Price Competition under Universal Service Obligations *

Axel Gautier† and Xavier Wauthy‡

March 8, 2010

Abstract

In industries like telecom, postal services or energy provision, universal service obligations (uniform price and universal coverage) are often imposed on one market participant. Universal service obligations are likely to alter firms’ strategic behavior in such competitive markets. In this paper we show that, depending on the entrant’s market coverage and the degree of product differentiation, the Nash equilibrium in prices involves either pure or mixed strategies. We show that the pure strategy market sharing equilibrium, as identified by Valletti et al. (2002) defines a lower bound on the level of equilibrium prices.

1 Introduction

Universal service obligations go along with the process of deregulation at work in most of the former public monopoly industries. Several important questions have been addressed in the literature. How should we define universal service obligations? What are the costs of universal service obligations (Panzar, 2000) and how should they be financed (Choné et al., 2002, Mirabel et al., 2009)? Which firms should be subject to universal service

---

*We are grateful to Nicolas Boccard, Jean-Christophe Poudou, Tommaso Valletti and an anonymous referee for their useful comments.

†CREPP, HEC-Université de Liège, Bat B31, Boulevard du Rectorat 7, 4000 Liège, Belgium, and CORE, Université Catholique de Louvain, Belgium. E-mail: agautier@ulg.ac.be

‡CEREC, FuSL, boulevard du jardin botanique, 43, 1000 Bruxelles, Belgium and CORE, Université Catholique de Louvain, Belgium. E-mail: xwauthy@fusl.ac.be
obligations (Hoernig, 2006)? In the present paper, we focus on the implications of universal service obligations for price competition in a deregulated industry.

Valletti et al. (2002) underline the fact that whenever universal service obligations involve a constraint of uniform pricing, this constraint creates a strategic link between otherwise segmented markets and induces a less aggressive pricing pattern by the incumbent. Since prices are strategic complements, equilibrium prices tend to increase overall and this in turn is likely to affect the extent of market coverage by incoming firms. Anton et al. (2002) establish a comparable result under quantity competition.

The argument is best summarized as follows. Think of a reference industry consisting of a collection of segmented submarkets (typically, the industry for postal services, with submarkets corresponding to delivery routes in different geographical areas). Suppose then that the historical operator is challenged by an entrant on a limited number of submarkets. Assume further that universal service obligations constraint the incumbent’s behavior: it must offer its services in all submarkets at a uniform price. At the price competition stage, the incumbent’s behavior is affected by the extent of the entrant’s market coverage. If the entrant is a low scale competitor, the incumbent firm is better off setting a price close to the monopoly price. In this case it enjoys near monopoly profits on the (relatively numerous) protected markets but possibly sells very little - or nothing- on the contested ones. If the entrant covers a larger set of submarkets, the incumbent is better off being more aggressive over the whole set of submarkets. In which case the profits lost on the protected markets are compensated by larger sales on the (relatively numerous) contested ones. Hence, by choosing the number of submarkets it challenges, the entrant controls the aggressiveness of the incumbent. Prices therefore decrease with the entrant’s coverage (Valletti et al. (2002), lemma 1). For that reason, the entrant will strategically limit its entry scale.

The present paper builds on this line of argument but goes a step further. It is conceivable indeed that, under universal service obligations, the incumbent decides to withdraw on the protected markets where it can charge the monopoly price and collect the corresponding monopoly profits. This strategy turns to be particularly attractive when competition is fierce on the contested markets (for example because products are close substitutes) and insulated markets are relatively numerous. However, this strategy proves to be quite difficult to sustain in equilibrium: if the incumbent ‘retreats’ on the insulated markets, the entrant is likely to price almost as a monopolist on the contested ones, which in turn is likely to trigger an aggressive response
by the incumbent. The presence of universal service obligations therefore tends to destabilize price competition. Formally speaking, taking this strategy into account may destroy the pure strategy Nash equilibrium (Valletti et al., 2002, Hoernig, 2002).

In this paper, we address precisely this point. In a Hotelling setup, we characterize the price equilibrium under universal service obligations and identify the range of parameters for which a pure strategy or a mixed strategy equilibrium respectively exists. For a relatively low market coverage by the entrant, the equilibrium is a quasi-monopoly (pure strategy) equilibrium where the incumbent retreats on the insulated markets and the entrant monopolizes the contested ones with a limit price. For a relatively high market coverage, at equilibrium, the incumbent challenges the entrant on the whole set of contested markets with an aggressive price. Finally, for intermediate values of the entrant’s coverage, the equilibrium is a mixed strategy one. Then, we characterize the optimal degree of market coverage by the entrant. In the Hotelling set-up, the entrant covers fewer markets when universal services obligations are imposed on the incumbent. Moreover, optimal coverage is such that the relevant price equilibrium is either the mixed strategy or the quasi-monopoly one. Thus universal service obligations unambiguously increase the price on the contested markets while they do not necessarily lead to a price below the monopoly price in the protected markets. Finally, in a more general set-up, we derive general conditions on market coverage and the degree of product differentiation for the existence of the pure strategy market-sharing equilibrium considered by Valletti et al. (2002).

