Day-Ahead Residual Demand Curve Forecasting in Electricity Markets

José Portela, Antonio Muñoz, Estrella Alonso
Outline

• Motivation

• Model definition

• Case study: Spanish electricity market

• Conclusion
MOTIVATION
MOTIVATION
Residual Demand Curves

Energy bids are defined by a quantity and a price
- Selling bids
- Buying bids

Residual Demand

\[ R(q) = D^{-1}(q) - S^{-1}(q) \]

Point of view of an entering company

Energy bids are defined by a quantity and a price.

- Selling bids
- Buying bids

Selling bids

Buying bids

Clearing market price

Energy

Price

D(q)

S(q)

Selling bids

Buying bids

Energy bids are defined by a quantity and a price.

- Selling bids
- Buying bids

Point of view of an entering company

Residual Demand

\[ R(q) = D^{-1}(q) - S^{-1}(q) \]
The residual demand function indicates the quantity that the company of interest can sell/buy at each market clearing price.

\[ \pi(q) = R(q) \cdot q - c(q) \]

MOTIVATION

Curve forecasting for bidding optimization

Allows supply function optimization by scenario generation

Profit function

Buy

Sell

Energy

Price

Clearing market price

\[ p_b \]

\[ p_s \]

\[ q_b \]

\[ q_s \]
MODEL DEFINITION
MODEL DEFINITION
Curves data

Observation of the curves are in discrete points along the energy axis

Observations of a series of curves

Observations of a curve

<table>
<thead>
<tr>
<th>q₁</th>
<th>...</th>
<th>qₙ</th>
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</thead>
<tbody>
<tr>
<td>p₁₁</td>
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<td>p₁ₙ</td>
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<td>Curve 1</td>
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<tr>
<td>pₙ₁</td>
<td>...</td>
<td>pₙₙ</td>
</tr>
</tbody>
</table>

Multivivariate data
MODEL DEFINITION

Curve prediction - Two approaches

\[ \begin{align*}
R_1 & \quad \ldots \quad R_{n-1} \quad R_n \\
\end{align*} \]

Dimensionality Reduction

(\text{Lee} \ & \ \text{Verleysen}, \ 2007) ; \ (\text{Maaten} \ \text{et al}., \ 2008)

- Obtain low dimension representation of the curves
- Forecasting models to predict a vector of reduced components
- Reconstruct forecasted components

Functional Data Analysis

(\text{Ramsay}, \ 2006) ; \ (\text{Ferraty} \ \& \ \text{Vieu}, \ 2006)

- Curves as functions of infinite dimensionality
- Forecasting by: Autoregressive Hilbertian models

\[ x \in L^2 \]
\[ q \in [a, b] \subset \mathbb{R} \]

Discrete functions

Continous functions
MODEL DEFINITION
Dimensionality Reduction (I)

Time series
Component 1

Time series
Component 2

Time

Component 1

Component 2

Forecasting

Reconstruction

Linear techniques: Linear transformation

Non-Linear techniques: K-means reconstruction

Linear techniques: Linear transformation

Non-Linear techniques: K-means reconstruction

Time

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MODEL DEFINITION
Dimensionality Reduction (II)

**EMBEDDING**

- Principal Component Analysis (PCA)
  - (Hotelling, 1933)

- Sammon’s Nonlinear Mapping
  - (Sammon Jr, 1969)

- Isomap
  - (Tenenbaum et al., 2000)

- Kernel PCA
  - (Schölkopf et al., 1998)

- Laplacian Eigenmaps
  - (Belkin & Niyogi, 2001)

- Locally Linear Embedding (LLC)
  - (Saul & Roweis, 2003)

**FORECASTING**

**ARIMA** (Box & Jenkins, 1970) & **VARIMA**

\[ \phi(L)y[t] = \theta(L)e[t] \]

Time series Transfer Function
(Pankratz, 1991)

\[ y[t] = c + \frac{\phi(L)}{\delta(L)} x[t-b] + \nu[t] \]

**Neural networks** (Haykin, 2004)
MODEL DEFINITION
Functional Data Analysis (I)

Definition

• Functions are assumed to elements of the Hilbert space $L^2$ of real square integrable functions on the interval $[a,b]$.

• The usual inner product is defined:

\[ \langle f, g \rangle = \int_a^b f(t)g(t) \, dt \]

• A distance is established

\[ d(f, g) = \sqrt{\langle f - g, f - g \rangle} \]

• Forecasting Hilbertian models:
  - Autoregressive (ARH) (Bosq, 2000)
  - Autoregressive with exogenous variables (ARHX) (Aneiros et al., 2011)
MODEL DEFINITION
Functional Data Analysis (II) - ARH

\( x_i(\cdot) \rightarrow \text{Curve function} \)

**ARH model**

\[
x_{i+1}(\cdot) = m(x_i(\cdot)) + \varepsilon_{i+1}(\cdot)
\]

**Estimation**

\[
\hat{x}_{i+1}(\cdot) = \hat{m}(x_i(\cdot))
\]

