Adverse Selection in a labor market with motivated workers.

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Abstract

In a labor market model of adverse selection, workers differ in their productivity levels. We introduce a second source of workers’ heterogeneity: their intrinsic motivation for the job. Intrinsic motivation is modeled as the monetary equivalent of a benefit workers obtain when they accept the job.

We show that, when workers are motivated, inefficiencies related to adverse selection decrease and the competitive equilibrium is characterized by a higher wage. Concerning the characteristics of labor supply we prove that, when productivity and motivation are positively correlated, one of two counter-intuitive effects occurs, for at least a sub-interval of possible wage levels: either average productivity of active workers is decreasing or average vocation of active workers is increasing in the wage.

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1 Introduction

Our main research question is the following: how does intrinsic motivation affect adverse selection in a vocation-based labor market?

When they are intrinsically motivated for a job, by performing their task workers receive a non-pecuniary benefit (a “vocational” premium), together with the wage. A typical example is the market for health services: nurses and physicians possibly derive utility not only from their wage, but also from the tasks they are performing (e.g., caring and helping patients). In the same way some school teachers receive a benefit from their relationship with kids and/or from the achievements of their students and many civil servants think...
their task is important per se (in this case we generally refer to public sector motivation). A "vocation-based labor market" is thus a market where workers' intrinsic motivation can be relevant, as in the previous example. Besley and Ghatak (2005) observe that motivation is familiar in sectors providing collective goods. Whereas a "non vocational labor market" is a market where workers' intrinsic motivation does not matter (workers do not receive any vocational premium by performing their task).

In the standard model of adverse selection (see for example Mas-Colell et al. 1995, chapter 13), workers differ in their productivity levels. We introduce a second source of heterogeneity: workers' vocation for the job. As in Heyes (2005) vocation is modeled as the monetary equivalent of a benefit workers can obtain when they accomplish their task and thus enters the workers' rationality constraint. We assume that vocation does not affect workers' production (neither as regards the number of units produced, nor as regards the quality of the output) but uniquely the benefit workers receive from accepting the job. Moreover, as in the standard model of adverse selection, (i) the opportunity cost of accepting the job is increasing in workers' productivity (in other words, the outside option is an increasing function of workers' skills); (ii) for institutional or informational reasons, firms offer a uniform wage in the market. In our setting this leads to two phenomena: (1) given a productivity level and a market wage, potential workers with high vocation are more likely to accept the job; this implies a (positive) selection effect on vocation. 1 (2) Given intrinsic motivation and a market wage, potential workers with low productivity are more likely to accept the job; this leads to the standard adverse selection effect on productivity.

Note that a theoretical justification for offering a uniform wage in some sectors producing collective goods is provided by the multitask principal-agent analysis. Holmstrom and Milgrom (1991) show that an optimal incentive contract can be to pay a fixed wage independent of measured performance when the agent's single task has several dimension to it.2

In this paper we consider the interaction of the two previous phenomena and investigate how the distribution of abilities and vocations in the population of potential workers affects the selection of workers hired in the market and the production inefficiency due to adverse selection. This allows us to characterize labor supply in a vocation-based market and to compare inefficiency caused by adverse selection in that market to the one arising in the standard non vocational labor market.

We first show that, given the wage rate, average productivity of workers accepting the job is higher in the vocation-based sector than in the non-vocational one. This implies that, with respect to the standard case of a non-vocational labor market, intrinsic motivation mitigates the inefficiency due to adverse

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1 The reason for writing the word positive in brackets will be clarified in the next section.

2 The authors explicitly refer to teachers' compensation. They show that the argument according to which "it is better to pay a fixed wage without any incentive scheme that to base teachers' compensation only on the limited dimensions of student achievement that can be effectively measured" (Holmstrom and Milgrom 1991, page 25) is valid.
lection. Moreover, the outside-option function plays a crucial role in defining the difference between average productivity of active workers in a labor market with motivated individuals and average productivity in the standard labor market where vocation does not matter. In particular, the steeper the function, the lower the difference between average productivity of workers accepting the job in the two markets. The intuition is the following: a steep outside option function implies that an increase in productivity is highly rewarded outside the market. When this is the case, the vocation-based market is able to attract only few high-skill / high-vocation workers. As a consequence intrinsic motivation is not so effective in reducing the production inefficiency due to adverse selection.

More interestingly are results concerning the impact of a wage increase on average productivity and average vocation of active workers that allow us to characterize labor supply in the vocation-based market. In the market where vocation does not matter, if wage increases (i) more workers accept the job, (ii) average productivity among workers hired by firms increases. We show that the second result is not necessarily true in the vocation-based market. In fact, a wage increase can deteriorate the average productivity of active workers.

More generally, as wage increases, we expect that average productivity rises (since more productive workers enter the market), and that average vocation falls (since less motivated workers enter the market). We show that both intuitive effects always occur if skills and vocations are independently distributed in the population, or if productivity and vocations are negatively correlated. However, when productivity and vocations are positively correlated, one of the two counter-intuitive effects always occurs: a wage increase can make either average productivity fall or average vocation rise. Which one of the two counter-intuitive effects occurs depends on the relative slope of two functions: the outside-option function and the regression line, the latter describing the positive linear dependence between productivity and vocation in the population of potential workers. The two counter-intuitive effects cannot occur simultaneously, however we cannot exclude that they occur sequentially. Our findings on the characteristics of labor supply generalize the result in Heyes (2005) that a wage increase can deteriorate the average vocation of workers hired by firms (which indeed corresponds to our intuitive effect on average vocation of active workers and is thus completely natural in our setting). In Heyes’ model two vocation levels exist and outside options are uniformly distributed and do not depend on productivity levels. We prove that a wage increase not only can deteriorate average vocation of active workers, but it can also and simultaneously deteriorate their average productivity.

Finally we compare competitive equilibria in the market where vocation does not matter and in the vocation-based market. Since average productivity of active workers is higher in the vocation-based sector than in the non-vocational one for every wage level, the equilibrium wage in the vocation-based market is weakly higher than the equilibrium wage in the market where vocation does not matter. Moreover, the flatter the outside-option function, the higher the difference between the two equilibrium wages.

This paper is related to the growing literature on workers’ intrinsic motiva-
tion and incentives which generally focuses on moral-hazard issues instead of on adverse selection\(^3\). Within that literature, the papers closest to our are the few considering the selection of motivated workers by employers. Handy and Katz (1998) show how nonprofit firms may screen out non motivated managers through a policy of lower wages, whereas Delfgaauw and Dur (2007) examine how the firm can attract and select highly motivated workers to fill a vacancy when workers’ motivation is private information. Those papers analyze firms screening motivated workers, we instead investigate "adverse selection in markets" when workers differ both on productivity and motivation.

In a companion paper (Barigozzi and Turati 2010), one of us considers a simple discrete version of the present model and focuses on the characteristics of labor supply (neither production inefficiencies caused by adverse selection nor competitive equilibrium issues are investigated). The results concerning the characteristics of labor supply are essentially confirmed also in the discrete case.

The rest of this work is organized as follows. Section 2 describes the model. In Subsection 2.1 we characterize adverse selection in the vocation-based market. In Section 3 we investigate how average vocation and average productivity change with the wage rate and in Section 4 we discuss market equilibrium issues. Section 5 concludes with some policy implications.

2 A simple labor market model with intrinsic motivation

We introduce intrinsic motivation in a simple model of adverse selection in the labor market.\(^4\)

Many identical potential firms can hire workers and each worker produces the same output. Workers use an identical constant returns to scale technology in which labor is the only input. The firms are risk neutral, seek to maximize their expected profits, and act as price takers. The price of the firms’ output is equalized to 1 (in units of a numeraire good). The risk neutral potential workers differ in the number of units of output they produce if hired in the market. The number of unit produced is denoted by \(\theta\), with \(\theta \in [\underline{\theta}, \bar{\theta}]\), and \(0 \leq \underline{\theta} \leq \bar{\theta} \leq \infty\).

We depart from the standard model by assuming that potential workers can have a vocation for the job. When they have a vocation, workers benefit from their intrinsic motivation for the specific task they are asked to accomplish. Vocation for the job is denoted by \(\gamma\), with \(\gamma \in [0, \bar{\gamma}]\).\(^5\)

As mentioned before, all workers produce the same output. The vocation parameter \(\gamma\) does not affect workers’ production (neither as regards the number of units produced, nor as regards the quality of the output) but uniquely the benefit workers receive from accepting the job, as will be explained in the following. A discussion on how to relax the latter assumption will be provided later on.