2 Price competition without universal service obligations

There is a continuum of identical local markets indexed by \( j, j \in [0, N] \). Universal service obligations consist of two constraints: a universal coverage constraint and a uniform price constraint. If universal service obligations are imposed on the incumbent, firm I, this firm must serve all the markets at a uniform price \( p_i \). The entrant, firm \( E \), is not constrained by universal service obligations and serves a subset of the \( N \) markets at price \( p_e \). Let \( n_e \) denote the index of the last market firm \( E \) has decided to compete in. The complete set of markets can thus be decomposed into two subsets: the set \([0, n_e]\) of contested markets and the complement \([n_e, N]\) of insulated ones. Serving a market \( j \in [0, N] \) involves a fixed cost \( g(j) \geq 0 \). Markets are ordered in such a way that \( g'(j) \geq 0 \). All operating costs are normalized to
Firm $I$ and $E$ sell differentiated products and we rely on the linear version of the Hotelling model to formalize differentiation.\footnote{In section 4 we extend the analysis to a more general model of product differentiation.} In each market $j \in [0, N]$, consumers’ type $x$ are uniformly distributed in the $[0, 1]$ interval according to their idiosyncratic taste. The indirect utility of a consumer with type $x$, buying a product $k$ is given by

$$U(x) = S - td(x, k) - p_j,$$

where $d(x, k)$ is a measure of the distance between the product’s characteristic and type $x$’s ideal product. If the consumer does not buy any product, his utility is defined as $U(x) = 0$. Incidentally, this amounts to assume either that consumers cannot turn to alternative markets where a comparable service would be offered, or that if they can, they are charged a price which leaves them no surplus. The incumbent offers a product with characteristic $x = 0$ and the entrant a product with type $x = 1$.

In each market, the monopoly payoff is given by $\pi^M = p_i \frac{S - p}{2}$ and this expression is maximized for $p = \frac{S}{2}$. We shall assume that $\frac{S}{t} > 2$. As a result, the monopoly price on each market is a corner solution: $p^M = S - t$. The monopoly price leaves the consumer located at a distance 1 from the monopolist indifferent between buying and not buying. This assumption is perfectly in line with the literature on Hotelling competition, which most often assumes full coverage by assuming that $S$ is \textit{large enough}.\footnote{Notice that this assumption of monopoly market coverage amounts to assume that no market expansion effect is expected as a result of competition. All of the market shares gained by the entrant are taken from the incumbent (the displacement ratio is equal to 1). As shown hereafter, this particular feature of the model reinforces that strategic value attached by the entrant to a voluntary limitation of the market coverage.}

On the contested markets, standard Hotelling competition takes place, taking $n_e > 0$ as given. Given consumers’ preferences, we may identify the indifferent consumer, denoted by $\tilde{x}$ who separates the firms’ market shares. By definition, he solves $S - t\tilde{x} - p_i = S - t(1 - \tilde{x}) - p_e$. Formally, we obtain $\tilde{x} = \frac{t - p_i + p_e}{2t}$. Demands addressed to Firms $I$ and $E$ are then defined as follows:

$$x_i^D(p_i, p_e) = \begin{cases} 0 & \text{iff } p_i \geq p_e + t \\ \tilde{x}_i & \text{iff } p_i \in [p_e - t, p_e + t] \\ 1 & \text{iff } p_i \leq p_e - t \end{cases}$$

(1)
\[ x^D_k(p_i, p_e) = \begin{cases} 
0 & \text{iff } p_e \geq p_i + t \\
1 - \bar{x}_i & \text{iff } p_e \in [p_i - t, p_i + t] \\
1 & \text{iff } p_e \leq p_i - t
\end{cases} \] (2)

\[ x^D_k(.) \] denotes the duopoly demand addressed to firm \( k \) in a contested market, \( \pi^j_k(.) \) is the operating profit of firm \( k \) on submarket \( j \) and \( \Pi_k(.) \) is the total operating profit of firm \( k \), from which the fixed cost must be substracted.

When there are no universal service obligations, firms’ optimal behavior can be characterized independently on each of the local market. We only have to distinguish between contested and insulated markets, depending on whether the entrant challenges the incumbent or not. Equilibrium prices on the contested markets solve:

\[ \phi^1_i(p_e) \equiv \arg\max_{p_i} p_i \tilde{x} = \frac{p_e + t}{2}, \] (3)

\[ \phi^1_e(p_i) \equiv \arg\max_{p_e} p_e (1 - \tilde{x}) = \frac{p_i + t}{2}. \] (4)

The Nash equilibrium in prices without universal service obligations is therefore characterized as follows:

**Proposition 1** When the incumbent is not subject to universal service obligations,

1. Equilibrium prices in the contested markets are given by \( p^1_i = t = p^1_e \).

2. The incumbent charges the monopoly price \( p^m_i = S - t \) on the insulated markets.

3. The entrant’s optimal coverage is given by: \( n^*_e = \text{Min}[g^{-1}(\frac{1}{2}), N] \). The incumbent’s optimal coverage is \( n^*_i = \text{Min}[g^{-1}(S - t), N] \).

