Bargaining and Networks in a Gas Bilateral Oligopoly

Matteo M Galizzi

University of Brescia e-mail: galizzi@eco.unibs.it; Queen Mary University of London e-mail: m.galizzi@qmul.ac.uk

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Abstract  In the context of international gas markets, we investigate the interaction between price formation and communication networks in a bilateral duopoly with heterogeneous buyers. Given the communication structure conveyed by a particular buyers-sellers network graph, prices are formed as the outcome of dynamic decentralized negotiations among traders. We characterize, for any network structure, the full set of sub-game perfect Nash equilibria in pure and stationary strategies (PSSPNE) of the non-cooperative bargaining game with random order of proposals and simultaneous responses. Depending on the inter-temporal discount factor and the dispersion of reservation values across buyers, negotiations may lead, even in a completely connected buyers-sellers network, to multiple equilibria, co-existence of different prices, delays in trade and inefficient allocations. The endogenous bargaining power of each trader as a function of her position in the communication network is derived by comparing traders’ payoffs across networks. Model’s predictions are then discussed in view of the empirical evidence on negotiations within infrastructural networks - international gas markets above all - and of the potential regulation policy implications.

Key words  Non-cooperative bargaining; buyer-seller networks; thin markets; international gas markets.

1 Motivation

On January 2009, during the coldest days of the year, Europe stared at what resembled the sequel of a scene already on stage three years before. Alike on 1st January 2006, 2009 New Year’s celebrations in western Europe
were shadowed by the news that gas extracting company Gazprom, backed by the Russian government (whose leaders often coincide with Gazprom’s top executives), began cutting off gas in pipelines to Ukraine. European Union countries were concerned about such an exacerbation of Ukraine-Russia conflict specially because about 80 percent of Russian gas exports to Western Europe were actually made through Ukraine.\footnote{In 2006, Russia sold only about 8 percent of Ukraine’s annual gas requirement, supplied at a subsidized price of about 50 US dollars, compared to an average international rate of around 230 US dollars, per 1000 cubic meters. From 80 billion cubic meters of natural gas consumed every year by Ukraine, 20 billion came from its own production, about 36 billion were bought from Turkmenistan, and as many as 17 billion were received from Russia as a transit fee for the gas Gazprom passed through Ukrainian pipeline network to Western Europe. In fact, from 2001 on, Ukraine received a payment in form of gas corresponding to the 15 percent of the gas passing through its pipes, a figure estimated around 100 billion cubic meters.}

Clearly Gazprom’s decision to reduce pressure in the pipelines did not help Russia and Ukraine to reach a compromise in their on-going negotiations over the revision of both the price of supplied gas and the transit fee. At the contrary, until January 19th, after many European countries saw an immediate drop in the supply of gas, Russia accused Ukraine to siphon off gas and Ukraine accused Russia to undersupply gas and falsely accuse of siphoning.

It was only on January 19th, after the intervention of the European Union, that the on-going negotiations between Ukraine and Russia finally reached an equilibrium: Premiers Putin and Tymoshenko agreed that, during 2009, Ukraine would pay for the Russian gas a price 20 percent cheaper than the one charged to European Union countries, a figure estimated at 360 US dollars for 1000 cubic metres, lying in between the initial offers of 450 and 250 US dollars, by Russia and Ukraine, respectively.

Even though the European Union officially welcomed the reached agreement in the negotiations as a positive factor of stability, many European countries still remain worringly concerned about the substantial problem still unsolved, namely the heavy dependency from Russia for the gas supplies. In fact, several other times in the past Gazprom and Russia have used exclusive access to distribution networks as a threat to enhance their bargaining power during negotiations over the gas price to be charged to European countries.

Actually, looking at the material web of gas pipelines connecting Russia to western European countries, one may be tempted to believe a commonly quoted idea: Gazprom strategically builds gas networks in order not only to enhance its own bargaining position in negotiations, but also to weaken the power of countries - with large domestic gas consumption - which are reluctant to accept its price conditions, or attracted by the possibility to diversify the portfolio of gas suppliers.
For instance, it is often argued that Gazprom signed with E.On Ruhrgas and Basf a partnership in Germany to build a direct gas pipeline - called North Stream - in the Northern Sea, in order to by-pass Ukraine, Belarus and Poland. One can also observe how effectively Gazprom has managed to exploit the inability of European countries to set up a common energy purchasing agency, and even to exacerbate competition between European states, by deriving country-specific branches from the main pipelines and by signing up individual contracts. Moreover, even very recently it has been claimed that the main reason beyond the agreement signed by Gazprom and ENI to build in partnership the South Stream pipeline from Russia to Bulgaria under the Black Sea, is Gazprom’s aim at blocking any other project - such as the EU-funded Nabucco and Transcaspian projects - of gas networks directly connecting Europe to alternative gas-extracting countries, like Turkmenistan, Azerbaijan, Uzbekistan or Iran.

How the negotiations may be affected by the shape of network architectures is thus one of the most central issue still to be fully investigated in modelling gas markets. Clearly this is also of more general interest, as, for instance, in any network infrastructure a company owning (or having the exclusive right to build) an important branch of the network passing in its country and directed to other domestic markets has clearly a better bargaining position than a terminal node of a foreign-owned pipeline. However, the interrelation between negotiations and networks becomes a much more serious issue in the specific case of the gas industry. This is due to two further features of the international gas markets.

Firstly, the one of gas is not a fully competitive market, but, at the contrary, shows all the salient characteristics of a bilateral oligopoly: a thin market where a very limited number of traders on both sides are likely to strategically affect both the formation of price and the choice of their trading partners. In fact, a handful of largest extracting countries sell natural or liquified gas to few major buying companies, mostly behaving as national distributing monopolies. According to the latest statistics on gas world import-export, Russia, Algeria, Canada, Norway, Qatar, Indonesia, Malaysia, Turkmenistan and Iran alone represent almost 80 percent of the world export, while two thirds of gas imports are concentrated in the purchases of national companies from less than a dozen of countries: Japan, South Korea, China, India, United States, France, Germany, Italy and Spain.