**Kernel Estimator**

\[
\hat{m}_h(x_i) = \sum_{j=1}^{n-1} \omega_h(x_i, x_j) \cdot x_{j+1}
\]

**Nadaraya-Whatson estimator**

\[
\omega_h(x_i, x_j) = \frac{k\left(\frac{d(x_i, x_j)}{h}\right)}{\sum_{p=1}^{n-1} k\left(\frac{d(x_i, x_p)}{h}\right)}
\]

\( h \) is optimized minimizing the cross-validation function

\[
cv(h) = \sum_{i=n-r}^{n-1} \left( d\left( x_{i+1}(\cdot), \hat{m}_{h,n-r}(x_i(\cdot)) \right) \right)^2
\]

\( k = \text{gaussian kernel function} \)
MODEL DEFINITION
Functional Data Analysis (III) - ARHX

ARHX model

\[ x_{i+1}(\cdot) = v_{i+1}^t \cdot \beta(\cdot) + m(x_i(\cdot)) + \varepsilon_{i+1}(\cdot) \]

Estimation

\[ \hat{x}_{i+1}(\cdot) = v_{i+1}^t \cdot \hat{\beta}(\cdot) + \hat{m}(x_i(\cdot)) \]

Estimators

\[ \hat{\beta}(\cdot) = (\tilde{V}_h^t \cdot \tilde{V}_h)^{-1} \cdot \tilde{V}_h^t \cdot \chi_h \]

\[ \hat{m}_h(x_i) = \sum_{j=1}^{n-1} \omega_h(x_i, x_j) \cdot (x_{j+1}(\cdot) - v_{i+1}^t \cdot \hat{\beta}(\cdot)) \]

Matrix definitions:

\[ W_h = (\omega_h(x_i, x_j))_{1 \leq i, j \leq n-1} \]

\[ V = (v_{ij})_{1 \leq i \leq n-1; 1 \leq j \leq r} \]

\[ \chi(\cdot) = (X_2(\cdot), \ldots, X_n(\cdot))^t \]

\[ \tilde{V}_h = (I - W_h)V \]

\[ \chi_h = (I - W_h)\chi \]
CASE STUDY
CASE STUDY
Residual Demand Curves – Spanish market

- Spanish Electricity market day-ahead residual demand curves.

- Curves are obtained in the interval $[-1, 1]$ GWh with a resolution of 50 MWh.

- Data range: Jan-March 2008
  - In-sample: Jan & Feb (1440 curves)
  - Out-of-sample: March (720 curves)

- Exogenous variables:
  - Demand
  - Wind energy production
  - Fossil fuel energy production
CASE STUDY
Residual Demand Curves – Spanish market

Subtraction of clearing price from residual demand curves in order to observe price variation

First principal component explains 99.5% variance and correlated with price
CASE STUDY
Embedding technique and dimensions

- Dimensionality reduction for in-sample data.
- Out-of-sample reduction for Out-of-sample data

Determining number of reduced dimensions

\[ \text{MSE}(k_{opt}) \text{ en función de } n_{\text{dims}} \]

- PCA
- Isomap
- KPCA Gauss
- LLE
- Laplacian
- Sammon
- KPCA Poly

Number of dimensions in embedded manifold

MSE

1 2 3 4 5 6

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
CASE STUDY
Reduced components PCA

Scatterplot

Time series

Variable: PCA1
Función de autocorrelación simple de: PCA1
Función de autocorrelación parcial de: PCA1

Variable: PCA2
Función de autocorrelación simple de: PCA2
Función de autocorrelación parcial de: PCA2

Variable: PCA3
Función de autocorrelación simple de: PCA3
Función de autocorrelación parcial de: PCA3
CASE STUDY
Models

Dimensionality Reduction

- 4 techniques selected.
- 3 embedded dimensions each
- 1 ARIMA, TF and MLP model for each reduced component
- Outputs from models are reconstructed

FDA

- Range \([a,b)\) of the curves is \([-1,1)\) GW
- 1 ARH and 1 ARHX model
- Optimize smoothness parameter \(h\) by minimizing cross-validation function.

• Forecast for all methods are 24 hour horizon.
• Estimated variables are used as “known values” when necessary.
## CASE STUDY

### Results

#### Results for out-of-sample period

<table>
<thead>
<tr>
<th>Dim Reduction</th>
<th>PCA</th>
<th>Isomap</th>
<th>Sammon</th>
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<td>MSE ARHX</td>
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</tbody>
</table>

**Graphs:**

- [Graph 1](#)
- [Graph 2](#)
- [Graph 3](#)
CONCLUSION
CONCLUSION
Evaluation and forward improvements

- An empirical comparative study has been presented in order to forecast residual demand curves.

- Two approaches have been compared:
  - Discrete: dimensionality reduction.
  - Continuous: Functional Data Analysis.

- Naïve method is outperformed. Using PCA embedding with transfer function forecasting shows the best results.

- The final objective is to be used to design bidding strategies. Methodology is to be developed to generate scenarios.
Thank you