One-to-Many Bargaining with Endogenous Protocol

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Abstract

This paper studies the bargaining between one central player and \( N \) peripheral players. In each period the central player chooses which peripheral player to bargain with, hence the bargaining protocol is endogenously determined. The peripheral players are heterogeneous in terms of their bargaining power. We characterize the set of equilibrium outcomes with two different types of contracts, namely, contingent contract and cash-offer contract. It is shown that different bargaining protocols may arise in equilibria sustaining different agreements. The central player can play off the weaker peripheral player against the stronger peripheral players. When the players become extremely patient, the central player’s equilibrium payoff can be arbitrarily close to what he can obtain in a bilateral bargaining with only the weakest peripheral player.

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1 Introduction

This paper studies the bargaining between one central player and \( N \) peripheral players on how to share the added value of a joint project, which requires the cooperation of all parties. The relevant real-life situations include the price negotiation between a real estate developer and multiple land owners, the wage negotiation between a firm and multiple unions, and the peace talk between Israel and the Arab countries, etc. A common feature of these situations is that the peripheral players do not bargain with each other directly.\(^1\)

In contrast to most of the literature that assumes an exogenously fixed bargaining protocol, i.e., the order of the pairwise bargaining and the length of each bargaining round, in our model the bargaining protocol is endogenously determined. More specifically, in each period of the bargaining, the central player chooses which peripheral player to bargain with. If no agreement is reached in the current period, the central player can either continue to bargain with the same peripheral player or move on to any other player in the subsequent periods.

Another new feature of the model is that the peripheral players are in general heterogeneous in terms of bargaining power. As in Rubinstein (1982), the right to propose is the source of the bargaining power, and the peripheral players differ on the probabilities with which they are recognized as the proposer when they bargain with the central player.

A binding contract is signed when an agreement is reached between the central player and a peripheral player. We consider two types of contracts, namely, contingent contract and cash-offer contract. With a contingent contract, the peripheral player receives the agreed payment only after the central player reaches agreements with all peripheral players and the project is finally implemented. With a cash-offer contract, the peripheral player is paid right away upon reaching an agreement. What type of contract is adopted has subtle influences on the set of equilibrium outcomes.

With contingent contracts, the bargaining model admits a rich set of equilibria. Firstly, the equilibrium outcomes under different fixed bargaining protocols can arise in equilibria

\(^1\)The two-sided multi-agent bargaining differs from the usual multilateral bargaining problem, which was first analyzed by Herrero (1985) and Sutton (1986).
with endogenously determined protocols. Various beliefs about the future actions off the equilibrium path sustain different agreements reached on the equilibrium path. For example, a sequence of agreements can be sustained by the belief that the central player will not switch to another peripheral player until an agreement is reached, and a different sequence of agreements can be sustained by the belief that the central player will alternate among all the remaining peripheral players. In each equilibrium the belief is correct, that is, it is indeed optimal for the central player not to switch or to alternate in case of temporary disagreement.

More interestingly, there exist equilibria in which the central player can play off one (the weaker) peripheral player against other (stronger) peripheral players, namely, the skimming equilibria. More specifically, the central player leaves an peripheral player only after his own offer is rejected, and returns after a certain number of bargaining periods without agreement. This creates a disparity on the rejection costs and reduces the relative bargaining power of that particular peripheral player. As the players become extremely patient, the central player’s highest equilibrium payoff from a skimming equilibrium can be arbitrarily close to what he can obtain in a bilateral bargaining with only the weakest peripheral player.

The main effect of using binding cash-offer contracts is that payments based on contracts signed in earlier bargaining rounds are sunk costs of the central player, and the surplus in the subsequent bargaining remains the same. It is shown that with cash-offer contracts, impasse is an equilibrium outcome when the number of peripheral players is large, whereas with contingent contract, impasse never occurs in equilibrium. Furthermore, in general there does not exist an equilibrium in which the central player alternates among peripheral players, which is often assumed as a fixed bargaining protocol.

As a plausible refinement, we subsequently focus on the Markov equilibria of the bargaining game. The notion of Markov equilibrium is more restrictive in our model than in the models with fixed protocols. Consequently, any Markov equilibrium must be efficient in our model in contrast to the existing literature discussed below. Again, what type of contract is adopted matters. If contingent contract is used, there exists an unique Markov equilibrium
in mixed strategies; if cash-offer contract is used, there are multiple Markov equilibria in pure or mixed strategies.

Our model is more realistic than those with exogenously fixed protocols as in real life bargaining is rarely conducted according to a pre-determined protocol. In reality, if the central player can commit to a certain bargaining protocol, he certainly will choose the one that maximizes his equilibrium payoff. Even without commitment, if the same central player has to bargain with different sets of peripheral players over time, the central player will have the incentive to build up the reputation for sticking to a certain protocol. To protect the peripheral players’ benefits, it would be helpful to impose certain regulations on bargaining protocol.

This paper is most closely related to two papers by Cai (2000, 2003). Cai (2000) studies the bargaining between a railroad company and \( N \) farmers with binding cash-offer contracts. The farmers are located on a circle with a fixed ordering. Each bargaining round, between the company and a farmer, consists of two periods, in which each party makes one offer. If no agreement is reached, the company moves on to the next farmer on the circle. It is shown that there are multiple equilibria with endogenously determined order of reaching agreements. Inefficient delay may arise in equilibria that satisfy a weak stationarity condition.

Cai (2003) studies a similar model with contingent contracts. It is shown that there are multiple Markov equilibria, some of which entails inefficient delay. A crucial feature of this model is that the central player moves down to another peripheral player after the rejection of his own offer. If, instead, in each bargaining round the peripheral player makes the last offer, then the multiplicity result will not hold.\(^2\) This observation shows how sensitive the equilibrium outcome is to the bargaining protocol.

Also related is Suh and Wen (2009), who study a bargaining model with endogenously determined protocol. Their model is a multilateral bargaining model in the sense that any

\(^2\)More precisely, in the model of Cai (2003), if the order of making offers is reversed in each bargaining round, then there is an unique equilibrium in which the \( n^{th} \) peripheral player in the queue obtains a share close to \( 1/2^n \) as \( \delta \) goes to 1. It is equivalent to the equilibrium with sequential protocol in our model.
pair of players can bargain with each other for a partial bilateral agreement. During the bargaining, the players have to agree on who will leave the bargaining after each round. When the players are sufficiently patient, there exist multiple equilibria including inefficient ones.

Horn and Wolinsky (1988) study the relation between worker substitutability and patterns of unionization. In a bargaining model with fixed protocol, they show that when the workers are strong complements, it is optimal for them to bargain separately. Our result suggests that the workers, even being perfect complements, may find it optimal to form an encompassing union to bargain jointly if they anticipate a skimming equilibrium in the separate bargaining.

The rest of the paper is organized as follows. In Section 2, we lay out the one-to-many bargaining model with endogenous protocol. In Section 3, we analyze the bargaining game with contingent contracts. Section 4 considers cash-offer contracts. Section 5 discusses the implication of our results on the pattern of unionization and Section 6 contains some further discussions.