2 See The Economist, 26th January 2008.

3 The European Union Nabucco project, in fact, is meant to bring gas from the Central and Caspian Asia to Europe through the Balkans, and would be the only pipeline from the region that does not cross Russian territory. This would be give Europe the only hope of more diversified gas supplies. In order to be effectively working, gas must arrive to Nabucco from either a trans-Caspian pipeline (the Transcaspian project), which is however blocked by Russia, or through Iran, a solution which is strongly opposed by USA. If Gazprom alternative pipeline South Stream were ever built, it would certainly make Nabucco uneconomic and would convince European Union to definitely abandon it.
The bilateral concentration of the international gas market is even higher if the existence of a global network of exclusive or primary partnership relationships is taken into account, which is further shaping gas trades within macro-regional areas: Japan, for instance, buys more than half of its liquefied imported gas from Indonesia and Malaysia; France relies on gas from Algeria for 80 percent of its import; from Russia is imported more than 90 percent of the gas consumed by Finland, Slovakia, Bulgaria, Lithuania and Czech Republic, and between 50 and 75 percent of the gas consumed by Greece, Austria, Hungary and Poland; Italy depend almost exclusively on gas supplies by Russia and Algeria.

Secondly, and consequently, in the international gas markets, prices are not simply reflecting the daily trading in an organized financial institution, but are the outcome of bilateral contracts and of, possibly intricate, decentralized negotiations. Most south-european countries, for instance, depend almost exclusively on gas supplies by just two national extracting companies, the russian Gazprom and the algerian Sonatrach, with which they bargain bilateral contracts specifying trading prices and conditions.

It seems interesting to explore at which extent the negotiations depend on the shape of the distribution network: what are the interrelations between a trader’s bargaining power and its position in the gas network?

The issue can be very intricate\footnote{For a fairly updated survey of the potential insights to gas markets from economic models of bargaining and networks, see Galizzi (2006).}. Here we just focus on the simpler case of a small buyers-sellers network with heterogeneous traders, in which fully decentralized negotiations take place. It can be seen as an exploratory analysis: of course, there remains a lot more to investigate.

2 Discussion of the model hypothesis and related literature

*Bilateral oligopolies* are characterized by a small number of traders on each side. Being both sides of the market rather concentrated and endowed by market power makes both buyers and sellers able to affect the prices at which they trade. Furthermore, due to the absence of serious searching costs, traders in such thin markets do not act anonymously and are usually able to affect to some extent the choice of their trading partners.

Examples of bilateral oligopolies, beyond the case of the international gas market, can be found in some of the basic commodities markets - such as the ones for the coffee, tobacco, hazelnuts\footnote{In the market for hazelnuts, about 60 percent of the orders come from Ferrero, a food italian company famous for producing Nutella, while a centralized agricultural agency of Turkey represents more than half of the supply.} - of some minerals, and, above all, in estimated 90 percent of the intermediate goods markets: just to name

\[\text{For a fairly updated survey of the potential insights to gas markets from economic models of bargaining and networks, see Galizzi (2006).}\]
some, the aerospace, aircrafts\textsuperscript{6} and shipping\textsuperscript{7} industries, the gigantic-size mechanical and electro-mechanical engineering, the infrastructural plants, the defence or pharmaceutical hi-tech\textsuperscript{8}.

As few pioneering studies (Bjornerstedt and Stennek (2004), Hendricks and McAfee (2005)), have recently pointed out, it is very unlikely that the traders on any side of a thin market may behave as price-takers. Rather, it seems reasonable to think at the price formation as the outcome of a complex of negotiations among traders. The mentioned studies have argued that bilateral oligopolies may be reduced to a collection of many bilateral monoplies: the prices, thus, may emerge as the outcome of many simultaneous Nash-bargaining cooperative solutions, or of many simultaneous bilateral negotiations each involving an exogenously matched pair of one seller and one buyer.

In this paper, on the contrary, we explore an alternative approach, by focusing on non-cooperative interdependent bargaining solutions. The aim of this work, in particular, is to investigate the role of communication networks on endogenous price formation in a thin market.

In the literature on non-cooperative bargaining in decentralized markets, in fact, it is traditionally assumed that buyers and sellers are pairwise matched through some random procedure, and that the order in which agents can make or respond to price offers is exogenously given. However, as

\textsuperscript{6} The US-based Boeing and the pan-european consortium Airbus are, in fact, accounting for substantially two thirds of the world supply of aircrafts. Contrary to what it may be believed, the demand side is also extremely concentrated: the market leader US-based ILFC, with about half of the demand share, and a company of the General Electric group, in fact, buy almost 70 percent of the world production of aircrafts and, then, sign long-term leasing contracts with most the airlines world-wide.

\textsuperscript{7} In the segment of the cruises, for instance, the shipping industry is substantially an oligopoly of three main producers facing three big buyers. In fact, about 98 percent of the supply side is represented by the italian Fincantieri, with 44.8 percent of the market, the norwegian Aker, which, with a share of 29.6 percent, also controls the former Alstom ship-building activities in France, and the german Meyer Werft with the 23.2 percent of the market. What is left of the market is then covered by a fringe of small companies. On the other hand, also the demand side of the market is highly concentrated, since most the purchases come from three cruising companies: from the bigger to the smaller market share, the US-based Carnival (also controlling Costa, Cunard, P\&O Cruises and Princess Cruises), the american Royal Caribbean and the swiss-italian MSC Cruises. Interestingly, moreover, the buyers tend to strategically differentiate their portfolio of business partners more than the producers do: for instance, Fincantieri builds almost exclusively for Carnival, Aker sells to Royal Caribbean the ships produced in Finland and to MSC Cruises the ones build in France, while Meyer Werft is the only supplier working for both Carnival and Royal Caribbean.