Notice that whenever \( g(j) = 0 \forall j \in [0, N] \), both firms cover all markets.\(^3\)

---

\(^3\)Imposing universal service obligations is relevant when there are markets that otherwise would not be served, i.e. when there are unprofitable markets: for some \( j \in [0, N] \), \( \hat{p} \) such that \( \pi^M(p) - g(j) \geq 0 \).
3 Price competition with universal service obligations

Whenever the incumbent is not subject to universal service obligations, its profit is additively separable between the \( n_e \) contested markets and the \( N - n_e \) insulated ones. This explains why optimal prices do not depend on market coverage in this case. However, this property does not hold under universal service obligations. The uniform pricing constraint creates a strategic link between the two types of markets because increasing market shares in the contested segment by decreasing the price involves an opportunity cost corresponding to those profits which are lost through this price decrease on the insulated segment. The characterization of a Nash equilibrium in prices is more involved because of this trade-off.

Under universal service obligations, the incumbent faces a positive demand on the contested markets only if \( p_i \leq p_e + t \). Thus, the operating profit of firm \( I \) is defined as follows:

\[
\Pi_i(p_i, p_e) = \begin{cases} 
(N - n_e)p_i & \text{if } p_i \geq p_e + t \\
(N - n_e)p_i + n_e \bar{x} & \text{if } p_i \leq p_e + t
\end{cases}
\]  

Given \( p_e \), the incumbent has two options: it may either set a relatively high price such that it is actually not active on the contested markets and focuses on the protected ones, or it names an aggressive price and shares contested markets with the entrant. In order to characterize the incumbent’s best reply, we need to consider two different strategy profiles, corresponding to the two branches of the monopoly profit (5) and compare the resulting payoffs to formally identify the relevant best reply.

**Lemma 1** The best reply correspondence of the incumbent is given by the following equation:

\[
BR_i(p_e) = \begin{cases} 
p^m_i = S - t & \text{if } p_e \leq \tilde{p}_e \\
\phi_i(p_e) = \frac{p_e + t}{2} + t(\frac{N}{n_e} - 1) & \text{if } p_e \geq \tilde{p}_e
\end{cases}
\]  

with \( \tilde{p}_e = 1 - \frac{2N}{n_e} + \frac{2\sqrt{2(N-n_e)n_e}}{n_e}\sqrt{S-t} \).

**Proof:** Notice that the payoff function in the lower branch of equation (5) is not necessarily concave in own price. There are two candidate price best replies for the incumbent: the monopoly price \( p^m_i \) defined along the first branch of (5). This first candidate ensures the incumbent to benefit at least from the monopoly profit on the \( N - n_e \) insulated markets. Let us
define this strategy as the security strategy, since it defines the lowest payoff the incumbent can guarantee to herself, whatever the entrant’s strategy i.e. its MinMax payoff. The second candidate, which we call the aggressive price, amounts to maximize profits along the second branch of equation (5). Let us denote this candidate by $\phi_i(p_e)$. By definition, it is defined by $\arg\max_{p_i}(N - n_e)p_i + n_e p_i \bar{x}$. Solving the first order condition along the second branch of equation (5) for $p_i$ we obtain:

$$\phi_i(p_e) = p_e + t \frac{N}{n_e} - 1. \quad (7)$$

It then remains to compare the profit obtained by the incumbent, given $p_e$ when playing either of these two best reply candidates. Intuitively, against a high price from the entrant, fighting on the contested markets is not too costly in terms of lost monopoly revenues on the protected ones, whereas the contrary prevails if the entrant’s price is low. Computations indicate that there exists a critical price $\bar{p}_e$ below which the incumbent prefers to retire on the protected markets and set the monopoly price, and above which the incumbent sells on both contested and protected markets at a uniform price. By definition, such a price $\bar{p}_e$ ensures that when the incumbent optimally replies to that price along the second branch of equation (5), i.e. the incumbent replies aggressively, the resulting payoff is identical to the payoff obtained when playing the security strategy, i.e. the monopoly payoff aggregated over the $N - n_e$ insulated markets. Formally, $\bar{p}_e$ solves

$$n_e \pi_i^D(\phi_i, \bar{p}_e) + (N - n_e) \pi_i^M(\phi_i) = (N - n_e) \pi_i^M(p_{m_i}). \quad (8)$$

Direct computations then yield

$$\bar{p}_e = 1 - \frac{2N}{n_e} + \frac{2\sqrt{2} \sqrt{(N - n_e)n_e \sqrt{S - t}}}{n_e}. \quad (9)$$