2 A Bargaining Model with Endogenous Protocol

There are $N + 1$ players, a central player A and $N$ peripheral players indexed by $i \in \{1, 2, ..., N\}$. Player A has a project with a commonly known surplus normalized to one. To undertake the project she needs the cooperation from each of the $N$ peripheral players. Hence, player A has to bargain with each peripheral player over a payment in exchange for his cooperation.

The bargaining takes place over time divided into periods of equal length. In each period $t \in \{0, 1, 2, ...\}$, player A chooses at first with whom to bargain. Then, either player A or the chosen player $i$ makes an offer and the other party responds with acceptance or rejection. The offer is simply the size of the payment that player $i$ should receive. If the offer is accepted, the two parties sign a binding contract and player A then moves on to bargain with other peripheral players; if it is rejected, the bargaining proceeds to the next period.
and player A chooses again with whom to bargain. After player A reaches agreements with all peripheral players, the project is implemented immediately and the surplus is realized.

In each bargaining period, the proposer is randomly selected. More precisely, the probability with which player \( i \) is recognized as the proposer is \( p_i \in (0, 1) \), and with probability \( 1 - p_i \), player A is recognized as the proposer. It is well known that the allocation of the right to propose determines the relative bargaining power in the noncooperative bargaining framework. Therefore, the recognition probability \( p_i \) is taken as a measure of the relative bargaining power of player \( i \) with respect to player A. The peripheral players are in general heterogeneous in terms of bargaining power.

A bargaining outcome is denoted by \( \left\{ (s_i, t_i)_{i=1}^N \right\} \), where \( s_i \in [0, 1] \) is the agreed payment to peripheral player \( i \), and \( t_i \) is the period in which this agreement is reached. Let \( T = \max_i (t_i) \) be the date at which player A reaches the last agreement. Since it takes at least \( N \) periods to reach agreements with all peripheral players, the bargaining outcome is inefficient if and only if \( T > N - 1 \). All players discount future payoffs with a common discount factor \( \delta \in (0, 1) \).

We consider two different types of contracts between the central player and each peripheral player, namely, contingent contract and cash-offer contract. With a contingent contract, the peripheral player receives the agreed payment only after the central player reaches agreement with all peripheral players and the project is finally implemented. With a cash-offer contract, the peripheral player receives his payment right away. In our model what type of contract is adopted is exogenously given. Conceivably one can study a model in which the contract type is also endogenously determined. This will be briefly discussed in Section 6.

With contingent contracts, from the outcome \( \left\{ (s_i, t_i)_{i=1}^N \right\} \), player \( i \)'s payoff is \( \delta^T s_i \) and player A’s payoff is \( \delta^T \left( 1 - \sum_{i=1}^N s_i \right) \). Moreover, if there is an impasse, i.e., \( T = \infty \), the project is not implemented and everyone gets a payoff of zero. With cash-offer contracts, player \( i \) receives the payment of \( s_i \) in period \( t_i \), thus his payoff is \( \delta^{t_i} s_i \), and player A’s payoff is \( \left( \delta^T - \sum_{i=1}^N \delta^{t_i} s_i \right) \). Now, if the bargaining enters an impasse before any agreement is reached, everyone gets a payoff of zero; if some agreements have been reached before the
bargaining enters an impasse, player A’s payoff is negative since the agreed payments have been made and become sunk cost. This, however, will not happen in any equilibrium.

The bargaining game described above is a well-defined extensive form game with perfect information and nature’s move. Histories and strategies are defined as usual. We adopt the subgame perfect equilibrium (henceforth equilibrium) as the solution concept. After the first $N - 1$ agreements are reached, the bargaining between player A and the remaining peripheral player becomes a Rubinstein game with randomly selected proposer. It is well known that the game has a unique subgame perfect equilibrium with immediate agreement, as stated in the following Lemma.

**Lemma 1** If player A has reached agreement with every peripheral player $j \neq i$ on $s_j$, then in the continuation game between player A and $i$, there is a unique equilibrium in which an agreement is reached without delay. With contingent contracts, player A (player $i$ resp.) offers $s_i = \delta p_i \left(1 - \sum_{j \neq i} s_j\right)$ ($\tilde{s}_i = [1 - \delta \left(1 - p_i\right)] \left(1 - \sum_{j \neq i} \tilde{s}_j\right)$ resp.), and the offer is accepted. With cash-offer contracts, player A (player $i$ resp.) offers $s_i = \delta p_i$ ($\tilde{s}_i = 1 - \delta \left(1 - p_i\right)$ resp.), and it is accepted.

With contingent contracts, the previously agreed payments are contingent on the final agreement. Hence, in the last round, on the bargaining table is the total surplus less the sum of the agreed payments. With cash-offer contracts, payments are made upon agreements and become “sunk” from the perspective of the remaining peripheral players. Hence, the total surplus in the last bargaining round is still one.

### 3 Contingent Contract

In this section, we characterize the equilibria of the bargaining game with contingent contracts. Our first result is that impasse is never an equilibrium outcome.

**Lemma 2** With contingent contracts, impasse is not an equilibrium outcome.

**Proof.** Denote by $R_k$ the remaining share after the first $k \in \{0, 1, ..., N - 1\}$ agreements, where $R_0 = 1$ by definition. Player A can offer any remaining peripheral player a share
slightly greater than $\delta R_k$, which will never be rejected. Since $p_i < 1$ for any $i$, it must be that $R_{N-1} > 0$ in any equilibrium. Then, by Lemma 1 player A’s equilibrium payoff must be positive.

Similarly, a peripheral player, if chosen by the central player to bargain after the first $k$ agreements, can make a demand slightly less than $(1 - \delta) R_k$, which will never be rejected. Since $p_i > 0$, the peripheral player $i$’s equilibrium payoff must be positive. ■

We now focus on the efficient equilibria. In an efficient equilibrium, an agreement is reached in every bargaining period. For expositional ease, the rest of this section focuses on the game with two peripheral players. Our results can be easily extended to the case with $N \geq 3$ peripheral players.

3.1 Equilibria with Different Underlying Protocols

Different equilibrium outcomes can be sustained by different conjectured underlying protocols. In the equilibrium, the conjecture is correct. In other words, it is optimal for the central player to stick to the underlying protocol. Two well-received one-to-many bargaining protocols are sequential protocol and alternate protocol. With sequential protocol, the central player bargains with the peripheral players one after another according to a certain order, and the central player never moves down to another peripheral player before he reaches an agreement with the current one; with alternate protocol, the central player alternates among all peripheral players, that is, if no agreement is reached in the current period, the central player will bargain with another peripheral player in the next period. In our model both protocols may arise endogenously in equilibrium.

We first describe an equilibrium, referred to as the *equilibrium with sequential protocol*, in which the central player will never switch between the peripheral players before the first agreement is reached. More precisely, before any agreement is reached, when each peripheral player $i$ bargains with player A in period $t$, believes that he will bargain with player A again

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3 As it is shown that the bargaining game has multiple efficient equilibria, it is straightforward to construct equilibria with inefficient delay.
in period $t + 1$ if no agreement is reached. On the equilibrium path, player $A$ randomly chooses one peripheral player and they reach an agreement immediately.