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\(^3\) Among others, Besley and Ghatak (2005), Francois (2000) and (2003), Siciliani (2009).


\(^5\) In alternative, one can interpret the parameter \(\gamma\) as potential workers’ taste for the job.
We call $F(\theta, \gamma)$ the cumulative distribution function (CDF) of the population of potential workers, and $f(\theta, \gamma)$ the probability density function (PDF). Moreover, $H(\theta)$ and $G(\gamma)$ respectively are the marginal cumulative distribution functions of productivity levels and vocations; with $h(\theta)$ and $g(\gamma)$ their marginal probability density functions. Finally, $E[\theta] = \int_0^\theta \theta h(\theta) d\theta = \mu_\theta$ and $E[\gamma] = \int_0^\gamma \gamma g(\gamma) d\gamma = \mu_\gamma$ respectively are the average productivity and the average vocation in the whole population of potential workers.

Workers seek to maximize the amount that they earn from their labor (in units of the numeraire good). A worker can choose to work either in the vocation-based labor market or outside. A worker with productivity $\theta$ can obtain $r(\theta)$ outside the vocation-based labor market. Thus $r(\theta)$ is the opportunity cost to a worker of productivity $\theta$ of accepting employment in the vocation-based sector. The function $r(\theta)$ can be interpreted either as a reduced form for workers’ utility or as the salary reached outside the vocation-based market. What is relevant is that the outside option depends on the worker’s productivity level.

By slightly abusing notation, the parameter $\gamma$ also corresponds to the monetary equivalent of the vocational premium workers obtain from the job in the vocation-based sector.

Potential applicants accept the job if and only if the total monetary benefit they receive from the job is larger than their outside option. The total monetary benefit to the worker is given by the wage rate $w$ plus the monetary equivalent of the vocational premium $\gamma$. Thus, a potential applicant with characteristics $(\theta, \gamma)$ accepts the job if and only if she receives a total benefit of at least $r(\theta)$ in the market (for convenience, we assume that the worker accepts if she is indifferent):

$$r(\theta) \leq w + \gamma$$

Interestingly, productivity $\theta$ is relevant both in the vocation-based labor market and outside, whereas the vocational premium $\gamma$ is uniquely obtained when the worker is hired by firms in the vocation-based market. In fact, a worker with characteristics $(\theta, \gamma)$ produces $\theta$ units of output if hired in the vocation-based market and obtains $r(\theta)$ if she stays out of that market. However, if staying out, she does not get any vocational benefit.

Inequality (1) shows that, all else equal, the higher the worker’s vocation, the higher their total benefit from the job. Note that, provided her vocation is sufficiently high and/or her outside option sufficiently low, a worker can decide to accept the job even when the salary is $w = 0$. This could correspond to the case of workers engaged in charity work.

As mentioned in the introduction, from inequality (1) we observe the following two phenomena:

1. given $\theta$ and $w$, potential workers with high vocation are more likely to accept the job; this implies a "positive selection" effect on vocation.\(^6\)

\(^6\)The words "positive selection" could suggest that firms are positively affected by the
2. given \( \gamma \) and \( w \), potential workers with low productivity are more likely to accept the job; this leads to the standard adverse selection effect on productivity.

While the first phenomenon is peculiar to the vocation-based labor market, the second one is not. Thus, in the following subsection we will compare the production inefficiency caused by adverse selection in the standard model where vocation does not matter, to the production inefficiency arising in the vocation-based labor market.

Later on we will call "standard non-vocational market" the market where vocation plays no role and workers are only characterized by their productivity \( \theta \), with \( \theta \in [\underline{\theta}, \overline{\theta}] \), and \( 0 \leq \underline{\theta} \leq \overline{\theta} \leq \infty \). Note that, in the standard non-vocational market, inequality (1) simply is \( r(\theta) \leq w \).

2.1 Adverse selection in the vocation-based labor market

Let us assume that workers’ productivity levels and vocations are unobservable by firms or that institutional constraints exist such that firms offer a uniform wage to potential workers. We also make the two following additional assumptions:

- \( r(\theta) \leq \theta, \ \forall \theta \in [\underline{\theta}, \overline{\theta}] \), i.e. production inside the market is higher than production outside the market for each worker. Or, production efficiency requires that all workers are hired by firms.
- \( r(\theta) \) is a strictly increasing function, i.e. workers more productive inside the market are also more productive outside. Which implies that skills are somehow rewarded outside the market. On the contrary, since the wage offered inside the market is uniform, skills are not rewarded there.\(^7\)
- We do not impose any concavity conditions on \( r(\theta) \) but we assume that the sign of \( r''(\theta) \) is constant.

The previous first two conditions represent standard assumptions assuring that adverse selection on productivity arises (both in the non-vocational market and in the vocation-based market). In particular, adverse selection on ability occurs since workers’ decision depends on the productivity level \( \theta \) in a way that adversely affects the firms: outside the market the payoff is greater for more selection of workers in the market. However, we assumed that intrinsic motivation has no direct impact on workers’ output. This explains the quotation marks.

\(^7\)If we interpret the outside option function \( r(\theta) \) as the salary workers can obtain in a non-vocational alternative labor market, we implicitly are assuming that some kind of screening mechanism is in place (only) outside the market. The vocation-based sector inability to screen workers’ productivity could depend on a more costly screening technology characterizing the sector. Or it could have institutional reasons: in sectors where workers can receive a vocational premium often services (health services, child care and education) are publicly provided and contracts for workers are standardized, i.e. they are uniformly designed on a national basis. (Think about contracts for teachers in every level of public school and contracts for nurses in public hospitals.)
capable workers, thus only relatively less capable workers are willing to accept the job at any given wage. Moreover, since production efficiency requires that all workers are hired by firms, employment is inefficiently low.

A clarification is useful at this stage. In this subsection we compare the production inefficiency caused by adverse selection on workers’ productivity in the vocation-based market to the same inefficiency arising in the standard market where vocation does not matter. When considering a standard market, the population of potential workers can be described by the same probability density function \( f(\theta, \gamma) \), provided that intrinsic motivation does not lead to any vocational premium when workers accept the job (in a sense, the parameter \( \gamma \) remains inactive).

From now on let us call marginal workers those workers who are indifferent between accepting or not the job. Moreover, let \( \tilde{\theta} \) be the productivity of marginal workers in the model where intrinsic motivation plays no role: \( \tilde{\theta} = r(\theta) - w \). Thus \( \theta = r^{-1}(w) \). In the same way, let \((\tilde{\theta}, \tilde{\gamma})\) be marginal workers in the model where vocation matters: \((\tilde{\theta}, \tilde{\gamma})\) such that \( r(\tilde{\theta}) = w + \tilde{\gamma} \). Thus, \( \tilde{\theta} = r^{-1}(w + \tilde{\gamma}) \). Note that, in the vocation-based sector and given wage \( w_0 \), all workers such that \( \gamma = r(\theta) - w_0 \) are indifferent between accepting and not accepting the job. According to where the function \( r(\theta) \) crosses the horizontal line \( \gamma = \tilde{\gamma} \), the highest productivity of workers accepting the job, \( \theta_{\text{max}} \), is lower or equal to \( \bar{\theta} \). Put differently, \( \theta_{\text{max}} \equiv \min \{r^{-1}(w_0 + \gamma), \tilde{\theta} \} \). In Figure 1 below, the set of potential workers is represented in the plane \((\theta, \gamma)\). The function \( r(\theta) - w_0 \) divides the set of potential workers in two regions and \( r(\theta) \) and \( w_0 \) are such that \( \theta_{\text{max}} = r^{-1}(w_0 + \gamma) < \bar{\theta} \). The shadowed area indicates all types accepting the job, the complementary region indicates types refusing the firms’ offer.

Note that in the standard market where vocation does not matter, active workers are simply indicated by the rectangle with sides \((r^{-1}(w_0) - \tilde{\theta}) \) and \( \tilde{\gamma} \).

**Lemma 1** Given a salary \( w_0 \), average productivity of active workers is higher in the vocation-based market than in the standard market where vocation does not matter: \( E_{VM}[\theta|w_0] > E_{SM}[\theta|w_0] \).