Three remarks are in order at this stage. First, in subgames where the coverage is close to zero, the price defined by $\phi_i(\cdot)$ tends to increase exponentially. In this case consumers stop buying and the demand is no longer equal to $\bar{x}$. Hence, the corresponding best reply must be computed as the solution to $U(\bar{x}) = 0$, which is relevant against the highest values of $p_e$. Second, whenever the entrant does not cover all markets ($n_e < N$), the incumbent is less aggressive on the contested markets compared to the case without universal service obligations: $\phi_i(\cdot) > \phi_i^1(\cdot)$. Moreover, $\frac{\partial \phi_i}{\partial n_e} > 0$. These results are well-known from Valletti et al. (2002). Finally and more
importantly, the best reply correspondence exhibits a downward jump at $\tilde{p}_e$: $\phi(\tilde{p}_e) < p_i^m$.

The profit realized by the entrant when it challenges $n_e$ markets is given by:

$$\Pi_e(p_i, p_e) = \begin{cases} n_e p_e & \text{if } p_e \leq p_i - t \\ n_e p_e (1 - \tilde{x}) & \text{if } p_e \geq p_i - t \end{cases} \tag{9}$$

Since the incumbent may choose to set the monopoly price as an optimal strategy, we must consider the possibility that the entrant excludes the incumbent from the contested markets by setting a limit price. This amounts to maximize profits along the first branch of the above equation. Since the payoff is strictly increasing along this branch, we may define this limit price as

$$p_e^L(p_i) = p_i - t.$$ 

The entrant’s optimal behavior is summarized in the following Lemma:

**Lemma 2** The best reply function of the entrant is given by the following equation:

$$BR_e(p_i) = Max[\phi^1_e(p_i), p_e^L(p_i)]. \tag{10}$$

**Proof:** There are two candidate best reply for the entrant: the limit price $p_e^L(p_i)$ and the profit-maximizing price defined along the second branch of the profit function. This latter candidate is identical to the price maximizer without universal service obligations and given by (4). Notice that the entrant’s profit function is concave in own price, therefore, there exists a unique best reply which is defined by the maximum of the two candidates, i.e. $p_e^L$, or $\phi^1_e(.)$.

\[\blacksquare\]

### 3.1 Equilibrium analysis

Depending on the entrant’s coverage, there are three possible and mutually exclusive equilibrium configurations: two pure strategy equilibrium and a mixed strategy equilibrium. We consider each of them in turn.

**Quasi-Monopoly Pure Strategy Equilibrium.** The first candidate is defined by $(p_i^m = S - t, p_e^L(p_i^m) = S - 2t)$. The incumbent monopolizes the protected markets and the entrant monopolizes the challenged markets with a limit price. This equilibrium is depicted in Figure 1. The entrant’s best reply exhibits a kink for the pair of prices where the non-negativity constraint on $x_i^D(.)$ becomes binding. Our equilibrium candidate lies above
this kink. A necessary condition for this candidate to be a valid one is that
\( \phi_i(S - 2t) \geq S - t \), i.e. the best reply defined by (7) against \( p_e = S - 2t \)
is above the monopoly price. This condition is satisfied whenever \( n_e \leq n_e^- = \frac{2tN}{S + t} \). A second necessary condition is that \( \phi^1_e(S - t) \leq S - 2t \), i.e the entrant’s best reply against \( S - t \) is not defined by the interior solution. This condition is satisfied whenever \( S \geq 4t \). When these two conditions are satisfied, \( (S - t, S - 2t) \) defines the unique Nash equilibrium.\(^4\)

\[ p_i^* = \frac{t}{3}(\frac{4N}{n_e} - 1), \]  
\[ p_e^* = \frac{t}{3}(\frac{2N}{n_e} + 1). \]  

\(^4\)This equilibrium may exist because the monopoly price \( p_i^m \) is a corner solution.

Figure 1: Quasi-monopoly pure strategy equilibrium

Market Sharing Pure Strategy Equilibrium. The second candidate equilibrium in pure strategies can be identified by solving the system \{\( p_i = \phi_i(p_e), p_e = \phi^1_e(p_i) \}\). We obtain:

\[ p_i^* = \frac{t}{3}(\frac{4N}{n_e} - 1), \]  
\[ p_e^* = \frac{t}{3}(\frac{2N}{n_e} + 1). \]  

Such an equilibrium is depicted in Figure 2. For this equilibrium to exist, it is necessary that the downward jump in firm \( i \)'s best reply, as identified in lemma 1, takes place for a low value of \( p_e \). More precisely, this equilibrium applies whenever \( p_e^* \geq \tilde{p}_e \), as depicted in 2. Formally, this inequality defines a critical number of local markets \( n_e^+ \) above which the competitive pure
strategy equilibrium exists. Computations indicate that \( n_e^+ \) is defined as:

\[
n_e^+ = \frac{(2(6\sqrt{3}\sqrt{N^2(S-t)(-3+3S-t(7+t))+N(9+18S-t(18+t))}))}{(9+72S+(-78+t)t)},
\]

with \( n_e^- < n_e^+ \). An interesting feature of this equilibrium is that prices \( p_k^* \) decrease with the entrant’s coverage and converge to the prices \( p_k^{1*} \) as the entrant reaches full coverage.