During the bargaining for the first agreement, player $i$ always asks for $1 - \delta (1 - p_i)$ and accepts any offer no less than $\delta p_i$; player $A$ always offers $\delta p_i$ and accepts any demand no greater than $1 - \delta (1 - p_i)$. After the first agreement is reached, player $A$ and the remaining peripheral player will immediately reach an agreement as specified in Lemma 1. Thus, there are two equilibria with sequential protocol differing on who reaches the first agreement with player $A$. Although each peripheral player prefers to be the first to bargain, player $A$’s expected payoff is $\delta (1 - p_1)(1 - p_2)$ in both equilibria. Hence, player $A$ is indifferent in choosing between two peripheral players to bargain for the first agreement and he also has no incentive to switch to another peripheral player during the bargaining. This justifies the peripheral player’s belief described at the beginning. Meanwhile, it also implies that any order of bargaining and agreements can be sustained in an equilibrium with sequential protocol. Hence, we have the following proposition.

**Proposition 1** Any ordering of the peripheral players can be sustained in an equilibrium with sequential protocol.

The following corollary shows that among all possible equilibria the central player obtains his lowest expected payoff from the equilibria with sequential protocol. By the same token, the peripheral player reaching the first agreement obtains his highest expected payoff in this equilibrium.

**Corollary 1** With contingent contracts, the central player’s expected equilibrium payoff is at least $\delta (1 - p_1)(1 - p_2)$, and the peripheral player $i$’s expected equilibrium payoff is at most $\delta p_i$.

**Proof.** From Lemma 1, we see that if player $A$ reaches the first agreement with player $i$ on a share of $s^*_i$, then his expected share in the continuation game is $(1 - p_j)(1 - s^*_i)$. If we can show that the expected share of player $i$ in any equilibrium is no greater than $p_i$, i.e.,
Es_i^* ≤ p_i, then it must be that player A’s expected share in any equilibrium is no less than (1 − p_1)(1 − p_2), i.e., Es_A^* ≥ (1 − p_1)(1 − p_2).

Denote as M_i the supremum of player i’s expected equilibrium share and denote as R_m the infimum of the expected remaining share after the first agreement. That is, Es_i^* ≤ M_i and Es_A^* ≥ (1 − p_j)R_m. The following inequalities are self-explanatory:

\[ R_m \geq (1 − p_i)(1 − \delta M_i) + p_i\delta R_m, \]
\[ M_i \leq (1 − p_i)\delta M_i + p_i(1 − \delta R_m). \]

With some rearrangement, we obtain M_i ≤ p_i. Thus, Es_A^* ≥ (1 − p_1)(1 − p_2), and it follows that player A’s expected equilibrium payoff is at least \( \delta (1 − p_1)(1 − p_2) \), and that player i’s expected equilibrium payoff is at most \( \delta p_i \).

Note that if the bargaining protocol is exogenously fixed, by which the central player is restricted to bargain with one peripheral player until an agreement is reached, then the outcome described above will be the unique equilibrium outcome. Although the sharp prediction is desirable, the equilibrium outcome is rather counterintuitive. If there are \( N \) peripheral players with identical bargaining power \( p_i = 1/2 \) as often assumed in the literature, the central player’s expected share is \( 1/2^N \), lower than all but one peripheral player. More importantly, the equilibrium outcome is not “renegotiation-proof”\(^4\). More specifically, player A, after reaching the last agreement, has the incentive to renegotiate the first agreement.

Next, we describe a renegotiation-proof equilibrium, referred to as equilibrium with alternate protocol, in which the central player alternates in choosing whom to bargain with. That is, if player A bargains with player \( i \) in period \( t \), then he will switch to player \( j \neq i \) in period \( t + 1 \) regardless whether an agreement is reached or not. In the bargaining for the first agreement, player \( i \) always demands a share of \( \hat{s}_i \) and accepts any offer no less than \( \tilde{s}_i \), and player A always offers \( \tilde{s}_i \) and accepts any demand no greater than \( \hat{s}_i \). Then, the following

\(^4\)Stole and Zwiebel (1996) explicitly incorporate renegotiation-proofness into their bargaining model and obtain an unique noncooperative equilibrium outcome that is equivalent to the Shaley value of the corresponding cooperative game.
equilibrium conditions holds:

\[ \tilde{s}_i = \delta p_i [(1 - p_j) (1 - \tilde{s}_j) + p_j (1 - \tilde{s}_j)] \]
\[ (1 - p_j) \hat{s}_i = \delta (1 - p_i) [(1 - p_j) (1 - \tilde{s}_j) + p_j (1 - \tilde{s}_j)], \]

by which we obtain

\[ \tilde{s}_i = \frac{\delta p_i (1 - p_j)}{1 - \delta p_1 p_2} \quad \text{and} \quad \hat{s}_i = \frac{(1 - \delta) + \delta p_i (1 - p_j)}{1 - \delta p_1 p_2}. \]

After the first agreement is reached, player A and the remaining peripheral player will immediately reach an agreement as specified in Lemma 1. Being the first one reaching agreement, player i’s expected share is

\[ E(s^{ij}) = \frac{p_i (1 - \delta p_j)}{1 - \delta p_1 p_2}, \]

where the superscript \( ij \) refers to the order of reaching agreement. If player i is the second one reaching agreement, his expected share is

\[ E(s^{ji}) = \frac{p_i (1 - p_j)}{1 - \delta p_1 p_2}. \]

Again, each peripheral player prefers to be the first one reaching agreement. However, as \( \delta \) tends to 1, the difference between being the first or the second one reaching agreement vanishes. It is also easy to check that player A’s expected share is

\[ E(s_A) = \frac{(1 - p_1) (1 - p_2)}{1 - \delta p_1 p_2}. \]

Hence, player A is indifferent in choosing between two peripheral players in period 0.

**Proposition 2** Any ordering of the peripheral players can be sustained in an equilibrium with alternate protocol.

Conceivably, there are equilibria with general alternate protocols, by which the central player bargains with each peripheral player i for \( T_i \) periods before switching to another peripheral player. The equilibrium agreements can be derived in a similar way as above. For
any finite $T_1$ and $T_2$, the equilibrium outcome converges to the same limit as the equilibrium with the basic alternate protocol (i.e., $T_1 = T_2 = 1$). Let $p_1 = p_2 = 1/2$, the equilibrium with alternate protocol induces equal split among the three players as $\delta$ tends to $1$.\footnote{This is the unique equilibrium outcome of the bargaining model in Horn and Wolinsky (1988).}

Can the central player’s equilibrium payoff be even higher? In Cai (2003), there is a Markov equilibrium in which player A’s payoff goes to $3/8$ and the peripheral player reaching the first agreement goes to $1/4$ as the discount factor goes to $1$. Recall that the crucial feature of his model is that the switch between the two peripheral players is made after the rejection of A’s offer. Hence, the peripheral player’s rejection is more costly than that of player A. More precisely, player A’s rejection causes one period of delay, and the peripheral player’s rejection causes three periods of delay as there will be no agreement in the next bargaining round. This translates into a relative bargaining power ratio of $1:3$. Therefore, as $\delta$ tends to $1$, a peripheral player’s equilibrium share can be forced down to $1/4$, whereas player A and the other peripheral player split the rest equally. With endogenous bargaining protocol, the central player can further exploit this “skimming” effect. We now describe a *skimming equilibrium*.