\textsuperscript{8} In a vary different context, moreover, examples of so-called thin markets may emerge every time the stocks or derivatives markets are systematically characterized by a restricted number of traders.
Chatterjee and Dutta (1998) observe, while these assumptions are acceptable when modelling large anonymous markets, they are less appropriate in thin markets where the search costs are usually low, and, particularly when agents are heterogeneous, traders may have interest in choosing their partner.

Chatterjee and Dutta (1998) provides a first insight into the role of competition for trading partners on the price prevailing in a thin market. They, in fact, investigate three main models of interdependent bargaining among two identical sellers and two heterogeneous buyers. All the models are based on a bargaining procedure with alternating offers between sellers and buyers, and differ just as the communication structure is concerned. In particular, the strategic interaction among traders is cast on three exogenously designed frames where offers are, respectively, public, privately targeted but publicly known or, finally, privately targeted and secret. The equilibria of the negotiation game typically imply multiple prices and delay.

The analysis of Chatterjee and Dutta (1998) raises two interesting, closely related, research questions.

The first concerns the opportunity of modelling negotiations in thin markets with an alternating order of proposers. Clearly, alternating offers is the most natural specification for any exclusive bilateral bargaining. On the other hand, the hypothesis of a random order of proposals has been typically adopted for the analysis of bargaining in large decentralized markets (see for instance Osborne and Rubinstein (1990), Gale (1986), De Fraja and Sakovics (2001)) in the specific sense that, at any instant of time, either side of the market could, equally likely, be entitled to make proposals. In fact, such a stylized mechanism is usually justified in view of the fact that it mimicks the neutral anonymity of markets and enables to draw a direct comparison with the outcome of a Walrasian competitive framework.

Here, in contrast, we argue that the probably most peculiar features shown by thin markets are the negligible, almost inexistent, searching frictions and the sheer role played by the identity of each individual trader. Therefore, it is difficult to reject the conjecture that the traders themselves, rather than the sides of the market, should be endowed by an \textit{ex ante} identical ability to strategically affect price formation. Therefore, in our thin market we imagine that, at any instant of time, any individual trader is equally likely to start a negotiation, by being selected to announce a proposal to the counterparts on the opposite side of the market.

The second question opened up by Chatterjee and Dutta (1998) is the investigation of whether and how the communication structure can affect strategic negotiations among traders. In fact, any possible set of communication restrictions can equivalently be thought as a network of potential links among agents: the existence of a communication link enables a pair of agents to negotiate.

The existence of physical infrastructural networks, altogether with their shape, in fact, play a crucial role in the distribution of bargaining power and in the feasibility of the implementation of trades among companies
both in international gas, oil and electricity thin markets. Furthermore, most the intermediate markets are endowed with an immaterial web of communication, reputation and trust links which is very likely to affect business relationships and negotiations.

Therefore, we aim in particular at drawing a preliminary picture of the *interrelations among bargaining* in thin markets with heterogeneous traders and specific architectures of the *buyers-sellers networks*.

The issue of endogenous formation of trading links has been already tackled by Kranton and Minehart (2001). On the other hand, sound descriptions of the negotiations' outcome *given a fixed network* structure has been provided by the works of Calvó-Armengol (2001, 2002, 2003a, 2003b) and Corominas-Bosch (2004). From this perspective, then, our work may be seen as lying at the crossroads between these two approaches, as concerns the case of decentralized thin markets. With respect to the first work, our paper introduces an explicit analysis of a structured bargaining process with interdependent strategic negotiations. With respect to those in the second group, on the other hand, our work contributes to extend the analysis of the interaction between network architectures and negotiations to the case of markets with *heterogeneous* traders and *fully decentralized* bargaining procedures, beyond the specific case of alternating offers with identical traders.

We study a simple model of endogenous price formation in a thin market where trading is restricted by the shape of the formed bipartite networks. In particular, we consider completely decentralized negotiations with random order of proposers in the simplest case of a *bilateral duopoly with impatient traders and heterogeneous buyers*: trade of a homogeneous asset between a seller and a buyer is possible only when a link is present between them.

The rest of the paper is organised as follows. Section 3 is a description of the model. In Section 4 we fully characterize the equilibria of the negotiations game within any fixed network structure. Section 5 contains a comparison of the bargaining position of each trader across networks, a discussion of our results and some considerations on the issues of network formation and experimental validation.

3 The Model

3.1 The market

In our bilateral duopoly two identical sellers, \( S_1 \) and \( S_2 \), own one identical indivisible asset - such as a gas bundle. Both sellers have the same null reservation value for the asset. We can think of them as the national exporting companies from two major gas extracting countries (for instance, the russian *Gazprom* and the algerian *Sonatrach*) endowed by comparable industrial strength in terms of financial means, extracting volumes, market shares and so on. We will refer to sellers as females.

In the thin market there are two heterogeneous buyers, \( B_1 \) and \( B_2 \), each of whom demands one single asset. The buyers' valuations are \( v_1 = 1 \)
and \( v_2 = \lambda \), respectively, with \( 1 > \lambda > 0 \). Analogously, we can think of them as two gas purchasing and distributing companies which are (almost) monopolists in two asymmetric national final markets (say, ENI in Italy, and Gaz de France in France or E.On in Germany), or as two asymmetric competitors in a domestic market (say, ENI, and Edison or A2A in Italy). In the following, we will refer to buyers as males, and to \( B_1 \) and \( B_2 \) as the strong and the weak buyer, respectively.