![Figure 2: Market sharing pure strategy equilibrium](image)

**Mixed Strategy Equilibrium.** For intermediate values of \( n_e \in [n_e^-, n_e^+] \) no pure strategy equilibrium exists. However, a mixed strategy equilibrium exists because payoffs are continuous.

Referring to Figure 3, we observe that \( p_e^* < \tilde{p}_e \). Thus, there exists no intersection between firm’s best replies, hence no pure strategy equilibrium. There exists however one natural candidate, mixed strategy, equilibrium to consider. Firm \( I \) that has a discontinuous best reply randomizes over two prices which are the two possible values of \( BR_i \) at the point of discontinuity. That is, firm \( I \) chooses \( p_i^m \) with probability \( \alpha \) and \( \phi_i(\tilde{p}_e) \) with probability \( (1-\alpha) \). The probability \( \alpha \) is chosen to ensure that playing the pure strategy \( \tilde{p}_e \) is indeed a best reply for firm \( E \).\(^{5}\) Formally, \( \alpha \) is defined by:

\[
\arg\max_{p_e} \alpha \pi_D(p_i^m, p_e) + (1-\alpha) \pi_D(\phi_i, p_e) = \tilde{p}_e.
\]

\(^{5}\)To the best of our knowledge, the structure of this equilibrium has been analyzed first by Krishna (1989) and developed afterwards in various contexts, see for instance Boccard and Wauthy (2003) for an application in the context of a Hotelling model.
Figure 3: Mixed strategy equilibrium

Summing up, we have established the following proposition:

**Proposition 2** Under Universal Service Obligations, one of the following mutually exclusive equilibrium applies:

- The incumbent sets the monopoly price $p^m_i$, while the entrant sets a limit price $p^L_e(p^m_i)$ such that it is the only active firm on the contested markets. This equilibrium prevails whenever $n_e \in [0, n^-_e]$.

- For intermediate values $n_e \in [n^-_e, n^+_e]$, a mixed strategy equilibrium prevails: the incumbent names $p^m_i$ with probability $\alpha$ and $\phi_i(\tilde{p}_e)$ with probability $(1 - \alpha)$, the entrant plays the pure strategy $\tilde{p}_e$.

- Whenever $n_e \in [n^+_e, N]$, a pure strategy equilibrium exists where the incumbent names $p^*_i$, the entrant names $p^*_e$. Firms share the contested markets.

### 3.2 Optimal coverage by the entrant

Because they alter equilibrium prices in the contested markets, universal service obligations also affect the entrant’s payoffs, hence entry behavior. From equilibrium payoffs in the pricing game, it is straightforward to show that the equilibrium prices and the profit obtained by the entrant in each covered market are weakly decreasing in $n_e$ (and strictly decreasing for $n_e \geq n^-_e$). Therefore the entrant has strategic reasons to limit its market coverage (Valletti et al., 2002). The following proposition establishes that, when coverage
is endogenous and when there are no fixed costs of serving markets, the optimal coverage is such that the resulting price equilibrium is the mixed strategy one. If serving markets is costly, it will decrease further the entrant’s coverage. Hence, in our example where local markets are represented by a Hotelling line, the interior ‘market sharing’ equilibrium is never the relevant one.

**Proposition 3** (i) The optimal coverage by the entrant $n^*_e$ is smaller than $n^+_e$. (ii) If $g(j) = 0 \forall j \in [0, N]$, then $n^*_e \in [n^-_e, n^+_e]$.

**Proof:** Consider first that $g(j) = 0 \forall j \in [0, N]$. We have: (1) For $n_e \in [0, n^-_e]$, the entrant’s profit $n_e(S - 2t)$ is strictly increasing in $n_e$. (2) For $n_e \in [n^-_e, N]$, the entrant’s profit $n_e p^*_e (1 - \tilde{x})$ is strictly decreasing in $n_e$. (3) The entrant’s payoff is continuous in $n_e$. In particular, we have $\lim_{n_e \to n^-_e} \alpha = 0$ (since at $n^-_e$, $p^*_e = \tilde{p}_e$) and $\lim_{n_e \to n^-_e} \phi_i(\tilde{p}_e) = p^*_i = p^m_i$. Therefore, the highest payoff will be reached for a $n_e$ in $[n^-_e, n^+_e]$. Finally, when market coverage is costly and $g'(j) > 0$, coverage will be reduced further.

Our findings can be illustrated with the help of a numerical example. Consider for instance that $N = 1$, $S = 5$, $t = 1$ and $g(j) = 0$, $\forall j$. Numerical computations yield the following outcomes: $n^-_e \simeq 0.33$, $n^+_e \simeq 0.83$ and $n^*_e \simeq 0.45$. In other words, the entrant approximately covers about 45% of the markets. Since, $g(j) = 0$, $\forall j$, the entrant would have covered 100% of the markets without universal service obligations.