Player A bargains with player 1 in period 0, and switches to player 2 if and only if his own proposal is rejected for the first time in period $t \geq 0$ and switches back after $T \geq 1$ periods of bargaining with player 2, during which no agreement is reached. Take this as the conjectured protocol, in the equilibrium, player A and player 1 reaches an immediate agreement on either

$$\tilde{s}_1 = \frac{1 - (1 - p_1) \delta^{T+1}}{p_1 + (1 - p_1) \sum_{t=0}^{T} \delta^t}$$

if player 1 proposes, or

$$\hat{s}_1 = \frac{\delta^{T+1} p_1}{p_1 + (1 - p_1) \sum_{t=0}^{T} \delta^t}$$

if player A proposes. Subsequently, player A and player 2 agree immediately as specified in Lemma 1.

The $T$ periods of fruitless bargaining with player 2 serves as a punishment on player 1. If player A deviates during the punishment by switching back early, the continuation...
equilibrium will be the one in which player A receives the lowest share as specified above. Player 2 has common interest with player A in executing the punishment against player 1. Hence, the length of punishment $T$ is chosen such that

$$\delta^{T+1} [(1 - p_1)(1 - \hat{s}_1) + p_1 (1 - \hat{s}_1)] \geq \delta (1 - p_1),$$

where the left hand side is the total payoff of player A and player 2 from following the equilibrium strategy and the right hand side is their highest total payoff from deviating. The condition can be simplified as:

$$\delta^T \geq (1 - p_1) + p_1 \left( \sum_{t=0}^{T} \delta^t \right)^{-1}. \quad (1)$$

Observe that $T$ is well-defined when

$$1 - \delta^2 \leq \delta p_1.$$

Thus, we have the following proposition:

**Proposition 3** *When $\delta$ is sufficiently close to 1, there exists a skimming equilibrium.*

Again, let $p_1 = p_2 = 1/2$, player 1’s expected share tends to $1/(T + 2)$ as $\delta$ tends to 1. For any $T \geq 1$, condition (1) is satisfied when $\delta$ is sufficiently close to 1. Thus, as $\delta$ tends to 1, the maximal $T$ tends to infinity, thus player 1’s minimal expected share tends to zero and player A’s maximal expected share tends to $1 - p_2$. Similarly, player A can play off player 2 against player 1 and his maximal expected share tends to $1 - p_1$ in the limit. Hence, when the players are extremely patient, the central player’s highest equilibrium payoff becomes arbitrarily close to what he can obtain in a bilateral bargaining with only the weakest peripheral player. Meanwhile, any peripheral player’s lowest equilibrium payoff tends to 0.

**Corollary 2** *As $\delta$ tends to 1, the upper bound of the central player’s expected equilibrium payoff tends to $1 - \min \{p_1, p_2\}$, and the lower bound of each peripheral player’s expected equilibrium payoff tends to 0.*
3.2 A General Formulation

Although we have established the upper and lower bounds of each player’s equilibrium payoff, our characterization of equilibrium bargaining protocols is not exhaustive. We now give a general formulation of bargaining protocol and show that there is a vast multiplicity of equilibrium bargaining protocols.

Each session of consecutive bargaining periods with a specific peripheral player is now referred to as a *bargaining round*. Denote an underlying bargaining protocol as a sequence of natural numbers \( \{K_s\}_{s=1}^{\infty} \), where \( K_s \) is the number of periods that the \( s^{th} \) round of the bargaining lasts before any agreement is reached. When \( s \) is odd (even resp.), the bargaining is between player A and player 1 (player 2 resp.) If during any round of the bargaining, there is an agreement reached, the central player immediately switches to another peripheral player to bargain over the remaining surplus.

Note that \( K_1 = 0 \) if player A chooses player 2 in period 0, and that if \( K_1 = 0 \), it must be that \( K_2 > 0 \). To accommodate sequential protocol, we allow \( K_s = \infty \). For example, \( K_1 = \infty \) (or \( K_1 = 0 \) and \( K_2 = \infty \)) if player A adopts the sequential protocol starting from player 1 (or player 2). The alternate protocol corresponds to \( K_s = 1 \) for any \( s \).

This formulation only specifies the central player’s reduced strategies. For example, if \( K_s = \infty \), then it does not specify \( K_{s'} \) for any \( s' > s \). This suffers no loss of generality though as we assume that following the central player’s deviation from an equilibrium protocol, the continuation equilibrium will be the one with sequential protocol. Since the central player obtains the same lowest equilibrium payoff in all equilibria with sequential protocol, this is the most severe equilibrium punishment. Thus, it sustains the largest possible set of equilibrium protocols.

Note that the formulation does not include the protocol adopted in a skimming equilibrium, in which whether to switch to another peripheral player depends on who has rejected an offer. In other words, a protocol in the form of \( \{K_s\}_{s=1}^{\infty} \) is history independent.

**Proposition 4** With contingent contract, any bargaining protocol in the form of \( \{K_s\}_{s=1}^{\infty} \) can be sustained in an equilibrium, and there is a unique sequence of equilibrium agreements.
associated with each protocol.

**Proof.** As we have explained, it is easy to sustain an arbitrary bargaining protocol in an equilibrium as any deviation from the protocol will be punished by switching to the equilibrium with sequential protocol, in which the central player’s payoff is minimized. See Appendix for the uniquely determined equilibrium outcome under an arbitrary protocol. ■

A related question is that what the equilibrium outcome is in a model with a fixed bargaining protocol in the form of \( \{ K_s \}_{s=1}^{\infty} \). For regular protocols such as sequential protocol or alternate protocol, it is easy to derive the equilibrium outcome. It becomes more complicated when the bargaining protocol is irregular. For example, with \( \{ K_s = s \}_{s=1}^{\infty} \), then number of periods in each bargaining round increases uniformly, it is not immediately clear what the equilibrium agreements would be. The main difficulty is that we cannot establish a closed loop of indifference conditions as in the sequential protocol or alternate protocol. Whereas our focus here is to characterize the set of bargaining protocols sustainable in equilibrium, our result also provides an answer to this question. A rather surprising finding is that any irregular bargaining protocol is associated with a unique equilibrium outcome that converges to the same limit as the equilibrium with alternate protocol.

### 3.3 Markov Equilibria

When a bargaining game has multiple subgame perfect equilibria, stationary or Markov equilibrium is often proposed as plausible refinements. In our model, the number of remaining peripheral players changes during the bargaining process, i.e., the game itself is not stationary, thus Markov equilibrium is the proper solution concept.

A Markov strategy is a strategy that depends only on payoff-relevant variables. In our model, these include the number of peripheral players who have not reached agreements.