**Proof.** See Appendix 6.1. ■

Adverse selection arises both in the vocation-based sector and in the standard market where vocation does not matter, however the previous lemma implies that adverse selection has a higher impact in the outcome of the latter market:

**Corollary 1** With respect to the standard case of a non-vocational labor market, intrinsic motivation reduces the production inefficiency due to adverse selection. In particular, the flatter the outside-option function \( r(\theta) \), the lower the inefficiency.
Proof. Given all positive $\gamma$, the larger $r^{-1}(w_0 + \gamma)$, the higher the amount of workers with $\theta > \hat{\theta} = r^{-1}(w_0)$ entering the market and, thus, the lower the production inefficiency due to adverse selection. Obviously, the amount of workers with $\theta > r^{-1}(w_0)$ entering the market is decreasing in $r'(\theta)$ (see again Figure 1).

Corollary 1 states that the outside-option function $r(\theta)$ plays a crucial role in defining how large is the difference between average productivity of active workers in a labor market with intrinsically motivated individuals and average productivity in the standard labor market where vocation does not matter. In particular, we saw that the flatter the curve, the higher average productivity of active workers in the vocation-based labor market. To intuit this result let us consider high-ability intrinsically motivated workers and their decision whether to accept the job in the vocation-based market: on the one side, in such a market they receive a wage $w$ which does not reward their high productivity; on the other side, they obtain their vocational premium. When the reservation wage function is flat, return to skills outside the vocation-based sector is low. This implies that, outside the vocation-based market, an increase in productivity has a low impact on workers’ payoff. Thus, working in the vocation-based market becomes more attractive. When, on the contrary, the reservation wage function is steep, return to skills outside the vocation-based sector is high such that working in the vocation-based market becomes less attractive.

As a final remark note that, when return to skills outside the vocation-based sector is so high that the reservation wage function is a vertical line, average productivity of active workers in the vocation-based labor market and average productivity in the standard labor market where vocation does not matter are exactly the same. Put differently, when the reservation wage function is a vertical line, vocation has no impact at all on workers’ rationality constraint:

Remark 1 When return to skills outside the vocation-based labor market is infinitely high ($r(\theta)$ is a vertical line), vocation does not affect workers’ decision whether to accept the job. Thus, production inefficiency due to adverse selection is the same in the vocation-based and in the standard labor market.

We can conclude that the impact of intrinsic motivation on the decision whether to accept the job in the vocation-based labor market is negatively related to return to skills outside the market ($r(\cdot)$). As already noticed, since productivity is not rewarded at all in the vocation-based market, the more it is rewarded outside the market, the less attractive is the job for high productivity workers.
3 Average productivity and average vocation of active workers as a function of the wage

We consider here how the characteristics of the pool of active workers change as the wage increases. This analysis will provide useful insights to understand labor supply in the vocation-based market. Then, in the next section, we will characterize the competitive equilibrium.

Note that, as wage marginally increases and becomes $w_0 + dw_0$, the indifference curve $\gamma_I(\theta, w_0) = r(\theta) - w_0$ shifts on the right (see again Figure 1). Intuitively, we expect that a wage increase has (1) a negative impact on average vocation of active workers since also workers with vocation lower than before enter the market, and (2) a positive impact on average productivity since also workers with productivity higher than before enter the market. However, vocation and productivity interact and together determine workers’ willingness to accept the job, so that also the counter-intuitive cases are possible: given a marginal increase in the wage rate, either average vocation can be increasing or average productivity can be decreasing in the wage rate, as we will see.

To intuit why, in the vocation-based market, average productivity of active workers can be decreasing in the wage, recall the expression for average productivity as it was derived in the proof of Lemma (1):

$$E_{VM}\left[\theta | w_0\right] = \frac{\int_0^\gamma \left[\int_0^{f^{-1}(w_0)} \theta f(\theta, \gamma) d\theta\right] d\gamma + \int_0^\gamma \left[\int_{f^{-1}(w_0)}^{f^{-1}(w_0+\gamma)} \theta f(\theta, \gamma) d\theta\right] d\gamma}{\int_0^\gamma \left[\int_0^{f^{-1}(w_0)} f(\theta, \gamma) d\theta\right] d\gamma + \int_0^\gamma \left[\int_{f^{-1}(w_0)}^{f^{-1}(w_0+\gamma)} f(\theta, \gamma) d\theta\right] d\gamma}.$$  

The denominator is always increasing in the wage $w$: the higher the wage, the higher the probability that workers accept the job. The numerator is always increasing in the wage too: the higher the wage, the higher workers’ average productivity for every value of $\gamma$. Thus, average productivity $E_{VM}\left[\theta | w_0\right]$ decreases with the wage when the denominator increases more than the numerator does, it increases when the opposite occurs.

The very same reasoning can be used to understand why, in the vocation-based market, average vocation of active workers $E_{VM}\left[\gamma | w_0\right]$ can be increasing in the wage.

We now show that average productivity of active workers is always increasing in the wage when the market is a standard one where intrinsic motivation does not matter. Then we analyze the case of the vocation-based sector.

Remark 2 In the standard labor market where vocation does not matter, (i) the impact of a marginal increase in wage on marginal workers’ productivity

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8Note that, when interpreting the outside option $r(\theta)$ as wage potentially received in an alternative, non-vocational labor market, we must assume that the vocation-based labor market is small with respect to the non-vocational one. Since price in the small market does not affect price in the big one, we can keep $r(\theta)$ fixed also when salary in the vocation-based labor market changes.
\[ \hat{\theta} = r^{-1}(w_0) \] is positive and equal to \( r'(\theta) \). (ii) Average productivity of active workers is monotonously increasing in the wage.

**Proof.** See the Appendix 6.2.

Remark 2 describes the positive effect on average productivity of active workers induced by a wage increase in the standard market where vocation does not matter. As we saw, if wage increases more productive workers accept the job and average productivity among active workers increases. We will now show that this is not necessarily true in the vocation-based market.

To prove that, in the vocation-based market, average productivity and average vocation of active workers can be decreasing and increasing in the wage respectively, at least for some intervals of the relevant wage rates, we define:

**Definition 1** Net reservation wage is the wage that makes the potential worker with characteristics \((\theta, \gamma)\) indifferent between accepting and not accepting the job in the vocation-based market:

\[ W(\theta, \gamma) \equiv r(\theta) - \gamma. \]

Note that, given \( \gamma \), net reservation wage increases with \( \theta \) whereas, given \( \theta \), net reservation wage decreases with \( \gamma \). Moreover, since \( r_0(\theta) > 0 \), among all potential workers, those with characteristics \((\theta, \gamma)\) accept the job for the lowest wage whereas those with characteristics \((\theta, 0)\) accept the job for the highest wage. In other words, starting from a wage below \( r(\theta) - \gamma \) and letting wage increase, workers with characteristics \((\theta, \gamma)\) are the first whereas workers with characteristics \((\theta, 0)\) are the last to enter the vocation-based market. More in details, for \( w < r(\theta) - \gamma \) no workers accept the job and average productivity and average vocation of active workers are both zero, for \( w = r(\theta) - \gamma \) average productivity of active workers is \( E_{VM}[\theta | w = r(\theta) - \gamma] = \hat{\theta} \) and average vocation is \( E_{VM}[\gamma | w = r(\theta) - \gamma] = \bar{\gamma} \). For \( w \geq r(\theta) \), since all workers already entered the market, average productivity of active workers is \( E_{VM}[\theta | w \geq r(\theta)] = \mu_\theta \) and average vocation is \( E_{VM}[\gamma | w \geq r(\theta)] = \mu_\gamma \). In the following we just want to investigate how average productivity and average vocation of active workers change for values of the wage belonging to the interval \( [r(\theta) - \gamma, r(\bar{\theta})] \).

Suppose that the market wage is \( w = r(\bar{\theta}) - \gamma \) such that only workers with characteristics \((\bar{\theta}, \gamma)\) entered the vocation-based market. Since workers with the lowest productivity level and the highest vocation are already in the market, as the wage marginally increases average productivity of active workers can not decrease and average vocation can not increase. As a consequence, the counter-intuitive results as for average productivity and for average vocation of active workers can only occur for values of the wage rate such that some different workers types entered the market already.

**Definition 2** \( E_\gamma [W(\theta, \gamma) | \theta] = r(\theta) - E_\gamma [\gamma | \theta] \) indicates average net reservation wage for workers with productivity \( \theta \).