We assume that all the valuations are common knowledge. Also, we assume that traders are impatient and discount their future payoffs at a common discount rate \( \delta \in (0, 1) \). Thus, if one unit of the good is exchanged in period \( t \) between the buyer \( i \) and the seller \( j \) at the price \( p \), then the payoff of the buyer will be \( \delta^{t-1}(v_i - p) \) and the payoff of the seller \( \delta^{t-1}p \).

The prices at which the goods are exchanged if trade takes place, are exclusively determined by endogenous bargaining among the players. In particular, we assume that all traders in the thin market negotiate according to a public offers bargaining procedure with random order of proposers. Moreover it is assumed there is no possibility of price discrimination.

The key feature of the model, however, is that trade may only take place between a buyer and a seller who are directly linked to each other. That is when an agent \( i \) on one side of the market has to respond to a price offer from traders belonging to the opposite side, he - or she - may only accept or reject a proposal from \( j \) such that \( g_{ij} = 1 \), where \( g_{ij} \) denotes the existence of a connection among agents \( i \) and \( j \). Analogous restrictions hold for proposal of price offers by agent \( i \), which are intended to be directed exclusively to counterparts \( j \) such that \( g_{ij} = 1 \). It is then helpful to denote with \( L(i) \) the set of traders on the opposite side of the market linked with agent \( i \). A network is said to be connected if there exists a path among any possible pair of traders.

It is immediate to see that in our thin market, only seven non-empty network architectures can emerge (see Figure 2.1). In fact they are the exclusive trade networks, where each agent on any side of the market is linked only with a single partner (a) - which nests all networks where just one buyer-seller pair is exclusively connected; the supply-short-side networks where only one seller is linked to both buyers (b); the demand-short-side networks where one, either strong or weak, buyer is connected to both the sellers (c-d); the two asymmetrically connected structures where either the weak buyer (asymmetric weak network, e) or the strong buyer (asymmetric strong network, f) is linked with both sellers, while the other buyer is connected only with one exclusive partner; and, finally, the complete connected bipartite graph (g).

We now describe more in detail a model for the bargaining process.

### 3.2 The negotiations

In the negotiation stage, at every round \( t \in \{1, 2, \ldots \} \), one trader is randomly selected to propose offers: each trader, independently of history of play, is
selected with equal probability \( \frac{1}{n} \), where \( n = 1, \ldots, 4 \), is the number of traders still active in the market.\(^9\) Traders are considered as active as long as there are still pairs of linked partners bargaining in the thin market.

Any round of the negotiation stage is composed by two phases. First takes place the *price-offer phase*: the agent who has been selected - say buyer \( B_2 \) - announces the price, \( p_{B_2} \in [0, \lambda] \) he is willing to pay for one unit of the asset, from any linked seller \( j = S_1, S_2 \in L(B_2) \), i.e. such that \( g_{B_2j} = 1 \).

Thereafter, the *price-response phase* occurs. Each seller \( j = S_1, S_2 \in L(B_2) \) responds, simultaneously and independently, to any price offer from \( B_2 \). A response is simply either acceptance or rejection of the buyer’s latest announced offer \( p_{B_2} \). In modelling individual strategic choice in the response game, we also assume the tie-breaking hypothesis by which, if any trader is perfectly indifferent about accepting or rejecting an offer, she (he) accepts it.

\(^9\) In real thin markets, in fact, at any period of negotiations, any trader has some chance to formulate the price offers to sell or to buy the asset at.
Therefore, if \textit{just one} trader on the opposite side - namely $S_1$ - is in fact \textit{linked} with $B_2$, the response phase reduces to an individual decision whether to accept or reject $p_{B_2}$ as in a standard bilateral negotiation.

If, at the contrary, \textit{both} traders on the opposite side - say $S_1$ and $S_2$ - are indeed \textit{linked} with the proposer, the response phase is modelled as a $2 \times 2$ simultaneous moves games. In such a $2 \times 2$ simultaneous moves game, sellers can end out in one of the four following situations: either one seller accepts $p_{B_2}$ while the other rejects it, or they both accept $p_{B_2}$, or, finally, both reject it.

First, if \textit{just one} of the linked sellers \textit{accepts} offer from $B_2$, she is matched with the weak buyer to trade at $p_{B_2}$, while the strong buyer and the remaining seller just enter a new round of negotiations \textit{if they are linked together}.

In fact, the peculiar feature of our framework implies that, once a buyer and a seller leave the market after trading, all the links connecting them with any of the other traders are immediately removed by the bipartite graph\footnote{Alike in Corominas-Bosch (2004).}. If, after such a removal, only isolated agents remain in the market, they all get automatically zero payoffs and the game ends.

If, at the contrary, a connected pair remains unmatched at the end of period $t$, then in period $t + 1$ they enter a further round of the negotiation stage, starting with a fresh random selection of the trader entitled to make proposals. Such a procedure is repeated so long as there are connected traders in the market.

It is worthwhile to underline a consequence of our bargaining procedure. Once just a single buyer and a single seller trade and leave the market, if the two remaining traders are linked each other, the subsequent negotiation stage reduces to a standard bilateral bargaining with random order of proposers. Therefore, from the following bilateral trade, the remaining players expect a surplus of approximately $\frac{1}{2}$ each in case the strong buyer is still in the market, or, alternatively, a surplus of $\frac{1}{2}$ whenever the weak buyer is, which correspond to the Binmore-Rubinstein bilateral bargaining payoffs.