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6See Appendix for the categorization of bargaining protocols into regular and irregular ones.
7Herrero (1985) shows that in the \( N \)-player Rubinstein bargaining game, there is a unique stationary subgame perfect equilibrium. The restriction to stationary strategies (to obtain uniqueness) is sometimes considered problematic (see, for example, the discussion in Osborne and Rubinstein (1990)).
with the central player and the total surplus that remains available. It should be noted that the central player’s strategy is Markovian if among the same set of remaining peripheral players and with the same available surplus, (i) he chooses the same one to bargain with or randomizes among them with the same probabilities in every period, and (ii) when bargaining with a specific peripheral player, he makes the same offer and accepts the same set of demands. A Markov equilibrium is a subgame perfect equilibrium in Markov strategies. Lemma 3 below establishes the efficiency of any Markov equilibrium.

**Lemma 3** A Markov equilibrium of the bargaining game with contingent contracts must be efficient.

**Proof.** Lemma 2 shows that with contingent contracts, impasse is not an equilibrium outcome. If a Markov equilibrium involves inefficient delay and the first agreement is reached in period $t > 0$ with player $i$, then player $A$ can deviate by truncating his strategy from period $t$. Then, the same sequence of agreements will be reached without any delay since every player is adopting a Markov strategy. The deviation is obviously profitable.

In Cai (2003), when there are two peripheral players, the game has three Markov equilibria for sufficiently large discount factor and one of them is inefficient. Lemma 3 suggests that Cai’s inefficient Markov equilibrium is an artefact of the assumed bargaining protocol. More precisely, in Cai (2003), although the order of reaching agreements is endogenously determined, the order of bargaining is, however, exogenously given. Delay occurs when the two orders are different.

**Lemma 4** With contingent contracts, there does not exist any Markov equilibrium in pure strategies.

**Proof.** In a Markov equilibrium in pure strategies, the central player will always bargain with the same peripheral player, say, player 1 for the first agreement. Thus, the outcome

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$^8$The definition of Markov equilibrium does not preclude a player from deviating to non-Markov strategies, but requires that no player can benefit from such deviations.
should be the same as that of the equilibrium with sequential protocol. However, if the central player deviates to player 2, the latter would expect that the central player will turn back to player 1 if no agreement is reached in the current period. Hence, player 2 is now willing to accept any offer greater than $\delta (1 - p_1) p_2$, and the central player is willing to accept player 2’s demand that is less than $1 - \delta (1 - p_2)$. This makes the deviation profitable.

Lemma 3 implies that the equilibria with sequential protocol or alternate protocol are not Markov equilibria. Especially, in an equilibrium with sequential protocol, once the central player deviates to bargain with another peripheral player, it is crucial that the latter believes that the central player will not switch again until an agreement is reached, whereas a Markov equilibrium (in pure strategy) requires that after any history the central player chooses the same peripheral player to bargain with. The next proposition specifies the unique Markov equilibrium in mixed strategies.

**Proposition 5** With contingent contracts, there is an unique Markov equilibrium in mixed strategies, in which the central player chooses each peripheral player with equal probability in each period.

**Proof.** See Appendix.

In the Markov equilibrium, the central player always randomly chooses a peripheral player with probability $1/2$ regardless their heterogeneous bargaining power. As $\delta$ tends to 1, the Markov equilibrium outcome converges to the same limit as the equilibrium with alternate protocol. Finally, the mixed strategy equilibrium is robust in the following sense. If the central player chooses peripheral player 1 with a probability $q > 1/2$, player 1 would ask for a greater offer, and player 2 would reduce his demand, which, in turn, induces the central player to reduce $q$.

The fact that there is no Markov equilibrium in pure strategies raises the question whether Markov equilibrium notion is too restrictive under the current setting. One may consider the weaker stationarity condition similar to that in Cai (2000). For example, we can take the identity of the current peripheral player as a state variable, then, whether to switch to
another peripheral player and which one to switch to are the decisions to make, which shall depend only on the state variable, not the details of the bargaining history. Clearly, the equilibria with either sequential or alternate protocol satisfy such a stationarity condition, whereas the skimming equilibria are not stationary.

4 Cash-Offer Contract

In many real life situations, only binding cash-offer contract is feasible. For example, in the railroad example, it is reasonable to assume that negotiating parties are limited to cash-offer contracts. As in the case with contingent contracts, for an arbitrary ordering of the peripheral players, there is an equilibrium with sequential protocol. Let player 1 be the first chosen peripheral player. Player A and player 1 are effectively bargaining over a surplus of \(1 - p_2\) as \(p_2\) will be the expected payment to player 2 in the continuation equilibrium after player A and player 1 reach an agreement. Thus, player A offers \(\delta p_1 (1 - p_2)\) and player 1 asks for \([1 - \delta (1 - p_1)] (1 - p_2)\). Again, player A obtains the same expected payoff among all the equilibria with sequential protocol.

Recall that with contingent contracts, the central player obtains his lowest equilibrium payoff in the equilibrium with sequential protocol. This does not hold with cash-offer contracts. The central player obtains a lower payoff from an equilibrium with alternate protocol when it exists, and impasse can also arise when there are three or more peripheral players. Also, with contingent contracts, each peripheral player prefers to be the first one reaching agreement in an equilibrium with sequential protocol, here, a peripheral player is better off being the last one reaching agreement.

It is also easy to construct skimming equilibria. The key step in constructing a skimming equilibrium is to ensure that it is optimal for the central player to execute the punishment imposed on a peripheral player. With cash-offer contracts, this actually becomes even easier since the lower bound of the central player’s equilibrium payoff is smaller than that with contingent contracts.

As we have mentioned, with cash-offer contracts, impasse can be an equilibrium outcome.
It is easy to see that impasse cannot occur when there are only two peripheral players. Nevertheless, the following lemma shows that if the two peripheral players are endowed with sufficiently high bargaining power, there is an equilibrium in which player A’s expected payoff is 0.

**Lemma 5** In the game with two peripheral players and \( p_1 + p_2 > 1 \), when \( \delta \) is sufficiently close to 1, there is an equilibrium in which player A’s expected payoff is 0.

**Proof.** Consider the following strategy profile. Player A randomly chooses a peripheral player in period 0. Before any agreement is reached, in each period \( t \), player A offers the chosen player \( i \) a payment of \( \delta (1 - p_j) \), and accepts player \( i \)’s demand if it is no greater than \( \delta (1 - p_j) \); the chosen player \( i \) asks for \( \delta (1 - p_j) \), and accepts any offer no less than \( \delta (1 - p_j) \); when there is a disagreement in period \( t \), player A switches to another peripheral player in period \( t + 1 \) if and only if the disagreement is caused by his own deviation. From this strategy profile, player A’s expected payoff is 0.

It is easy to see that the chosen peripheral player cannot benefit from any possible deviation. If player A offers player \( i \) less than \( \delta (1 - p_j) \), it is optimal for player \( i \) to reject when \( \delta^2 p_i > \delta (1 - p_j) \). As \( p_1 + p_2 > 1 \), this is satisfied when \( \delta \) is sufficiently close to 1. ■

When there are three peripheral players and \( p_i + p_j > 1 \) for any \( i \neq j \), the above lemma shows in the subgame with two remaining peripheral players, there is an equilibrium in which the central player’s expected payoff is zero. With this as the continuation equilibrium, impasse is an equilibrium outcome in the whole game because if the central player agrees with any peripheral player on a positive payment, his expected payoff becomes negative.