We assume a linear dependence in mean between \( \theta \) and \( \gamma \), implying that \( E_\gamma [\gamma | \theta] = \gamma(\theta) = a + b\theta \). This relation can be rewritten through the regression \( \gamma = a + b\theta + \tilde{\varepsilon} \) where \( \tilde{\varepsilon} \) is an error term with zero conditional mean and \( b = \)
such that high-productivity / high-vocation and low-productivity / low-vocation
the market before workers with low productivity and low vocation (\(w\) are such that workers with low productivity and low vocation enter the market on average. Since net reservation wage of workers that, on average, ask for a higher wage to enter the market as productivity increases. Let us consider the inverse function of \(W_A(\theta)\). \(W_A^{-1}(w)\) indicates the productivity of workers that, on average, have net reservation wage equal to \(w\). If \(W_A(\theta)\) is monotonically increasing in a given sub-interval of \([\theta_0, \theta]\), \(W_A^{-1}(w)\) is monotonically increasing too. This means that, as wage increases, the productivity of types who are, on average, indifferent to that wage increases too, so that more productive workers enter the market on average. Since net reservation wage is \(r(\theta) - \gamma\), intuitively \(W_A(\theta)\) is increasing when, on average, a marginal increase in productivity has a higher impact than a marginal increase in vocation on workers’ net reservation wage. Whereas, when \(W_A(\theta)\) is decreasing in a given sub-interval of \([\theta_0, \theta]\), on average workers ask for a lower wage to enter the market as productivity increases. If \(W_A(\theta)\) is decreasing, \(W_A^{-1}(w)\) is monotonically decreasing too. This means that, as wage increases, the productivity of types who are, on average, indifferent to that wage decreases too, so that less productive workers enter the market on average. Intuitively, \(W_A(\theta)\) is increasing when, on average, a marginal increase in vocation has a higher impact than a marginal increase in productivity on workers’ net reservation wage.

The proposition below is based on the following idea. Suppose \(\text{cov}(\theta, \gamma) > 0\) such that high-productivity / high-vocation and low-productivity / low-vocation potential workers belong to the regression line \(\gamma(\theta) = a + b\theta\). If net reservation wages are such that workers with high productivity and high vocation enter the market before workers with low productivity and low vocation \((r(\theta) - 0 > r(\bar{\theta}) - \bar{\gamma})\), average productivity of active workers necessarily decreases when the wage reaches the value \(w = r(\bar{\theta})\), and the counter-intuitive result for average productivity of active workers occurs. If, on the contrary, net reservation wages are such that workers with low productivity and low vocation enter the market before workers with high productivity and high vocation \((r(\bar{\theta}) - 0 < r(\bar{\theta}) - \bar{\gamma})\), average vocation of active workers necessarily increases when the wage reaches the value \(w = r(\bar{\theta}) - \bar{\gamma}\), and the counter-intuitive result for average vocation of active workers occurs. The same reasoning can be reproduced by comparing net reservation wage of a given workers type \((\theta_0, \gamma_0)\) on the regression line with net reservation wage of workers with characteristics \((\bar{\theta}, \bar{\gamma})\). If net reservation wages are such that workers \((\bar{\theta}, \bar{\gamma})\) enter the market before workers \((\theta_0, \gamma_0)\), average productivity of active workers necessarily decreases when the wage reaches the value \(w = r(\theta_0) - \gamma_0\). On the contrary, if net reservation wages are such that workers \((\bar{\theta}, \bar{\gamma})\) enter the market after workers \((\theta_0, \gamma_0)\), average vocation of active workers necessarily increases when the wage reaches the value \(w = r(\bar{\theta}) - \bar{\gamma}\).
Proposition 1. If $\text{cov}(\theta, \gamma) > 0$ and for at least a sub-interval of possible wage levels: average productivity of active workers is decreasing in the wage when $r'(\theta) < b$, average vocation of active workers is increasing in the wage when $r'(\theta) > b$.

Proof. The regression line is $\gamma(\theta) = a + b\theta$ and is defined for $\theta \in [\bar{\theta}, \bar{\theta}]$. We are considering the case where $b > 0$. Given that $\theta \in [\bar{\theta}, \bar{\theta}]$ and $\gamma \in [0, \bar{\gamma}]$, $b = \frac{\bar{\gamma}}{\bar{\theta} - \bar{\theta}}$, with $\gamma(\bar{\theta}) = 0$ and $\gamma(\bar{\theta}) = \bar{\gamma}$. Here $W'_A(\theta) = r'(\theta) - b \geq 0$, or the average net reservation wage function can be either increasing or decreasing in productivity. 

(i) We consider first the case where it does not exist a productivity level $\theta_0 \in [\bar{\theta}, \bar{\theta}]$ such that $r'(\theta_0) = b$. Either $r'(\theta) < b \forall \theta \in [\bar{\theta}, \bar{\theta}]$ or the opposite. 

(a) Suppose that $r'(\theta) < b \forall \theta \in [\bar{\theta}, \bar{\theta}]$ which implies $W'_A(\theta) < 0 \forall \theta \in [\bar{\theta}, \bar{\theta}]$, or the average net reservation wage function is monotonically decreasing in productivity. Let us consider the indifference curve $\gamma_I(\theta, w_0) = r(\theta) - w_0$ passing through the point $(\bar{\theta}, 0)$, that is $\gamma_I(\theta, \bar{\theta}) = r(\theta) - r(\bar{\theta})$, and compare it with the regression line $\gamma(\theta) = a + b\theta$ in correspondence of the value $\theta = \bar{\theta}$ (see Figure 2).

Insert Figure 2 here

Since $r'(\theta) < b \forall \theta \in [\bar{\theta}, \bar{\theta}]$, the value of $\gamma_I(\theta, \bar{\theta})$ and of $\gamma(\theta)$ for $\theta = \bar{\theta}$ must be such that: $\gamma_I(\bar{\theta}, \bar{\theta}) < \gamma(\bar{\theta})$. The previous inequality means $r(\bar{\theta}) - r(\bar{\theta}) < \bar{\gamma}$ and can be rewritten as:

$$r(\bar{\theta}) - \bar{\gamma} < r(\bar{\theta}) - 0.$$  \hspace{1cm} (3)

Inequality (3) indicates that net reservation wage of workers with characteristics $(\bar{\theta}, 0)$ is larger than net reservation wage of workers with characteristics $(\bar{\theta}, \gamma)$. In words: workers with low-productivity and low-vocation enter the market after workers with high-productivity and high-vocation. Moreover, since $\text{cov}(\theta, \gamma) > 0$, potential workers close to both the previous types have a large mass in the population. Thus, both average productivity and average vocation must decrease when market wage reaches the value $w = r(\bar{\theta})$. This means that the counter-intuitive effect for average productivity necessarily occurs. (ib) Now, suppose that $r'(\theta) > b \forall \theta \in [\bar{\theta}, \bar{\theta}]$ which implies $W'_A(\theta) > 0 \forall \theta \in [\bar{\theta}, \bar{\theta}]$, or the average net reservation wage function is monotonically increasing in productivity. Take again the indifference curve passing through the point $(\bar{\theta}, 0)$: $\gamma_I(\theta, \bar{\theta}) = r(\theta) - r(\bar{\theta})$ and compare it with the regression line $\gamma(\theta) = a + b\theta$ in correspondence of the value $\theta = \bar{\theta}$. Since $r'(\theta) > b \forall \theta \in [\bar{\theta}, \bar{\theta}]$, $\gamma_I(\bar{\theta}, \bar{\theta}) > \gamma(\bar{\theta})$ which means $r(\bar{\theta}) - r(\bar{\theta}) > \bar{\gamma}$. Thus:

$$r(\bar{\theta}) - \bar{\gamma} < r(\bar{\theta}) - \bar{\gamma}.$$  \hspace{1cm} (4)

Inequality (4) show that net reservation wage of workers with characteristics $(\bar{\theta}, \bar{\gamma})$ is larger than net reservation wage of workers with characteristics $(\bar{\theta}, 0)$. In words: workers with high-productivity and high-vocation enter the market after workers with low-productivity and low-vocation. Again, since $\text{cov}(\theta, \gamma) > 0$, potential workers close to both the previous types have a large mass in the population. Thus, both average productivity and average vocation must increase
when market wage reaches the value \( w = r(\bar{\theta}) - \bar{\gamma} \). This means that the counter-intuitive effect for average vocation of active workers necessarily occurs.