If, on the contrary, \textit{both} sellers $j = S_1, S_2 \in L(B_2)$ \textit{accept} $B_2$ ’s offer of $p_{B_2}$, then they access a random tie-breaking selection to sort out who is going to trade with $B_2$: any of them is randomly picked with $\frac{1}{2}$ probability and matched to trade with $B_2$. As above, the strong buyer and the seller who has not been selected in the tie-break just enter a new round of negotiations only \textit{if they are linked together}. If, at the contrary, \textit{they are not linked together}, they are forced to leave the market with zero payoffs.

Finally, if \textit{both} sellers \textit{reject} $p_{B_2}$, all traders access a further round of negotiations with a new selection of the player entitled to make offers.

Which one of the four above situations occurs only depends upon which Nash equilibrium of the $2 \times 2$ simultaneous moves game is reached. In general, traders’ optimal response correspondences are such that in theory any of the four outcomes can be supported as a Nash equilibrium of the response game.
Clearly, a trader’s best response function depends upon her (his) continuation payoff and on her outside option in case of a rejection (or a random tie-break) as depending on her position in the graph.

In fact, each of the four outcomes of the $2 \times 2$ simultaneous moves game can occur within a specific set of conditions on the level of the announced price $p_B$ in terms of the responders’ continuation payoffs and outside options and, ultimately, of the values assumed by the primitive parameters $\delta$ and $\lambda$.

Thus, the different levels of the announced price $p_B$ imply the occurrence of different, possibly multiple $^{11}$ Nash equilibria of the response game. Hence, given any expected equilibrium behaviour in the response game $^{12}$, it is then possible to move back to the price offer phase and to work out the proposer’s optimal choice. In general, if proposer is a buyer (seller), the optimal choice will be the lowest (highest) price offer implying a Nash equilibrium in which at least one of the traders on the opposite side accepts that price in the subsequent response phase, provided it can guarantee at least the proposer’s continuation payoff.

### 3.3 The solution

The negotiation game is solved given a fixed network structure. In particular, the negotiation game is an infinite horizon dynamic game of complete and imperfect information: in fact, players’ payoff functions are common knowledge and, although, at each move in the game, the players know the

$^{11}$ By looking at the specific best response functions in the response games, sometimes it is possible to order such restrictions in a mutually exclusive way so that a given level of the announced price may be corresponding to one and just one Nash equilibrium in the response game. Some other times, however, multiple equilibria for a given price offer may arise in the response game. Typically, it may be the case that $B_2$ proposes an offer $p_B$ which is simultaneously compatible with more than one set of conditions and mutual best responses: thus, for instance, two alternative Nash equilibria coexist, one where both sellers accept $p_B$, the other where both sellers reject it. In such a case we will always provide a full characterization of all the resulting multiple equilibria in the response game.

$^{12}$ Moreover, it is possible that there are levels of the announced price $p_B$ for which no pure-strategies Nash equilibrium can be guaranteed in the response game. In such a case, we need to specify what happens in the negotiations game. Although the finiteness of the game clearly ensures the existence of a mixed strategies equilibrium, our exclusive focus on pure strategies equilibria implies we need to describe what follows any node at which a proposal does not match any set of conditions for a pure-strategies Nash equilibrium. We assume that in such a case traders just enter a further bargaining stage with a new draw of the proposer. Thus making an offer which can not substain pure strategies Nash equilibrium strategies in the response phase is fully equivalent to making unacceptable offers, as simply implies accessing a further round of negotiations. Therefore, in such a case all the traders just get their own continuation payoffs.
full history of the play thus far, the price-response phases in the negotiation stage are simultaneous-moves games. Therefore in the following analysis we solve the negotiations game for its subgame-perfect Nash equilibria using backward induction.

Furthermore, given the overall complexity of the present game, we will only focus on the subgame-perfect Nash equilibria in pure and stationary strategies (PSSPN equilibria).

4 Bargaining in Networks

Here we solve for the bargaining sequential game between the traders in the thin markets, given the existence of a fixed bipartite network structure.

Note that the case of the exclusive trade networks (a) intuitively can be thought as a minor variant of the model of bilateral negotiations with random order of proposers: in fact, as in a Rubinstein bilateral negotiation, in the limit case $\delta \rightarrow 1$, we should expect that the strong buyer and his matched seller always get in expected terms a payoff of $\frac{1}{2}$, while the weak buyer and his matched seller each earn an expected surplus of $\frac{1}{2}$.

Thus, in the following we will start describing the negotiations game in the case one single pair of traders is linked, then gradually moving, through more and more connected bipartite graphs in which traders can still be isolated or asymmetrically connected, up to the complete network where any pair of traders is linked together.

Within any network where the strong buyer is not isolated, there can not be PSSPNE in which the bargaining process keeps on going on forever. Indeed, as the discounted payoffs of all the traders would be zero in such a case, there is certainly a profitable deviation at least by the strong buyer. In fact, whenever he is selected to make an offer, $B_1$ can always propose a price equal to $\delta$, which, being the highest price both sellers may ever gain in the following rounds, will be immediately accepted in the subsequent response phase. In turn, $\delta < 1$ ensures the strong buyer a strictly positive payoff, and then a profitable deviation from the perpetual disagreement situation.

Finally, notice that, by a standard argument by theory of infinite horizon dynamic games of complete information (see for instance Osbourne and Rubinstein (1990), Fudenberg and Tirole (1996)), a stationary dynamic game may be fully characterized by describing any of its strategically equivalent subgames.

In particular, define $S_i$-games the subgames of the original game of negotiations among the four traders in a given network, starting whenever the seller $S_i$ is randomly selected to make offers. Analogously define $B_i$-games the subgames of the original game that start when buyer $B_i$ is randomly selected to make offers. Hence, being for any given network structure, all the $S_i$-games and all the $B_i$-games strategically equivalent by the stationarity hypothesis, the analysis of the PSSPN equilibria in the original overall game perfectly corresponds to the investigation of the PSSPN equilibria in any of the $S_1$-games, $S_2$-games, $B_1$-games and $B_2$-games.
In the following sections, we will provide a full description of the equilibria for each network structure. For two cases we also provide proofs in the Appendix. The remaining proofs follow analogous arguments and are omitted. They are available on request from the author.