In the game with two peripheral players, there are multiple equilibria from which the central player obtains different payoffs. Same as in Cai (2000), we can construct an equilibrium for the game with three peripheral players, in which the central player’s expected payoff is zero. This is true even for \( \sum_{i=1}^{3} p_i < 1 \). Thus, impasse is always an equilibrium outcome when \( N \geq 4 \).

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\(^9\)In such an equilibrium, the peripheral player \( i \) reaching the first agreement may receive an expected payoff greater than \( p_i \), which is impossible if player \( i \) belongs the two remaining peripheral players.
Proposition 6 When $\delta$ is sufficiently close to 1, impasse is an equilibrium outcome when either (i) there are at least four peripheral players, or (ii) there are three peripheral players and $p_i + p_j > 1$ for any $i \neq j$.

Proof. See Appendix.

When there are many peripheral players, impasse becomes a possible outcome. This gives the peripheral players the incentive to merge. As intuition usually suggests that weak negotiating parties have the incentive to merge to achieve a high collective bargaining power, here we see that negotiating parties with high bargaining power are also willing to merge so that an agreement becomes possible.

Our next proposition shows that with cash-offer contracts, there might not exist equilibria with alternate protocol.

Proposition 7 With cash-offer contracts, there exist equilibria with alternate protocol for any $\delta \in (0, 1)$ only when $p_1 = p_2 \leq 1/2$.

Proof. See Appendix.

Being the second peripheral player reaching agreement, player $i$ receives an expected payment of $p_i$. Thus, in an equilibrium with alternate protocol, if player $i$ is chosen to bargain in period 0, he will not accept any offer less than $\delta^2 p_i$. When $p_1 + p_2 > 1$, there does not exist an equilibrium with alternate protocol when $\delta$ is sufficiently close to 1 because the central player’s payoff from such an equilibrium would be negative. When $p_1 = p_2 \leq 1/2$, there is an equilibrium with alternate protocol for any $\delta \in (0, 1)$. The central player is indifferent in choosing which peripheral player to bargain for the first agreement.

More interestingly, when $p_1 \neq p_2$ and $p_1 + p_2 \leq 1$, there does not exist an equilibrium with alternate protocol for any $\delta \in (0, 1)$. The reason is that the central player always finds it optimal to start with the stronger peripheral player, say, player 1. Thus, after a disagreement with player 1, player A will not switch to player 2 in the following period. This deviation cannot be punished as player A is already receiving the lowest possible payoff from the assumed equilibrium with alternate protocol.
With cash-offer contracts, the equilibrium with sequential protocol is a Markov equilibrium. Yet the equilibrium differs from that with contingent contracts on what happens off the equilibrium path. More precisely, if the central player deviates to bargain with another peripheral player, the latter believes that the former will switch back in the following period, and thus, no agreement is reached and the deviation is not profitable.

**Proposition 8** With cash-offer contracts, there are multiple Markov equilibria for any $\delta \in (0, 1)$: (i) every equilibrium with sequential protocol is a Markov equilibrium in pure strategy; (ii) there is also a unique mixed strategy Markov equilibrium when $p_1 + p_2 \leq 1$.

**Proof.** Part (i) has been explained above. See Appendix for Part (ii). ■

In the mixed strategy Markov equilibrium, player A randomizes in choosing the peripheral player in each period, and each player $i$ is chosen with probability

$$q_i^* = \frac{1 - p_i}{(1 - p_1) + (1 - p_2)}.$$

As $\delta$ tends to 1, player $i$'s expected payoff tends to $p_i$ and player A’s expected payoff tends to $1 - p_1 - p_2$.

Note that player $i$’s demand depends on his belief about the probability that he is chosen in each period. If his perceived probability is higher than $q_i^*$, his demand will be lower, then it becomes profitable for player A to further increase the probability of choosing player $i$. Thus, the mixed strategy equilibrium is not robust here.

5 **Implication on Unionization**

Horn and Wolinsky (1988) study the bargaining between an employer and two group of workers, and the employer alternates in meeting with one of the two groups to bargain. Their bargaining game has an unique subgame perfect equilibrium. More importantly, they show that when the two groups of workers are close substitutes, they prefer to form an encompassing union and bargain collectively; when they are strong complements, they prefer to form two independent unions and bargain separately. In this paper we focus on the pure
bargaining problem which corresponds to the special case where the workers are perfect complements. Our result suggests that in separate bargaining, the employer may play off one group of workers against the other group to reduce the overall wage payment, and thus, the workers, even being strong complements, still have the incentive to bargain collectively.

To be more precise, consider two unions with possibly different bargaining power, say, \(0 < p_1 \leq p_2 < 1\). Note that \(p_i\) is the relative bargaining power of union \(i\) when it bargains with the employer independently. When the two unions merge, it is not immediately clear how the bargaining power of the merged union is determined. More generally, denote as \(p = g(p_1, p_2) \in (0, 1)\) the bargaining power of the merged union. It is reasonable to assume that \(g(p_1, p_2) \geq p_i\) and \(\partial g(p_1, p_2)/\partial p_i \geq 0\). As a special case, here we further assume that \(p = \max\{p_1, p_2\} = p_2\).

Consider the following two-stage game. In the first stage, the two unions decide whether or not to merge. A wage bargaining game with endogenous protocol is played in the second stage. If the unions bargain separately, the minimum total wage in equilibrium approaches \(p_1\) as \(\delta\) tends to 1. If they merge and bargain collectively, there is a unique equilibrium in which the expected total wage is \(p_2\) for any \(\delta \in (0, 1)\). Hence, if the two unions have different bargaining power, i.e., \(p_1 < p_2\), then, when \(\delta\) is close to 1, there is a subgame perfect equilibrium in which an encompassing union is formed.

Note that when the two unions have the same bargaining power, it can never be optimal for them to merge. In other words, our result suggests that unions with highly asymmetric bargaining powers are more likely to merge.

6 Further Discussions

We study a one-to-many bargaining model, in which the central player is endowed with the power of choosing whom to bargain with in each period. Different bargaining protocols may arise as part of the equilibrium strategy of the central player. In our model, the control over the protocol is highly asymmetric. Conceivably, we can have a model in which the peripheral players also have some control over the protocol. For example, at the beginning
of each period, every peripheral player decides whether or not to make himself available for bargaining. The central player then decides which available peripheral player to bargain with. The set of equilibrium outcomes will be expanded in favor of the peripheral players.

An alternative way of modelling is to consider a two-stage game. The players choose a specific bargaining protocol in the first stage, and the bargaining is conducted in the second stage with the chosen protocol. If the central player is endowed with the power of dictating the bargaining protocol, he certainly will choose the one that gives him the highest equilibrium payoff. This, however, does not seem to be a realistic description of real-life situations. If, instead, the players bargain over the choice of the bargaining protocol, they would have to agree on the protocol of such a bargaining as well, which leads to a hierarchical problem.