(ii) Now let us consider the case where it exists a productivity level \( \theta_0 \in [\underline{\theta}, \bar{\theta}] \) such that \( r'(\theta_0) = b \). Suppose that the function \( r(\theta) \) is concave so that \( r'(\theta) \) is decreasing. Given \( \theta_0 \in [\underline{\theta}, \bar{\theta}] : r'(\theta_0) = b, r'(\theta) > b \) for \( \theta < \theta_0 \) and \( r'(\theta) < b \) for \( \theta > \theta_0 \). Thus, \( W_A'(\theta) > 0 \) for \( \theta \in [\underline{\theta}, \theta_0] \) and \( W_A'(\theta) \leq 0 \) for \( \theta \in [\theta_0, \bar{\theta}] \). We call \( \gamma_0 \) the point corresponding to \( \theta_0 \) on the regression line: \( \gamma_0 = a + b\theta_0 \). We apply here the same reasoning as in part (ia) before for the interval \( \theta \in [\theta_0, \bar{\theta}] \). Let us consider the indifference curve \( \gamma_I(\theta, w_1) = r(\theta) - w_1 \) passing through the point \((\theta_0, \gamma_0)\), that is \( \gamma_I(\theta, \theta_0) = r(\theta) - r(\theta_0) + \gamma_0 \) and compare it with the regression line \( \gamma(\theta) = a + b\theta \) in correspondence of the value \( \theta = \bar{\theta} \) (see Figure 3).

As insert Figure 3 here

Since \( r'(\theta) \leq b \forall \theta \in [\underline{\theta}, \bar{\theta}], \gamma_I(\bar{\theta}, \theta_0) = r(\bar{\theta}) - r(\theta_0) + \gamma_0 \leq \gamma(\bar{\theta}) = \bar{\gamma} \) which means:

\[ r(\bar{\theta}) - \bar{\gamma} < r(\theta_0) - \gamma_0. \tag{5} \]

The previous inequality shows that net reservation wage of workers with characteristics \((\theta_0, \gamma_0)\) is larger than net reservation wage of workers with characteristics \((\bar{\theta}, \bar{\gamma})\). In words: workers with productivity \( \theta_0 < \bar{\theta} \) and vocation \( \gamma_0 < \bar{\gamma} \) enter the market after workers with productivity \( \bar{\theta} \) and vocation \( \bar{\gamma} \). Both workers types \((\theta_0, \gamma_0)\) and \((\bar{\theta}, \bar{\gamma})\) belong to the regression line, so that workers types close to them have a large mass. Thus, both average productivity and average vocation must decrease when market wage reaches the value \( w = r(\theta_0) - \gamma_0 \). This means that the counter-intuitive effect for average productivity of active workers necessarily occurs.

Suppose now that the function \( r(\theta) \) is convex so that \( r'(\theta) \) is increasing. Given \( \theta_1 \in [\underline{\theta}, \bar{\theta}] : r'(\theta_1) = b, r'(\theta) < b \) for \( \theta < \theta_1 \) and \( r'(\theta) \geq b \) for \( \theta \geq \theta_1 \). Thus, \( W_A'(\theta) < 0 \) for \( \theta \in [\underline{\theta}, \theta_1] \) and \( W_A'(\theta) \geq 0 \) for \( \theta \in [\theta_1, \bar{\theta}] \). The same reasoning than in part (ib) before can be applied in the interval \([\theta_0, \bar{\theta}]\) using as a reference point \((\theta_0, \gamma_0)\) on the regression line, and considering the indifference curve passing through \((\theta_0, \gamma_0)\). Since \( r'(\theta) \geq b \forall \theta \in [\theta_0, \bar{\theta}], \gamma_I(\bar{\theta}, \theta_0) = r(\bar{\theta}) - r(\theta_0) + \gamma_0 \geq \gamma(\bar{\theta}) = \bar{\gamma} \) which means:

\[ r(\bar{\theta}) - \bar{\gamma} > r(\theta_0) - \gamma_0. \tag{6} \]

The previous inequality shows that net reservation wage of workers with characteristics \((\theta_0, \gamma_0)\) is larger than net reservation wage of workers with characteristics \((\bar{\theta}, \bar{\gamma})\). In words: workers with productivity \( \theta_0 < \bar{\theta} \) and vocation \( \gamma_0 < \bar{\gamma} \) enter the market before workers with productivity \( \bar{\theta} \) and vocation \( \bar{\gamma} \). Both workers types \((\theta_0, \gamma_0)\) and \((\bar{\theta}, \bar{\gamma})\) belong to the regression line, so that workers types close to them have a large mass. Thus, both average productivity and average vocation must increase when market wage reaches the value \( w = r(\bar{\theta}) - \bar{\gamma} \). This means that the counter-intuitive effect for average vocation of active workers necessarily occurs. ■

Obviously, case (i) in the proof of Proposition 1 always occurs if \( r''(\theta) = 0 \).
Remark 3 In a vocation based market, if \( \text{cov}(\theta, \gamma) \leq 0 \), average productivity of active workers is monotonically increasing and average vocation is monotonically decreasing in the wage.

Proof. Let us consider average net reservation wage \( W_A(\theta) \equiv r(\theta) - \gamma(\theta) \) with \( W'_A(\theta) = r'(\theta) - \gamma'(\theta) \). If \( \text{cov}(\theta, \gamma) \leq 0 \) then \( \gamma'(\theta) \leq 0 \). This implies \( W'_A(\theta) > 0 \ \forall \theta \in [\bar{\theta}, \bar{\gamma}] \), or the average net reservation wage function is monotonically increasing in the productivity. Thus, as we remarked before, as wage increases the productivity of types who are, on average, indifferent to that wage increases too, so that more productive workers enter the market on average. Remind that the first workers to enter the market have characteristics \((\bar{\theta}, \bar{\gamma})\), the last ones \((\bar{\theta}, 0)\) and, since \( \text{cov}(\theta, \gamma) \leq 0 \), all types close to \((\bar{\theta}, \bar{\gamma})\) and \((\bar{\theta}, 0)\) have a large mass in the population of potential workers. On the contrary workers with characteristics close to \((\bar{\theta}, 0)\) and \((\bar{\theta}, \bar{\gamma})\) have low weight in the population and, when the wage is sufficiently high for them to enter the market, they do not change the means of average productivity and average vocation of active workers very much. As a consequence it does not matter whether workers of type \((\bar{\theta}, 0)\) or of type \((\bar{\theta}, \bar{\gamma})\) enter the market the first. Since \( \text{cov}(\theta, \gamma) \leq 0 \), as wage increases productivity and vocation of new workers entering the market move in opposite direction and average productivity of active workers increases whereas average vocation decreases in the wage. ■

From Proposition 1 and Remark 3 it is clear that \( \text{cov}(\theta, \gamma) > 0 \) and \( r'(\theta) < b \) are necessary and sufficient conditions for the counter-intuitive effect for average productivity of active workers to occur. Whereas \( \text{cov}(\theta, \gamma) > 0 \) and \( r'(\theta) > b \) are necessary and sufficient conditions for the counter-intuitive effect for average vocation to occur. In general:

Corollary 2 \( \text{cov}(\theta, \gamma) > 0 \) is a necessary and sufficient condition for counter-intuitive effects to occur.

Proposition 1 implies that, as the wage increases, the two counter-intuitive effects cannot simultaneously occur: either average productivity of active workers decreases or average vocation increases. The proposition does not exclude however, that the two counter-intuitive effects sequentially occur when there exists a productivity level \( \theta_0 \in [\bar{\theta}, \bar{\gamma}] \) such that \( r'(\theta_0) = b \). The fact that the two counter-intuitive effects cannot simultaneously occur can be intuitively explained recalling the previous corollary: \( \text{cov}(\theta, \gamma) > 0 \) is a necessary condition for both of them. Suppose the counter intuitive effect for average productivity occurs such that, as wage increases, the new workers entering the market are less productive, on average, than those already inside. Since \( \text{cov}(\theta, \gamma) > 0 \), the new workers must also be characterized by less vocation, on average, than

\[ \text{cov}(\theta, \gamma) > 0 \]

\[ r'(\theta) < b \]

For the two counter-intuitive effects to sequentially occur we need specific assumptions on the joint probability distribution of \( \theta \) and \( \gamma \) when it exists a productivity level \( \theta_0 \in [\bar{\theta}, \bar{\gamma}] \) such that \( r'(\theta_0) = b \). In particular we need conditions such that, cases (ib) and (ia) also replicate in the interval \([\bar{\theta}, \theta_0]\) when \( r(\theta) \) is concave and when it is convex, respectively. (See the proof of Proposition 1).
before. In the same way, take the case where the counter intuitive effect for average vocation occurs such that, as wage increases, the new workers entering the market have higher vocation, on average, than those already inside. Since \( \text{cov}(\theta, \gamma) > 0 \), the new workers must also be characterized by more productivity, on average, than before.

Following Proposition 1, we now perform two simulations with a bivariate Normal random variable to obtain the counter-intuitive effects. The graphical representation of the two simulations is then provided. To keep things as simple as possible we take \( r(\theta) = \theta \) in both of them, so that we consider case (i) of the proof.