4.1 The Exclusive-trade network

The exclusive-trade network (a) corresponds to a market where two separate pairs of traders negotiate in a mutually exclusive partnership: for instance, Gazprom has an exclusive partnership with ENI, while Sonatrach deals with Gaz de France only. In particular, imagine that seller $S_1$ is linked with the strong buyer only, while $S_2$ is exclusively connected with the weak buyer. Although, in our framework each trader, at any round, has an identical $\frac{1}{4}$ probability to make offers, this situation corresponds to a case where two parallel bilateral negotiations are taking place simultaneously and independently, since any trader has just a potential partner to trade with. It is not surprising, then, that the equilibrium outcome of the bargaining process within each pair of traders is equivalent to the one of a pairwise Binmore-Rubinstein negotiation with random order of proposers.

**Proposition 1** For any discount rate $\delta \in (0,1)$ and reservation price $\lambda \in (0,1)$, there exists a unique PSSPN equilibrium of the negotiation game in the exclusive trade network such that

- $B_1$ proposes $S_1$ a price $p^*_B_{S_1} = \delta V(S_1)$, which is accepted by $S_1$,
- $B_2$ proposes $S_2$ a price $p^*_B_{S_2} = \delta V(S_2)$, which is accepted by $S_2$,
- $S_1$ proposes $B_1$ a price $p^*_S_{B_1} = 1 - \delta W(B_1)$, which in the response phase is accepted by $B_1$,
- $S_2$ proposes $B_2$ a price $p^*_S_{B_2} = \lambda - \delta W(B_2)$, which in the response phase is accepted by $B_2$.

Traders’ expected continuation payoffs by entering a new stage of the negotiations game are

$$
\begin{align*}
W(B_1) &= \frac{1+\delta}{4} \\
W(B_2) &= \frac{(1+\delta)\lambda}{4} \\
V(S_1) &= \frac{1+\delta}{4} \\
V(S_2) &= \frac{(1+\delta)\lambda}{4}
\end{align*}
$$

**Proof** In the Appendix.

Notice that in the limit case as $\delta \to 1$, the expected payoffs approach the values $W(B_1) \to V(S_1) \to \frac{1}{2}$, $W(B_2) \to V(S_2) \to \frac{\lambda}{2}$. Also notice that the exclusive trade network naturally nests two other subgraphs. In fact, by straight adaptations of the above arguments, it is possible to characterize the PSSPN equilibria of the negotiations game within the strong and the weak couple network configurations. By the former we simply mean
a market in which only the strong buyer and a seller - say $S_1$ - are connected, while the weak buyer and the remaining seller - $S_2$ - are isolated traders. The latter case is the analogous graph where $S_2$ is connected with the weak buyer while the remaining traders are isolated. The corresponding equilibria are described in the Appendix.

4.2 The Supply-short-side network

In the supply-short-side networks (b) only one seller $S_i$ with $i = 1, 2$ is linked to both buyers while $S_{-i}$ is an isolated trader. This trading structure mimicks all the market configurations where an exclusive seller is naturally endowed by the capability to elicit a significant extraction of surplus from two competing buyers. The leading market position of Gazprom, which provides gas to all the european countries, is perhaps the better example.

However, as discussed above, a peculiar trait of the present bargaining framework is that, as long as some not isolated agents still remain in the market, any trader is entitled with an identical probability to propose offers. Hence, in such a communication network - alike in the (c) and (d) below - agents incur delays in trade with at least $\frac{1}{4}$ probability, namely, at least after any $S_{-i}$ selection.

Another peculiar feature of the present supply-short-side structure is that the buyers are in a symmetric position concerning the number of accessible connections: both buyers can access an identical number of partners. Thus, it is with no lack of generality that henceforth we assume that the strong buyer’s continuation payoff does not differ from the weak buyer’s by more than some upper bound, equal to the original difference in their reservation prices. This is expressed by Condition $k$,$^{13}$

$$\delta [W (B_1) - W (B_1)] \leq 1 - \lambda, \quad (1)$$

by which the discounted value of the difference in the buyers’ expected continuation payoffs can not exceed the relative distance between their primitive reservation prices.

Finally, given the symmetry across sellers, we hereafter characterize the equilibrium offers within a supply-short-side network where $S_2$ is linked with both buyers while $S_1$ is isolated, like in (b), to easily extend the corresponding findings to the reverse positions of the sellers. As reported in the Appendix, we show that three equilibria are possible in the supply-short-side network.

Proposition 2 For any discount rate $\delta \in (0, 1)$ and reservation price $\lambda \leq \lambda < \bar{\lambda}$, there exists a PSSPN equilibrium SS1 of the negotiation game in the supply-short-side network such that

$^{13}$ It is worthwhile to clarify that we are not actually forcing the equilibrium payoffs to satisfy this property. Rather we assume it at the beginning of our analysis and we then check it a posteriori, selecting the equilibria whose computed continuation values actually satisfy it.
- $B_1$ proposes $S_2$ a price $p^*_B = \delta V(S_2)$ which is accepted by $S_2$
- $B_2$ proposes seller $S_2$ a price $p^*_B = \delta V(S_2)$ which is accepted by seller $S_2$
- for any offer proposed by $S_1$, the expected payoffs for the traders are just their continuation payoffs
- $S_2$ proposes both buyers a price $p^*_S = 1 - \delta W(B_1)$ which is accepted only by $B_1$.