We assume that what type of contracts are available to the players is exogenously given. The assumption seems to be a realistic description of most real-life situations. However, one would wonder what happens if the contract type is also endogenously determined. We argue that when the players can choose between two types of contracts during their bargaining, the contingent contract is more likely to be adopted. The reason is simple. Suppose that the first agreement is enforced by a cash-offer contract, one can always replace it with a contingent contract in different terms so that both the central player and the involved peripheral player will be better off when they are sufficiently patient. This is simply because the payment in a contingent contract will not become sunk cost, and it can force the remaining peripheral players to accept smaller offers.
Appendix

Proof of Proposition 4.

Firstly, in either Case 1: $K_{s^*} = \infty$ for some $s^*$ or Case 2: there exist $s^*$ and $l \geq 1$ such that $K_{s+2l} = K_s$ for any $s \geq s^*$, we can easily derive the equilibrium outcome using backward induction. Cases 1 and 2 are referred to as regular protocols. More specifically, in Case 1, starting from period $t^* = 1 + \sum_{s=1}^{s^*-1} K_s$, the central player adopts the sequential protocol, which determines a unique continuation equilibrium outcome. Then, in each period $t < t^*$, the randomly selected proposer makes an offer such that the responder is indifferent between accepting and rejecting. Thus, by backward induction, we can uniquely determine the equilibrium outcome. In Case 2, starting from period $t^*$, the central player adopts a general alternate protocol, which also has a unique equilibrium outcome. Then, backward induction applies again. It is easy to see that in a general alternate protocol, as $\delta \to 1$, the equilibrium outcome converges to the same limit as the equilibrium with the basic alternate protocol.

When $\{K_s\}_{s=1}^{\infty}$ does not belong to the above two cases, the main difficulty is that one cannot establish a closed loop of indifference conditions as in the equilibrium with sequential protocol or alternate protocol. We refer to such protocols as irregular protocols. Again, there are two different cases.

Case 3. For any $T > 0$, there exist $s^*$ such that $K_s > T$ for any $s > s^*$. The example with uniformly increasing length of bargaining rounds (i.e., $K_s = s$) belongs to this case. In other words, the bargaining protocol approaches the sequential protocol as $s \to \infty$.

Case 4. There exists $T^*$ such that $K_s \leq T^*$ for any $s > 0$. Then there must exist a general alternate protocol $P^*$ and an unbounded sequence of $l_n$ ($l_n$ is even) consecutive bargaining rounds (or, segment of length $l_n$) such that the bargaining protocol $\{K_s\}_{s=1}^{\infty}$ is consistent with $P^*$ on each segment. In other words, the bargaining protocol approaches a general alternate protocol as $s \to \infty$.

Below it is shown that as $\delta \to 1$, the equilibrium outcomes associated with irregular protocols converge to the same limit as the equilibrium with the basic alternate protocol.
Without loss of generality, assume that $K_1 > 0$.

Denote as $(x^*_i, y^*_i)$ as the equilibrium offers made in the $l^{th}$ period of the $s^{th}$ round of the bargaining, where $1 - x^*_i$ is the offer to the peripheral player involved in the current round and $1 - y^*_i$ is his demand. Let $B^*_i = (1 - p_i) x^*_i + p_i y^*_i$, where $i = 1$ (2 resp.) if $s$ is odd (even resp.). In each period, the randomly selected proposer makes an offer so that the responder is indifferent between accepting and rejecting. It is easy to establish that

$$B^*_{Ks} = (1 - p_i) + \frac{\delta p_1 p_2 (1 - p_i)}{1 - p_j} B^*_{s+1}$$

and

$$B^*_{s-1} = (1 - \delta) (1 - p_i) + \delta B^*_i,$$

by which we can obtain

$$B^*_1 = (1 - p_i) + \frac{\delta^{K_1} p_1 p_2 (1 - p_i)}{1 - p_j} B^*_2.$$

Hence, we have

$$B^*_1 = (1 - p_i) \left[ 1 + \sum_{n=0}^{\infty} \delta^{D_n} (p_1 p_2)^{n+1} \right],$$

where $D_n = \sum_{l=0}^{n} K_{s+l}$. As $\delta \to 1$, $B^*_1 \to (1 - p_i) / (1 - p_1 p_2)$, and it follows that

$$\lim_{\delta \to 1} (1 - x^*_1) = \lim_{\delta \to 1} (1 - y^*_1) = \frac{p_1 (1 - p_2)}{1 - p_1 p_2},$$

which is the limit of player 1’s share in the equilibrium with alternate protocol.

**Proof of Proposition 5.** (Mixed-Strategy Equilibrium: Contingent Contracts)

Formally, a mixed-strategy Markov equilibrium can be written as $\{q, (x_1, y_1), (x_2, y_2)\}$, where $q$ is the probability that player A chooses player 1 in each period, $1 - x_i$ is the offer to player $i$, and $1 - y_i$ is player $i$’s demand. In the equilibrium, each player is indifferent between (1) accepting the current offer which leads to the conclusion of the bargaining in the following period, and (2) rejecting the current offer, in which case the bargaining is concluded two periods later.
Thus, the following conditions hold:

\[
\begin{align*}
1 - x_1 &= \delta q [(1 - p_1)(1 - x_1) + p_1 (1 - y_1)] + \delta (1 - q) p_1 [(1 - p_2)x_2 + p_2y_2] \\
1 - x_2 &= \delta q p_2 [(1 - p_1)x_1 + p_1 y_1] + \delta (1 - q) [(1 - p_2)(1 - x_2) + p_2(1 - y_2)] \\
(1 - p_2)y_1 &= \delta [qE_{A_1} + (1 - q)E_{A_2}] \\
(1 - p_1)y_2 &= \delta [qE_{A_1} + (1 - q)E_{A_2}]
\end{align*}
\]

where

\[
\begin{align*}
E_{A_1} &= (1 - p_2) [(1 - p_1)x_1 + p_1 y_1] \\
E_{A_2} &= (1 - p_1) [(1 - p_2)x_2 + p_2 y_2].
\end{align*}
\]

For \(0 < q < 1\), player A has to be indifferent between choosing player 1 and 2. That is,

\[E_{A_1} = E_{A_2}.
\]

Hence, we have here five unknowns and five equations. After tedious algebra, we find there is a unique solution with \(q^* = 1/2\). Note that player A’s randomization probability does not depend on the discount factor and the distribution of the bargaining power. As \(\delta \to 1\), player A’s expected payoff goes to \(\lambda (1 - p_1)(1 - p_2)\) and player i’s expected payoff goes to \(\lambda p_i (1 - p_j)\), where \(\lambda = (1 - p_1 p_2)^{-1}\).

**Proof of Proposition 6.**

It suffices to construct an equilibrium for the game with \(N = 3\), in which player A’s expected payoff is 0. We have shown that in the game with \(N = 2\), there are multiple equilibria in which player A obtains different expected payoffs. Denote two equilibria in the subgame with two remaining peripheral players \(i\) and \(j\) as \(E_1 (i, j)\) and \(E_2 (i, j)\). Denote as \(U_s (i, j)\) player A’s expected payoff from \(E_s (i, j)\). Without loss of generality, assume that \(U_1 (i, j) > U_2 (i, j) > 0\).