In order to compute the expected values of \( \gamma \) and \( \theta \) given the wage \( w \) we recur to Monte Carlo integration. We first consider the experiment showing the counterintuitive effect as for average productivity (average productivity is decreasing in the wage).

- We draw 200,000 simulations from a truncated bivariate Normal random variable with \( \bar{\theta} = 20, \bar{\gamma} = 30, \bar{\tau} = 20 \). Means are \( \mu_\theta = 25 \) and \( \mu_\gamma = 10 \). The slope of the regression line is: \( b = \frac{\mu_\gamma - \mu_\theta}{\tau} = 2 \). The two marginals are defined on \([0, +\infty)\), we take \( \text{Var}(\theta) = 2.7 \) and \( \text{Var}(\gamma) = 12.2 \). \( \text{Cov}(\gamma, \theta) = 5.4 \) and correlation is \( \rho_{\gamma \theta} = \frac{\text{Cov}(\gamma, \theta)}{\sqrt{\text{Var}(\theta)\text{Var}(\gamma)}} = 0.93 \). We compute \( E[\theta|w] \) and \( E[\gamma|w] \) for different values of \( w \).

Results are shown in Figures 4. Average productivity of active workers is decreasing in the wage for \( w \geq 10 \). In fact, as stated in Proposition 1, when \( r'(\theta) = 1 < b = 2 \forall \theta \in [\bar{\theta}, \bar{\gamma}] \) high-ability/high-vocation workers enter the market before low-ability/low-vocation workers and the counterintuitive effect as for average productivity occurs. Moreover, as we expected, average vocation of active workers is monotonically decreasing in the wage.

Interestingly enough and as stated by Remark 3, if we take the very same exercise and uniquely change the sign of the covariance (from positive to negative), both intuitive effects occur and average productivity of active workers is increasing whereas average vocation is decreasing in the wage.\(^{10}\)

We now slightly change the previous distribution to show the counterintuitive effect for average vocation (average vocation is increasing in the wage). In particular the maximum level for vocation is reduced from 20 to 5, so that average vocation in the population of potential workers becomes \( \mu_\gamma = 2.5 \). Also variance of \( \gamma \) and \( \text{cov}(\gamma, \theta) \) are reduced w.r.t. before. More importantly, the slope of the regression line becomes \( b = 1/2 \).

\(^{10}\)We omit the graphical representation of this latter simulation. It is however available upon request to the authors.
• We draw 200,000 simulations from a truncated bivariate Normal random variable with \( \theta = 20, \beta = 30, \gamma = 5 \). Means are \( \mu_\theta = 25 \) and \( \mu_\gamma = 2.5 \). The two marginal are defined on \([0, +\infty)\), we take \( \text{Var}(\theta) = 2.7 \) and \( \text{Var}(\gamma) = 0.76 \). \( \text{Cov}(\theta, \gamma) = 1.34 \) and correlation is \( \rho_{\theta\gamma} = \frac{\text{Cov}(\theta, \gamma)}{\sqrt{\text{Var}(\theta)\text{Var}(\gamma)}} = 0.93 \).

We compute \( E[\theta|w] \) and \( E[\gamma|w] \) for different values of \( w \).

Results are shown in Figure 5. Average vocation of active workers is increasing in the wage for \( w \geq 20 \). In fact, as stated in Proposition 1, when \( r'(\theta) = 1 > b = 1/2 \) \( \forall \theta \in [2, 30] \) low-ability/low-vocation potential workers enter the market before high-ability/high-vocation workers and the counterintuitive effect as for average vocation occurs. Moreover, as we expected, average productivity of active workers is monotonically increasing in the wage.

Insert figure 5 here

As regards inefficiencies induced by adverse selection, Remark 2 and Proposition 1 imply:

**Corollary 3** Let us consider production inefficiency caused by adverse selection. In the vocation-based market, the standard positive effect on average productivity generated by a wage increase can be reversed when \( \text{cov}(\theta, \gamma) > 0 \).

Corollary 1 stated that the production inefficiency due to adverse selection is mitigated by intrinsic motivation since, given the same wage, more productive workers enter the vocation-based market than they enter the non-vocational one. Vocation is thus beneficial since it makes both the amount of trade between firms and workers and the production to increase. However Proposition 1 shows that, as wage increases, a reduction of average productivity is possible, implying that, in the vocation-based market, the production inefficiency caused by adverse selection can actually be worse as wage increases. However, Lemma 1 shows that for every value of the wage a higher production level is obtained in the vocation-based market than in the standard one. As a consequence, we can conclude that the positive effect of vocation on market productivity persists despite the possible negative impact of a wage increase on average productivity of active workers.

From a policy perspective, a wage increase is clearly desirable when average productivity of active workers is increasing in the wage. This is always the case when \( \text{cov}(\theta, \gamma) < 0 \) and when \( \text{cov}(\theta, \gamma) > 0 \) and \( r'(\theta) > b = \frac{\text{cov}(\theta, \gamma)}{\text{var}(\theta)} \). In the latter case we showed that average productivity of active workers is always increasing in the wage and average vocation of active workers is increasing in the wage for at least a subinterval of \( [r(\theta) - \gamma, r(\theta)] \). Given condition \( r'(\theta) > b \), when \( \text{cov}(\theta, \gamma) > 0 \) it is more likely that the undesirable counter-intuitive effect on average productivity is avoided the lower the covariance, the higher the slope of the outside-option function and the higher the variance of productivity among potential workers. In other words, the desired outcome is more likely when
return to skills outside the market is high. Note that, if we relax the assumption that vocation has no impact on the workers’ outcome, for example by assuming that it positively affects quality, than the case where average productivity and average vocation of active workers are both increasing in the wage is obviously a very desirable case.

More generally, in this section we analyzed how the distribution of abilities and vocations in the population of potential workers affects the characteristics of labor supply in a vocation-based market. Interestingly, we proved that not only average vocation (as Heyes 2005 already showed), but also average productivity of active workers may decrease as wage increases. Unfortunately average productivity and average vocation of active workers can deteriorate together, this represents a very negative phenomenon especially in the case where vocation positively affects workers’ outcome in some way. Our results depend on how productivity and vocation together interact to determine workers’ net reservation wage.

3.1 The distribution of productivity and vocation in the population of potential workers

We proved that counter-intuitive phenomena in labor supply occur when productivity and vocation are positively correlated. Two empirical studies finding evidence on the existence of such a positive correlation are Freeman (1997) and Becchetti et al. (?).

Freeman considers volunteers workers, or the ones who are ready to work for nothing. In our model they are the ones with either a very high motivation or a very low outside option, or both. He shows that volunteers are indeed high productivity workers characterized by a high opportunity-cost to engage in the working-for-nothing activity. This proves that potential workers with characteristics close to \((\theta, \gamma)\) have a large mass in the population so that a positive correlation between productivity and vocation exists at least for high-productivity levels.

Becchetti et al. (?) consider wage differentials in a sample of workers in the cooperative not for profit sector in Italy and find that more intrinsically motivated workers are also more productive. They argue that, even though a static negative correlation between intrinsic motivation and wages exists due to the fact that intrinsic motivation work as a compensating mechanism, the latter phenomenon is dominated by the dynamic effect by which intrinsic motivation causes or is a signal of higher productivity.

On a completely different line, Dohmen et al. (2007) show that higher cognitive ability is associated with more patience. The positive correlation between cognitive ability and patience could, however, explains a positive correlation between productivity and vocation for the following reasons. On the one hand high ability workers could better internalize the long term benefits of working in a vocation-based market. On the other side, high ability workers could be more prone to cooperation and this can amplify the benefit from vocation.
4 Market equilibrium

Consider first the case where workers’ productivity levels and vocations are observable by firms and no institutional constraints exist which oblige firms to offer a uniform wage to all workers. Firms can offer to workers their productivity level minus the vocational premium: \( w(\theta, \gamma) = \theta - \gamma \). In this case the set of workers accepting employment in the vocation-based market is \( \{ (\theta, \gamma) : r(\theta) \leq \theta \} \).

Note that, in this allocation firms obtain positive profits since they offer the workers a salary which is lower than workers’ productivity. However, it is easy to check that the previous allocation cannot be an equilibrium. In fact, like in Bertrand competition, each firm will try to attract workers by offering them a salary \( w(\theta, \gamma) = \theta - \gamma + \varepsilon \), where \( \varepsilon > 0 \) is a small wage supplement. At the end, firms will pay workers a wage equivalent to their marginal productivity and, the full information equilibrium wage will be \( w_{FI}(\theta, \gamma) = \theta \). As a consequence, hired workers finally obtain all the surplus. Thus, in the full information equilibrium, the set of workers accepting employment in the vocation-based sector is \( \Theta_{FI} = \{ (\theta, \gamma) : r(\theta) \leq \theta + \gamma \} \).