In the present setup, the strategic effect of limited coverage is particularly neat. When limiting its market coverage, the entrant relaxes price competition; this positively affects local profits and therefore possibly compensates for the smaller number of covered markets. But in addition, the equilibrium price differential between the incumbent and the entrant also increases. Since the displacement ratio is equal to 1 (i.e. the market is fully covered), this implies that firm $E$’s market share increases in all of the contested markets. This second effect clearly pushes incentives towards a more limited coverage.

### 3.3 Price equilibrium and optimal coverage with and without universal service obligations

In this section we compare the nature of equilibrium depending on whether universal service obligations apply or not. Universal service obligations change the nature of price competition because they create a strategic link between the markets which are contested by the entrant, $[0, n_e]$, and those
which are shielded from competition, \([n_e, N]\). This strategic link has the following consequences on the incumbent’s pricing behavior. First, the incumbent has the option to withdraw on the protected market, leaving the contested ones to the entrant. Second, if it decides to compete on the whole set of markets, it is less aggressive in the price game because any price decrease that would induce higher profits in the contested markets goes along with a profit reduction in the protected ones. As a result, whatever the entrant’s price, the incumbent’s best reply necessarily involves a higher price under universal obligations than without them whenever the set of protected markets is non-empty (compare (6) with (4)). As for the entrant, its best reply is invariant to the presence of universal service obligations except for the fact that the entrant may use a limit price. And if it does so, its price is strictly higher with universal service obligations. Consequently, equilibrium prices on the contested markets must be higher under universal service obligations. This property is obviously satisfied in the mixed strategy equilibrium since in such an equilibrium the lower bound of the incumbent’s price is precisely defined by \(\phi_2(\tilde{p}_e) > \phi_1^1(\tilde{p}_e)\).

Summing up, if we compare the market outcome with and without universal service obligations, we can conclude that, with universal service obligations,

1. Prices are strictly higher on the contested markets.

2. Prices are not necessarily lower on the protected markets.

3. The incumbent has a higher coverage and the entrant may have a higher or a lower coverage.

The incumbent’s higher coverage is a direct consequence of the universal coverage constraint. For the entrant, the impact of universal service obligations on coverage is actually twofold. The entrant realizes a higher profit on each covered market and this stimulates market expansion. At the same time, the entrant has strategic reasons to limit its coverage in order to maintain higher prices. The impact of universal service obligations on the entrant’s coverage depends on the relative importance of these two effects. Obviously though, in the particular case where \(g(j) = 0, \forall j \in [0, N]\), market coverage is strictly lower (proposition 3).

These conclusions echoed those of Valletti et al. (2002) who considered a similar problem but with a focus on the ‘market sharing’ pure strategy equilibrium.\(^6\) As we have just shown, this equilibrium may not be a valid

\(^6\)The only qualitative difference with their paper pertains to the evolution of the price
candidate in the Hotelling context. In the next section we attempt to generalize the approach. In particular we study how the existence of the market sharing pure strategy equilibrium depends on the degree of product differentiation and on the entrant’s coverage.

4 Existence of a pure strategy equilibrium: The general case

We assume that firm $I$ and $E$ sell differentiated products; they compete simultaneously in prices, taking $n_e > 0$ as given. Production costs are normalized to zero. $x^D_k(p_i, p_e)$ and $x^M_k(p_k)$ are assumed to be well behaved. In particular, there exists a unique well-defined monopoly solution and, in case of duopoly, goods are demand substitutes.

Define

$$p^m_i \equiv \arg\max_{p_i} \pi^M_i(p_i), \quad (13)$$

$$\phi^1_k(p_j) \equiv \arg\max_{p_k} \pi^D_k(p_k, p_j). \quad (14)$$

Without universal service obligations, the monopoly price $p^m_i$ prevails on the insulated markets and equilibrium prices $(p^{1*}_i, p^{1*}_e)$ applies on the contested ones.

With universal service obligations, firm $I$’s MinMax payoff is equal to $(N - n_e)\pi^M_i(p_i)$. Let us then denote by $\hat{p}_i(p_e)$ the solution to equation $x^D_i(\hat{p}_i, p_e) = 0$, i.e. $\hat{p}_i(p_e)$ defines the critical price above which the incumbent faces no demand on the contested markets. Then, given $p_e$, the payoff of the incumbent is formally defined by

$$\Pi_i(p_i, p_e) = \begin{cases} (N - n_e)\pi^M_i(p_i) + n_e\pi^D_i(p_i, p_e) & \text{if } p_i \leq \hat{p}_i(p_e) \\ (N - n_e)\pi^M_i(p_i) & \text{if } p_i \geq \hat{p}_i(p_e) \end{cases} \quad (15)$$