Traders’ expected continuation payoffs by entering a new stage of the negotiation game are

$$
\begin{align*}
W(B_1) &= \frac{4(1-\delta)}{55^{3/4} - 205^{3/4} + 16} \\
W(B_2) &= \frac{16^{3/4} - 40^{3/4} + 96^{3/4} - 64}{55^{3/4} - 205^{3/4} + 16} \\
V(S_1) &= 0 \\
V(S_2) &= 3^{3/4} - 5^{3/4} + 20^{3/4} + 16
\end{align*}
$$

In the first equilibrium, the connected seller is able to fully exploit most the surplus from the negotiation, leaving both buyers with payoffs close to zero. Notice that in the limit case $\delta \rightarrow 1$, the payoffs would approach the values $W(B_1) \rightarrow V(S_1) \rightarrow 0$, $V(S_2) \rightarrow 1$ and $W(B_2) \rightarrow \frac{1}{2} (\lambda - 1)$. As $\lambda \leq \lambda < \bar{\lambda}$, the latter expression is always non-negative for any $\delta \in (0, 1)$.

**Proposition 3** For any discount rate $\delta \in (0, 1)$ and reservation price $\lambda \geq \bar{\lambda}$, there exists a PSSPN equilibrium SS2 of the negotiation game in the supply-short-side network such that

- $B_1$ proposes $S_2$ a price $p^*_B = \delta V(S_2)$ which is accepted by $S_2$
- $B_2$ proposes seller $S_2$ a price $p^*_B = \delta V(S_2)$ which is accepted by seller $S_2$
- for any offer proposed by $S_1$, the expected payoffs for the traders are just their continuation payoffs
- $S_2$ proposes both buyers a price $p^*_S = \lambda$ which is accepted by both buyers.

Traders’ expected continuation payoffs by entering a new stage of the negotiation game are

$$
\begin{align*}
W(B_1) &= \frac{-96^{3/4} + 12 - 4\lambda}{2(55^{3/4} - 163^{3/4} + 16)} \\
W(B_2) &= \frac{3^{3/4} - 16^{3/4} + 16}{4\lambda (1-\delta)} \\
V(S_1) &= 0 \\
V(S_2) &= \lambda/35
\end{align*}
$$

In the second equilibrium the connected seller is not able to fully exploit all the potential surplus from the trade. At the contrary, she can just appropriate from negotiations at most the weak buyer’s reservation price, leaving the strong buyer with a positive surplus tending to half the difference between the reservation prices. In fact, in the limit case $\delta \rightarrow 1$, the payoffs tend to $W(B_1) \rightarrow \frac{1}{2} \lambda$, $V(S_2) \rightarrow \lambda$, $W(B_2) \rightarrow V(S_1) \rightarrow 0$.

Finally, for relatively low values of $\lambda$, there exists an alternative equilibrium where the weak buyer decides indeed to make unacceptable offer to avoid paying excessively onerous prices.
Proposition 4 For any discount rate $\delta \in (0, 1)$ and any reservation price $\lambda < \frac{1}{2}$, there exists a PSSPN equilibrium $SS_3$ of the negotiation game in the supply-short-side network such that:

- $B_1$ proposes $S_2$ a price $p^*_B = \delta V(S_2)$ which is accepted by $S_2$
- $B_2$ proposes seller $S_2$ any unacceptable price $p^*_B < \delta V(S_2)$ which is rejected by seller $S_2$
- for any offer proposed by $S_1$, the expected payoffs for the traders are just their continuation payoffs
- $S_2$ proposes both buyers a price $p^*_S = 1 - \delta W(B_1)$ which is accepted only by $B_1$.

Traders’ expected continuation payoffs by entering a new stage of the negotiation game are

\[
\begin{align*}
W(B_1) &= \frac{1}{2(2-\delta)} \\
W(B_2) &= 0 \\
V(S_1) &= 0 \\
V(S_2) &= \frac{1}{2(2-\delta)}
\end{align*}
\]

In the limit case $\delta \to 1$, payoffs tend to $W(B_1) \to V(S_2) \to \frac{1}{2}$, $W(B_2) \to V(S_1) \to 0$. When asymmetry among buyers is particularly sharp, the weak buyer chooses to abstain from active trading so that negotiations mimic in fact bilateral bargaining among the seller and the strong buyer only. Intuitively this equilibrium outcome is due to the fact that the seller’s continuation payoff is so high compared to $\lambda$ that the weak buyer is better off by choosing to not compete with the strong buyer.

4.3 The $B_1$-short-side and $B_2$-short-side networks

In the $B_1$-short-side networks (c) only the strong buyer $B_1$ is linked to both sellers while the weak buyer is an isolated trader. This trading network captures all the market structures where an exclusive large purchaser is naturally endowed by the power to exploit the existing competition between two homogeneous sellers.\(^{14}\)

As reported in the Appendix, there are two equilibria in the negotiations within this network.

Proposition 5 For any discount rate $\delta \in (0, 1)$ and reservation price $\lambda \in (0, 1)$, there exists a PSSPN $A$-equilibrium of the negotiation game in the $B_1$-short-side network such that:

- $B_1$ proposes the sellers a price, $p^*_B = 0$, accepted by both sellers
- for any offer proposed by $B_2$, the expected payoffs for the traders are just their continuation payoffs

\(^{14}\) The case may be probably thought as the secret ambition by any national incumbent gas-distributor, such as ENI when it illustrates its plans to transform Italy into a gas distribution hub for the Mediterranean Sea.
– $S_1$ proposes the strong buyer a price $p^*_S = 1 - \delta W(B_1)$ which is accepted by $B_1$.
– $S_2$ proposes the strong buyer a price $p^*_S = 1 - \delta W(B_1)$ which is accepted by $B_1$.