As a consequence, hired workers finally obtain all the surplus. Thus, in the full information equilibrium, the set of workers accepting employment in the vocation-based sector is \( \Theta_{FI} = \{ (\theta, \gamma) : r(\theta) \leq \theta + \gamma \} \). Note that, since firms’ profits are zero in equilibrium, in this model social welfare corresponds to workers’ surplus. In turn, workers’ surplus is maximized if all workers with characteristics \( (\theta, \gamma) \) such that \( r(\theta) \leq \theta + \gamma \) are hired in the market (this implies that the full information equilibrium is efficient). Moreover, we assumed that \( r(\theta) \leq \theta \forall \theta \in [\theta_l, \theta_r] \), this implies that \( r(\theta) \leq \theta + \gamma \forall \theta \in [\theta_l, \theta_r] \) necessarily holds. In words, social welfare in the vocation-based market is maximized if all potential workers are hired in the market. Or, efficiency in the standard labor market also implies efficiency in the vocation-based one.

As we already know, when firms offer a uniform wage rate \( w \) to all workers, the set of types who are willing to accept the job is \( \Theta(w) = \{ (\theta, \gamma) : r(\theta) \leq w + \gamma \} \).

We adopt Mas-Colell et al.’s competitive equilibrium concept which is based on rational expectations on the part of the firms. That is, in the equilibrium firms correctly anticipate the average productivity of those workers who accept employment.

**Definition 3** **Competitive equilibrium.** In the competitive vocation-based labor market with unobservable workers’ productivity levels and vocations, a competitive equilibrium is a wage rate \( w^* \) and a set of active workers \( \Theta(w^*) \) such that:

\[
    w^* = E_{VM} [\theta|w^*]
\]

Note that the equilibrium wage is equal to the average productivity of those workers who accept the job. This again is a consequence of the fact that all workers produce the same outcome and vocation has no direct effect on production.

Importantly, given Definition 1 and Lemma 1:

**Corollary 4** If the two equilibria exist, the competitive equilibrium wage in the vocation-based market is weakly higher than the competitive equilibrium wage.
in the market where vocation does not matter. In particular, the flatter the outside-option function $r(\theta)$, the higher the difference between the two equilibrium wages.\footnote{A necessary condition for a competitive equilibrium to exist in the standard labor market where vocation does not matter is that $r(\theta) \leq \theta \quad \forall \theta$. In fact, under the previous inequality, it is possible that a value of $w$ exists such that $w = E_{SM}[\theta | w]$. Whereas, the previous inequality is not a necessary condition for existence of a competitive equilibrium in the vocation-based market.}

**Proof.** When $w = r(\theta)$ only type $\theta$ enters the standard market where vocation does not matter and $E_{SM}[\theta | w = r(\theta)] = \theta$, whereas when $w = r(\overline{\theta})$ all workers enter the market and $E_{SM}[\theta | w = r(\overline{\theta})] = \mu_{\theta}$. From Remark 2 we know that $E_{SM}[\theta | w]$ is increasing in $w$ for wages between $r(\theta)$ and $r(\overline{\theta})$ (see the dotted increasing line in Figure 6). As we already observed, in the vocation-based market where $\theta \in [\underline{\theta}, \overline{\theta}]$ and $\gamma \in [0, \gamma]$; the first workers type to enter the vocation-based market is $(\theta, \gamma)$ for the wage rate $w = r(\theta) - \gamma$ and $E_{VM}[\theta | w = r(\theta) - \gamma] = \theta$, whereas when $w = r(\overline{\theta})$ all workers enter the market and $E_{VM}[\theta | w = r(\overline{\theta})] = \mu_{\theta}$. From Proposition 1, $E_{VM}[\theta | w]$ can be non monotonic in $w$ for wages between $r(\theta) - \gamma$ and $r(\overline{\theta})$. However, Lemma 1 implies that for every value of $w$ average productivity of active workers is weakly higher in the market where intrinsic motivation matters: $E_{VM}[\theta | w] \geq E_{SM}[\theta | w]$ $\forall w \in [r(\underline{\theta}), r(\overline{\theta})]$. Thus, if we consider a graph where the wage is represented on the horizontal axes and average productivity is on the vertical one (see Figure 6), the function $E_{VM}[\theta | w]$ lies necessarily above the function $E_{SM}[\theta | w]$. According to Definition 3, equilibrium wage is found by locating the wage rate at which the functions $E_{VM}[\theta | w]$ and $E_{SM}[\theta | w]$ cross the 45° line. At this point the conditions $w^{*} = E_{VM}[\theta | w^{*}]$ and $w^{**} = E_{SM}[\theta | w^{**}]$ are respectively satisfied. From the previous reasoning it follows that the function $E_{VM}[\theta | w]$ necessarily crosses the 45-degree line above the point where the function $E_{SM}[\theta | w]$ does. Figure 6 shows both the case where $E_{VM}[\theta | w]$ is monotonically increasing in the wage and the case where the counter-intuitive effect for average productivity occurs. A unique equilibrium is assumed to exists in both markets and in all cases. Remind that, in the case where average productivity of active workers is decreasing in the wage, $\text{cov}(\theta, \gamma) > 0$ and $r'(\theta) < b$.

Moreover, a flat outside option implies that many workers with productivity higher than $\theta = r^{-1}(w)$ accept the job for every value of $w$ and this means a large increase in average productivity of hired workers in the vocation-based market w.r.t. the non-vocational one. Which in turn leads to a larger equilibrium wage.

Insert figure 6 here

This proves again that production inefficiencies due to adverse selection are lower in a market with intrinsically motivated workers. In particular, workers’ surplus is higher in the competitive equilibrium of a vocation-based market.

Note that this result is counter-intuitive. Since motivated workers are ready to accept employment for a lower wage, we could expect a lower equilibrium

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wage in a vocation-based labor market than in a standard one. However, since average productivity is higher in a vocation-based market than in a standard one for every value of the wage and given Definition 3, we find instead the opposite result.

Note that the competitive equilibrium need not be unique. If multiple equilibria exist, they can be Pareto ranked: the equilibrium with the highest wage dominates all the others. To eliminate multiplicity of equilibria and obtain, as unique equilibrium, the equilibrium characterized by the highest wage, we can follow Mas-Colell et al. and model the market interaction using a two stages game between firms and workers. In this game firms are more sophisticated than in the competitive equilibrium where a price-taking behavior is implicitly assumed: here firms can change the offered wage but choose not to do it in equilibrium. In the first stage firms simultaneously announce their wage offer, in the second one workers decide whether to accept the offer. A unique subgame perfect equilibrium is obtained.

In reality vocation-based sectors seem not to be characterized by higher wages with respect to non-vocational ones, quite the opposite. See, for example, Weisbrod (1983) and Preston (1989) looking at the profit / no profit wage differential. However some disagreement persists since Leete (2001) demonstrates that, when more attention is put at industry and occupation levels, the non profit negative wage gap persists only in few cases. Indeed the profit / no profit gap could be a consequence of the fact that wages in vocation-based sectors are not necessarily the result of a competitive process.

When, in vocation-based sectors, wages are uniform for institutional reasons so that the level of the uniform wage is the result of a political instead of a competitive process, then our analysis of the characteristics of labor supply developed in section 3 is particularly important. For example understanding how, in vocation-based markets, labor supply changes as the wage increases is relevant when wages are institutionally defined and workers shortages exist, as in the case of the market for nurses.

5 Conclusions

We analyzed inefficiencies caused by adverse selection arising in vocation-based labor markets, that are markets where workers’ intrinsic motivation for the task to be performed can be relevant. We showed that intrinsic motivation makes average productivity of active workers increase with respect to the case of a standard labor market where vocation does not matter, so that workers’ motivation allows production inefficiency to decrease. This implies that competitive equilibrium in a vocation-based labor market is characterized by a higher wage than in a standard labor market where vocation does not matter. Considering the characteristics of labor supply, we provided necessary and sufficient conditions for average productivity of active workers to decrease and average vocation to increase in the wage at least for a sub interval of possible wage levels (we called them the counter-intuitive effects). We also showed that the two phenomena
cannot simultaneously occur. However average productivity and average vocation of active workers can simultaneously decrease with the wage such that a deep worsening of the composition of labor supply as the wage increases in vocation-based sectors is possible.