There are two local maximizers:

- $\phi_i(\cdot)$ along the first branch of equation (15),
- $p^m_i$ along the second branch of (15).

on the insulated markets that unambiguously decreases in the market sharing equilibrium but not in the other two equilibrium configurations.
The extent to which the first maximizer dominates the second one obviously depends on the extent of market coverage, i.e. on \( n_e \). More fundamentally, the lack of concavity is likely to destroy the existence of a pure strategy equilibrium. Notice that a sufficient condition ensuring that this lack of concavity is not problematic consists in assuming that \( x_i^D(p^m_i, 0) > 0 \). In this case indeed, the non-negativity constraint cannot be binding in the relevant domain of prices since firm \( I \) will never quote a price above the monopoly price while firm \( E \) will not sell at loss.\(^7\)

To characterize firm \( I \)’s best reply, we must compare the payoffs along the two profiles:

\[
n_e \pi^D_i(\phi_i(p_e), p_e) + (N - n_e) \pi^M_i(\phi_i(p_e)) = (N - n_e) \pi^M_i(p^m_i). \tag{16}\]

Which can be rewritten as follows:

\[
\frac{n_e}{N - n_e} \pi^D_i(\phi_i(p_e), p_e) + \pi^M_i(\phi_i(p_e)) = \pi^M_i(p^m_i).
\]

Because of strategic complementarity, the left-hand side of the equation is continuous and strictly increasing in \( p_e \) in the relevant domain whereas the right-hand side is constant. Moreover \( \pi^M_i(\phi_i) \leq \pi^M_i(p^m_i) \). Accordingly, there exists at most one solution to the above equation. Let us denote this solution by \( \tilde{p}_e \). The incumbent’s best reply correspondence therefore writes as follows:

\[
BR_i(p_e) = \begin{cases} 
  p^m_i & \text{if } p_e \leq \tilde{p}_e \\
  \phi_i(p_e) & \text{if } p_e \geq \tilde{p}_e 
\end{cases} \tag{17}
\]

Since \( p^m_i > \phi_i(\tilde{p}_e) \), the best reply correspondence exhibits a downward discontinuity at \( \tilde{p}_e \).

As for the entrant’s behavior, two strategy profiles are a priori possible. It can either compete with the incumbent on all contested markets or it can choose to quote a limit price. The first strategy corresponds to \( p_e = \phi^L_i(p_i) \), the second one to a limit price \( p^L_e \) defined as the solution of \( x_i^D(p_i, p_e) = 0 \). The second strategy applies whenever the non-negativity constraint is binding for firm \( I \) at prices \( \phi^L_i(p_i) \). The entrant’s best reply function is therefore kinked and defined as:

\[
BR_e(p_i) = Max[\phi^L_i(p_i), p^L_e(p_i)]. \tag{18}\]

\(^7\)Valletti et al. (2002) implicitly assume that this condition is satisfied, which indeed can be interpreted on as putting a lower bound on the degree of product differentiation.
There are a priori two pure strategy equilibrium candidates: the market sharing pure strategy equilibrium and the quasi-monopoly pure strategy equilibrium. It is immediate to establish that the second candidate can be ruled out whenever the equilibrium monopoly price is interior, i.e. whenever the monopoly payoff function is differentiable at $p^m_i$. In this case indeed, at $p^m_i$, the derivative of the payoffs on the insulated markets is zero whereas it is strictly negative on the contested ones. As a consequence, firm $i$’s best reply must be $\phi_i(p^L_e)$. We are then left with a unique interior pure strategy equilibrium candidate. Because firm $I$’s best reply is discontinuous, this equilibrium may not be a valid candidate either. The next Lemma summarizes the structure of the price equilibrium under universal service obligations. The first part of the Lemma (non-existence of a pure strategy equilibrium) follows from the discontinuity in $I$’s best reply whereas the second part (existence of a mixed strategy equilibrium) follows from the payoffs’ continuity (Glicksberg, 1952).

**Lemma 3** Whenever $p^*_e < \bar{p}_e$ the Market Sharing Pure Strategy Equilibrium does not exist. When this equilibrium does not exist, there always exists a mixed strategy equilibrium.

It is also easy to prove that average prices in a mixed strategy equilibrium are strictly above the pure strategy ones. Under which conditions does the market sharing pure strategy equilibrium exist? This question is in particular prompted by Valletti et al. (2002) who exclusively focus on that equilibrium. The answer depends on the entrant’s market coverage and the degree of product differentiation. When products are sufficiently differentiated, the pure strategy equilibrium exists (Valletti et al., 2002 and Hoernig, 2006) while, for homogenous products, the unique equilibrium is the mixed strategy equilibrium whenever the incumbent has a strictly positive MinMax payoff (Hoernig, 2002).