Consider the following strategy profile in the game with three peripheral players:
(1) Player A randomly chooses a peripheral player in period 0.

(2) In each period $t$, player A offers $\delta U_2(i, j)$ to the chosen player $k$ and accepts player $k$’s demand if and only if it is not greater than $\delta U_1(i, j)$.

(3) If no agreement is reached in period $t$, player A will stay with player $k$ in period $t + 1$.

(4) In each period $t$, the chosen player $k$ asks for $\delta U_1(i, j)$ and accepts any offer no less than $V_k$, where $V_k$ is player $k$’s continuation payoff given this strategy profile. More precisely,

$$V_k = \sum_{l=0}^{\infty} \delta^{l+2} p_k (1 - p_k)^l U_1(i, j) = \frac{\delta^2 p_k U_1(i, j)}{1 - \delta (1 - p_k)}.$$ 

(5) If player A’s offer is accepted in period $t$ by player $k$, the continuation equilibrium is $E_2(i, j)$; if player $k$’s demand is accepted, the continuation equilibrium is $E_1(i, j)$.

With this strategy profile, the first agreement is reached when player $k$ is recognized as a proposer for the first time, and then $E_1(i, j)$ is played. Player A’s expected payoff is 0. To verify that the strategy profile is indeed an equilibrium, first observe that player A is indifferent in choosing either peripheral player to bargain for the first agreement as his expected payoff is always 0. Second, player A cannot make an offer greater than $\delta U_2(i, j)$ because if such an offer is made and accepted, player A’s expected payoff will be negative. Hence, it remains to show that

$$\delta U_2(i, j) < V_k = \frac{\delta^2 p_k U_1(i, j)}{1 - \delta (1 - p_k)}.$$ 

When $\delta$ is sufficiently close to 1, the above condition is satisfied.

**Proof of Proposition 7.**

Suppose that there is an equilibrium with alternate protocol, which can be formally written as $\{(x_1, y_1), (x_2, y_2)\}$, where $1 - x_i$ is the offer to player $i$, and $1 - y_i$ is his demand.
Then, the following equilibrium conditions hold:

\[
\begin{align*}
1 - x_1 &= \delta^2 p_1 \quad \text{and} \\
1 - x_2 &= \delta^2 p_2 \\
\delta (1 - p_2) - (1 - y_1) &= \delta \left[ (1 - p_2) \left( \delta (1 - p_1) - \delta^2 p_2 \right) + p_2 (\delta (1 - p_1) - (1 - y_2)) \right], \\
\delta (1 - p_1) - (1 - y_2) &= \delta \left[ (1 - p_1) \left( \delta (1 - p_2) - \delta^2 p_1 \right) + p_1 (\delta (1 - p_2) - (1 - y_1)) \right]
\end{align*}
\]

from which we can solve for \( (x_1, y_1) \) and \( (x_2, y_2) \).

It is easy to check that player A’s expected payoff is

\[
E u_{12}^A = \frac{\delta (1 - p_1) (1 - p_2) - \delta^2 p_1 p_2 (1 - p_1) - \delta^3 p_1 p_2 (1 - p_2)}{1 - \delta^2 p_1 p_2}
\]

if player 1 is the first one to reach agreement, and it is

\[
E u_{21}^A = \frac{\delta (1 - p_1) (1 - p_2) - \delta^2 p_1 p_2 (1 - p_2) - \delta^3 p_1 p_2 (1 - p_1)}{1 - \delta^2 p_1 p_2}
\]

if player 2 is the first one to reach agreement. It is easy to check that

\[
\lim_{\delta \to 1} E u_{12}^A = \lim_{\delta \to 1} E u_{21}^A = (1 - p_1 - p_2)
\]

and that

\[
E u_{12}^A \geq E u_{21}^A \quad \text{if and only if} \quad p_1 \geq p_2.
\]

Hence, when \( p_1 + p_2 > 1 \), the equilibrium with alternate protocol does not exist when \( \delta \) is sufficiently close to 1 as the central player would obtain a negative payoff from such an equilibrium.

More importantly, if \( p_1 \neq p_2 \), the equilibrium with alternate protocol does not exist even when \( p_1 + p_2 < 1 \). This is because player A always finds it optimal to reach the first agreement with the stronger peripheral player, say, player 1. Thus, after a disagreement with player 1, player A will not switch to player 2 in the following period. This deviation cannot be punished because player A already receives his lowest possible payoff from the assumed equilibrium with alternate protocol.
Proof of Proposition 8. (Mixed-Strategy Equilibrium: Cash-offer Contracts)

Formally, a mixed-strategy Markov equilibrium can be written as \(\{q, (x_1, y_1), (x_2, y_2)\}\), where \(q\) is the probability with which the central player A chooses player 1, \(1 - x_i\) is the offer to player \(i\) and \(1 - y_i\) is his demand. Similar to the case with contingent contracts, the following equilibrium conditions hold:

\[
\begin{align*}
1 - x_1 &= \delta q \left[ (1 - p_1) (1 - x_1) + p_1 (1 - y_1) \right] + \delta^2 (1 - q) p_1 \\
1 - x_2 &= \delta (1 - q) \left[ (1 - p_2) (1 - x_2) + p_2 (1 - y_2) \right] + \delta^2 q p_2 \\
\delta (1 - p_2) - (1 - y_1) &= \delta \left[ q E_{A_1} + (1 - q) E_{A_2} \right] \\
\delta (1 - p_1) - (1 - y_2) &= \delta \left[ q E_{A_1} + (1 - q) E_{A_2} \right],
\end{align*}
\]

where

\[
\begin{align*}
E_{A_1} &= (1 - p_1) \left[ \delta (1 - p_2) - (1 - x_1) \right] + p_1 \left[ \delta (1 - p_2) - (1 - y_1) \right] \\
E_{A_2} &= (1 - p_2) \left[ \delta (1 - p_1) - (1 - x_2) \right] + p_2 \left[ \delta (1 - p_1) - (1 - y_2) \right].
\end{align*}
\]

In the mixed strategy equilibrium, player A is indifferent between choosing player 1 and 2. That is, \(E_{A_1} = E_{A_2}\), which can be simplified as

\[
(1 - p_1) x_1 + p_1 y_1 - \delta p_2 = (1 - p_2) x_2 + p_2 y_2 - \delta p_1.
\]

Solving for \(q\), we obtain

\[
q^* = \frac{1 - p_1}{(1 - p_1) + (1 - p_2)}.
\]

As \(\delta\) tends to 1, player \(i\)’s expected payoff tends to \(p_i\) and player A’s expected payoff tends to \(1 - p_1 - p_2\). This is an equilibrium for any \(\delta \in (0, 1)\) when \(p_1 + p_2 \leq 1\).

Here, the mixed-strategy equilibrium is unstable. Take \(q\) as the peripheral players’ common belief about player A’s strategy. Let \(1 - x_i(q)\) and \(1 - y_i(q)\) be player \(i\)’s least acceptable offer under this belief. If \(q > q^*\ (q < q^*\ resp.)\), given \(x_i(q)\) and \(y_i(q)\), the central player finds it profitable to further increase (reduce resp.) \(q\).
References