Hour findings have relevant policy implications. An important shortage in the labor market for nurses is documented in almost all developed countries (Antonazzo et al., 2003; Shields, 2004; Simoens et al., 2005). Other vocation-based markets are experimenting similar problem. For example, teacher shortage and concerns over the quality of the teaching force is documented by Corcoran et al. (2004) for the US market. As nurses and teachers represent crucial inputs in the production of essential collective goods as health care and education respectively, it is not surprising that many countries have started working at identifying possible policies to solve the shortage. At a micro level policy options include for instance, economic disincentives for early retirement, or also improvements in the pecuniary and non-pecuniary components of compensation. Clearly a wage increase has been indicated as the more obvious policy measure.

Our results show that an increase of the uniform wage must be carefully considered as a policy measure to reduce market shortage and should be preceded by an empirical analysis of the characteristics of the population of potential workers. In fact, if productivity and vocation are negatively correlated in the population then, as wage increases, we can expect a monotonic improvement in average productivity and a monotonic worsening in average vocation of active workers. However, if productivity and vocation are positively correlated two different effects can emerge. The most desirable one is when both average productivity and average vocation of active workers increase in the wage. The less desirable one is when both average productivity and average vocation of active workers decrease in the wage. The last result occurs when the correlation between productivity and vocation is sufficiently high and/or when return to skills out of the vocation-based market are sufficiently low. Since, as we discussed in sub-section 3.1, some studies suggest that a positive correlation between productivity and vocation exists, the latter undesirable effects of a wage increase should be taken seriously.

A deep empirical analysis aimed at understanding how productivity and vocation are distributed in the population of potential workers for different vocation-based markets appears an important future step to understand the characteristics of labor supply in those markers.

6 Appendix

6.1 Proof of Lemma 1.

Given \( w_0 \), the marginal workers’ productivity in the market where vocation does not matter is \( \theta = r^{-1}(w_0) \). For the same wage, the productivity level of marginal workers in the model with vocation is \( \tilde{\theta} = r^{-1}(w_0 + \tilde{\gamma}) \). We can compare productivity of the marginal workers in the two markets. Since the function \( r(\cdot) \)
is strictly increasing and $\tilde{\gamma} \geq 0$, it is $r^{-1}(w_0 + \tilde{\gamma}) \geq r^{-1}(w_0)$. Thus $\tilde{\theta} = \hat{\theta}$ for $\tilde{\gamma} = 0$ and, $\tilde{\theta} > \hat{\theta}$ for $\tilde{\gamma} > 0$. In different words: for every strictly positive $\tilde{\gamma}$ and $r(\tilde{\theta}) < w_0 < r(\hat{\theta})$, in the model where vocation matters marginal workers have higher productivity than in the standard model.

We now compare average productivity of active workers in the vocation-based market and in the market where vocation does not matter. Let us consider the case where $\tilde{\theta} > r^{-1}(w_0 + \tilde{\gamma})$ or $\tilde{\theta} > \theta_{\text{max}}$ as in Figure 1. The same reasoning can be applied when $\overline{\theta} < r^{-1}(w_0 + \tilde{\gamma})$ or $\overline{\theta} = \theta_{\text{max}}$.

The probability that workers enter the standard market where vocation does not matter at a given salary $w_0$, is:

$$A = \int_0^{r^{-1}(w_0)} \int_0^{\theta} f(\theta, \gamma) d\theta d\gamma,$$

whereas the probability that workers enter the vocation-based market conditioning on $w_0$ is $A + B$, where:

$$B = \int_0^{r^{-1}(w_0 + \gamma)} \int_0^{\theta} f(\theta, \gamma) d\theta d\gamma.$$

In particular, the expected value of $\theta$ given the salary $w_0$ in the standard market where vocation does not matter, is:

$$E_{SM}[\theta | w_0] = \frac{\int_0^{r^{-1}(w_0)} \theta f(\theta, \gamma) d\gamma}{A} = \frac{A'}{A}$$

and the expected value of $\theta$ in the vocation-based market given $w_0$ is

$$E_{VM}[\theta | w_0] = \frac{\int_0^{r^{-1}(w_0)} \theta f(\theta, \gamma) d\gamma + \int_0^{r^{-1}(w_0 + \gamma)} \theta f(\theta, \gamma) d\gamma}{A + B} = \frac{A' + B'}{A + B}.$$

We now prove that $E_{VM}[\theta | w_0] \geq E_{SM}[\theta | w_0] \forall w_0$, or:

$$\frac{A' + B'}{A + B} \geq \frac{A'}{A}.$$

The previous condition can be rewritten as follows:

$$\frac{B'}{B} \geq \frac{A'}{A}. \quad (7)$$

The ratio $\frac{B'}{B}$ is the expected value of $\theta$ in the interval $(r^{-1}(w_0), r^{-1}(w_0 + \tilde{\gamma}))$, whereas $\frac{A'}{A}$ is the expected value of $\theta$ in $(\overline{\theta}, r^{-1}(w_0))$. The two expected values lie respectively in the two intervals that are not overlapping and then $\frac{B'}{B} \in (r^{-1}(w_0), r^{-1}(w_0 + \tilde{\gamma}))$ and $\frac{A'}{A} \in (\overline{\theta}, r^{-1}(w_0))$. Inequality (7) is thus always valid, for any given $w_0$, provided that the probabilities $A$ and $B$ are different from zero.
6.2 Proof of Remark 2.

(i) Marginal workers in the non-vocational sector are $\hat{\theta} : r(\hat{\theta}) - w = 0$. By totally differentiating the previous equation with respect to $\hat{\theta}$ and $w$: $r'(\hat{\theta})d\theta - dw = 0$. Since the function $r(\cdot)$ is increasing, the first claim is obtained. Obviously, the higher the slope of the outside option function $r(\hat{\theta})$, the lower the impact of a wage increase on the productivity level of marginal types. (ii) From the proof of Lemma 1, the average productivity of active workers in the non-vocational sector when wage is $w_0$ can be written as follows:

$$E_{SM}[\theta|w_0] = \int_0^1 \left[ \int_{\theta_2}^{r^{-1}(w_0)} \theta h(\theta, \gamma) \, d\theta \right] \, d\gamma$$

Since $h(\theta) = \int_0^\infty f(\theta, \gamma) \, d\gamma$, we can write:

$$E_{SM}[\theta|w_0] = \frac{\int_0^{r^{-1}(w_0)} \theta h(\theta) \, d\theta}{\int_0^{r^{-1}(w_0)} h(\theta) \, d\theta}$$

We now calculate the derivative of $E_{SM}[\theta|w_0]$ with respect to the wage rate and we show that it is always increasing:

$$\frac{\partial}{\partial w_0} E_{SM}[\theta|w_0] = \frac{h(r^{-1}(w_0)) \frac{\partial r^{-1}(w_0)}{\partial w_0} \left[ \theta h(\theta, \gamma) - h(\theta) \right] \, d\theta}{\left[ \int_0^{r^{-1}(w_0)} h(\theta) \, d\theta \right]^2}$$

The sign of $\frac{\partial}{\partial w_0} E_{SM}[\theta|w_0]$ has the same sign of the numerator $(N)$ of the previous expression. $N$ can be rewritten as:

$$N = h(r^{-1}(w_0)) \frac{\partial r^{-1}(w_0)}{\partial w_0} \left[ \int_{\theta_2}^{r^{-1}(w_0)} [r^{-1}(w_0)h(\theta) - \theta h(\theta)] \, d\theta \right]$$

Since the function $r(\cdot)$ is increasing and $\theta \in [\hat{\theta}, r^{-1}(w_0)]$, $N$ is always non-negative.

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Figure 1: the set of potential workers and the curve defining marginal workers given salary $w_0$.

Figure 2: proof of Proposition 1 when $r'(\theta) < b$ for all the dominium of $\theta$. 
Figure 3: proof of Proposition 1 when \( r(\theta) \) is a concave function and \( \theta_0 \) belonging to the dominium exists such that \( r'(\theta_0) = b \).

Figure 4. Simulations: counter-intuitive effect as for average productivity when \( r(\theta) = 0 \) and slope of the regression line larger than 1.
Figure 5. Simulations: counter-intuitive effect as for average vocation. when \( r(\theta) = \theta \) and slope of the regression line lower than 1.

Figure 6: market equilibrium in the vocation-based and in the non-vocational (dotted curve) labor market. In the vocation-based labor market the counter-intuitive effect for average productivity of active workers can occur (non-monotonous curve).