following lemma says that when the outside OS project has one high-quality module only, the firm will choose to compete through an open business model if user innovation \( \sigma \) is strong or the value of service \( z \) is high.

**Lemma 8.** When the OS project has only one module with higher quality than the corresponding firm’s module, there is a function \( \sigma(z) > 0 \), with \( \sigma'(z) < 0 \) such that firms choose \( M_{\hat{a}} \) or \( M_{\hat{b}} \) for \( \sigma < \sigma(z) \) and \( O_{\hat{a}b} \) for \( \sigma > \sigma(z) \).

**Proof.** We are examining cases in which one of the OS modules is better than that of the firm, and the other module is worse. We know that the firm will always adopt the better OS module. What we have to determine is in which situation it will choose to open its own module. There are two effects of opening a module. On the one hand, user innovation increases the quality of \( C \) and OS products. This effect is always positive (\( v_c \) always increases more than \( v_o \)), and will be larger when \( \sigma \) or \( z \) are larger. On the other hand, the quality of the OS product increases because now it has access to a better module. This effect is always negative. Therefore, for OS to be preferred to MS, \( \sigma \) and \( z \) have to be large enough. Finally, a lower \( z \) implies that the corresponding \( \sigma \) necessary for OS to be optimal has to be larger.

The intuition is the same as that given in the earlier lemmas for why the open business model dominates when \( \sigma \) and/or \( z \) are high.
In conclusion, we can see that the analysis of these cases confirms our previous findings.