Let us measure product differentiation by a parameter $\delta \in [\underline{\delta}, \overline{\delta}]$. The lower bound corresponding to homogeneous products and the higher bound to independent demands. We already established that a pure strategy equilibrium always exists whenever $x^D_i(p^m_i,0) > 0$ which implicitly defines a bound on $\delta$ above which existence is non-problematic. The following proposition characterizes the type of equilibrium prevailing for each possible value of $\delta$ and $n_e$.

---

8This condition is not satisfied in the Hotelling setup developed in the previous section.
Proposition 4  (i) For each $n_e \in (0, N)$, there exists a degree of product differentiation $\tilde{\delta} < \delta$ such that for $\delta \leq \tilde{\delta}$, the Market Sharing Pure Strategy Equilibrium fails to exist.  

(ii) $\tilde{\delta}$ is decreasing in $n_e$.

Proof: The pure strategy equilibrium fails to exist whenever $p_e^* \leq \tilde{p}_e$. Consider any given $n_e \in (0, N)$. When the degree of product differentiation $\delta$ varies, the equilibrium and the cut-off prices vary in opposite directions: $\frac{\partial p_e^*}{\partial \delta} > 0$ and $\frac{\partial \tilde{p}_e}{\partial \delta} < 0$. Moreover, when products are almost homogeneous, we have $p_e^* < \tilde{p}_e$: $\lim_{\delta \to \delta} p_e^* = 0$ and $\lim_{\delta \to \delta} \tilde{p}_e > 0$. Combined with the fact that whenever $x_i^D(p_m^1, 0) > 0$ the corresponding equilibrium is the pure strategy equilibrium, we have proven part (i).

We have thus identified a locus $\tilde{\delta}(n_e)$ characterized by $p_e^*(\tilde{\delta}(n_e), n_e) = \tilde{p}_e(\delta(n_e), n_e)$. For a given $\delta$, when $n_e > \tilde{\delta}^{-1}(n_e)$, the corresponding equilibrium is the pure strategy equilibrium. As a matter of fact, both $p_e^*$ and $\tilde{p}_e$ are decreasing in $n_e$ and $\lim_{n_e \to N} p_e^* = p_e^{*L} > \lim_{n_e \to N} \tilde{p}_e = \tilde{p}_e^L(p_m^1)$. Therefore, for a given $\delta$, the mixed strategy equilibrium applies for the lowest coverage and the pure strategy equilibrium for the highest one. This proves that the locus $\tilde{\delta}(n_e)$ is decreasing in $n_e$.

Proposition 4 associates to all possible degrees of product differentiation and possible coverages the corresponding equilibrium type in the price game. Figure 4 illustrates the proposition. The pure strategy equilibrium does not exist when the incumbent’s MinMax payoff is high (low coverage by the entrant) and when competition is fierce (little product differentiation). And, when the coverage increases (and thus, the incumbent’s MinMax payoff decreases), it is possible to sustain the pure strategy equilibrium for more homogeneous products.

![Figure 4: Equilibrium type in the price game](image)

17
5 Final Remarks

In this paper, we have analyzed the impact of universal service obligations on the intensity of price competition and on the extent of market coverage by the entrant. Previous papers, most notably Valletti et al. (2002), emphasize the strategic link that results from the imposition of universal service obligations on the incumbent firm: universal service obligations weaken price competition because they penalize the incumbent from fighting in the contested markets through the monopoly revenues lost on the protected ones. The entrant in turn may take benefit from this strategic link by controlling for the incumbent’s aggressiveness through its own choice of market coverage. We push this intuition to its end by showing that, under low market coverage, the willingness to retreat in the protected markets actually leads to the non-existence of an equilibrium (in pure strategies).\textsuperscript{9} We show that this problem is almost a generic one: whatever the extent of market coverage, there exist products’ characteristics for which the non-existence problem arises. We also show that in a mixed strategy equilibrium, prices are higher on average. As a consequence, neglecting the existence of these mixed strategy equilibrium amounts to underestimate the anti-competitive consequences of universal service obligations.

Universal service obligations typically constraint the incumbent to offer its products for sale in all segments of the market. Obviously though, it may happen that the price differential is so large that the incumbent faces no demand at all on the contested markets. In any industry where such a configuration makes sense, our analysis is relevant. Such market configurations are expected to prevail with homogeneous products, for vertically differentiated products\textsuperscript{10} and horizontal ones. Moreover, the example we develop, based on a Hotelling framework, relies on a unit demand set-up. This set-up essentially describes a market where consumers rely on a unique provider, at which they possibly buy several units, i.e. a market where benefiting from the service requires a form a affiliation and where there is no real benefits to be obtained from multiple affiliations. Needless to say, this is a reasonable description of markets for postal services, energy provision, telecoms etc.... These models are the most prone to generate the 'higher price' mixed strategy equilibrium we identify as a consequence of universal service obligations.

\textsuperscript{9}A noticeable exception is Hoernig (2002) who addresses the problem in the particular case of homogeneous goods.

\textsuperscript{10}Armstrong (2008) develops such a stylized model where the incumbent may loose all its clients in one region after market opening.
References