Traders’ expected continuation payoffs by entering a new stage of the negotiation game are

\[
\begin{align*}
W(B_1) &= \frac{1}{4-\delta} \\
W(B_2) &= 0 \\
V(S_1) &= \frac{1}{4-\delta} \left(1 - \frac{\delta}{4-\delta}\right) \\
V(S_2) &= \frac{1}{4-\delta} \left(1 - \frac{\delta}{4-\delta}\right)
\end{align*}
\]

**Proposition 6** For any discount rate $\delta \in (0, 1)$ and reservation price $\lambda \in (0, 1)$, there also exist two symmetric PSSPN R-equilibria of the negotiation game in the $B_1$-short-side network such that

– $B_1$ proposes the sellers a price $p^*_B = \min\{\delta V(S_1), \delta V(S_2)\}$ accepted by both sellers
– for any offer proposed by $B_2$, the expected payoffs for the traders are just their continuation payoffs
– $S_1$ proposes the strong buyer a price $p^*_S = 1 - \delta W(B_1)$ which is accepted by $B_1$.
– $S_2$ proposes the strong buyer a price $p^*_S = 1 - \delta W(B_1)$ which is accepted by $B_1$.

Traders’ expected continuation payoffs by entering a new stage of the negotiation game are

\[
\begin{align*}
W(B_1) &= \frac{8 - 5 \delta}{78 - 36\delta + 32} \\
W(B_2) &= 0 \\
V(S_1) &= \frac{8(1-\delta)}{78 - 36\delta + 32} \\
V(S_2) &= \frac{8(1-\delta)}{78 - 36\delta + 32}
\end{align*}
\]

Notice that, although implying different traders’ payoffs, both equilibria converge to the same values at the limit case of absence of impatience: as intuition suggests, as $\delta \to 1, W(B_1) \to 1, W(B_2) \to V(S_1) \to V(S_2) \to 0$, so that the strong buyer is able to extract all the potential surplus from the trade.

In the Appendix, analogous equilibria for the $B_2$-short-side networks (d) are also reported.

### 4.4 The Asymmetric Weak Network

We now investigate the equilibrium prices and outcomes from the negotiations within network architectures asymmetrically connected as the buyers are concerned, like (e). In particular, we first consider the network where
the weak buyer is connected to both sellers, while the strong buyer is only linked to seller $S_2$. As a consequence, both the strong buyer and seller $S_1$ are exclusively connected with, respectively, $S_2$ and $B_2$.\footnote{Clearly the negotiation game within such a network is strategically equivalent to the one taking place in the homologous graph obtained re-labelling the nodes by switching $S_1$ with $S_2$.}

Alike for the previous network configurations (and for the complete network characterized below) where the buyers are in a symmetric position as the number of accessible connections is regarded, even in the present asymmetric weak network we may think at Condition $k$,

$$\delta [W(B_1) - W(B_2)] \leq 1 - \lambda,$$

as a convenient restriction.

In fact, if Condition $k$ is a reasonable restriction on buyers’ expected continuation payoffs for any symmetric structure, it must \textit{a fortiori} be a necessary feature of the latter within an asymmetric weak network. In fact, in such a case, the payoff advantage enjoyable by the strong buyer should be even less remarkable as the weak buyer is in fact endowed with a relatively sounder bargaining position concerning the number of accessible partners.\footnote{Therefore, also in the present network configuration we consider buyers’ continuation payoffs as suggested by Condition $k$: $1 - \delta W(B_1) \geq \lambda - \delta W(B_2)$. It is worthwhile to emphasize that we are not actually forcing the equilibrium payoffs to satisfy this property. Rather we assume it at the beginning of our analysis and we then check it \textit{a posteriori}, selecting the equilibria whose computed continuation values actually satisfy it.}

An analogous consideration on the supposedly better trading opportunities entailed by traders in more connected nodes lies behind a corresponding assumption on the sellers’ continuation payoffs, $V(S_2) \geq V(S_1)$.

We now provide a full description of the unique equilibrium. Details and proofs are in the Appendix.

\textbf{Proposition 7} For any discount rate $\delta \in (0, 1)$ and reservation price $\lambda \in (0, 1)$, there exists a unique PSSPN equilibrium $\mathcal{AW}$ of the negotiation game in the asymmetric weak network such that

- $B_1$ proposes $S_2$ a price $p_{B_1}^* = \delta V(S_2)$ which, in the response phase, is accepted by $S_2$.
- $B_2$ proposes both sellers a price $p_{B_2}^* = \delta V(S_1)$ which is accepted only by $S_1$.
- $S_1$ proposes $B_2$ a price $p_{S_1}^* = \lambda - \delta W(B_2)$, which is accepted by $B_2$.
- $S_2$ proposes both buyers a price $p_{S_2}^* = 1 - \delta W(B_1)$ which is accepted only by $B_1$. 

Traders’ expected continuation payoffs by entering a new stage of the negotiation game are

\[
\begin{align*}
W(B_1) &= \frac{1+4}{4} \\
W(B_2) &= \frac{\lambda(1+\delta)}{4(1+\delta)} \\
V(S_1) &= \frac{\lambda(1+\delta)}{4} \\
V(S_2) &= \frac{1+4}{4}.
\end{align*}
\]

The traders’ payoffs of the unique equilibrium in the negotiations game within an asymmetric weak network are interesting. Somehow contrarily to what one could expect, they suggest that the weak buyer is never able to take advantage of his most connected location in order to get better trading opportunities than the strong buyer. In fact, for any impatience rate and weak buyer’s reservation price, the strong buyer is always better off than the weak. Notice that in the limit case as \( \delta \to 1 \), \( W(B_1) \to V(S_2) \to \frac{1}{2} \), \( W(B_2) \to V(S_1) \to \frac{\lambda}{2} \).

4.5 The Asymmetric Strong Network

We now consider the network \((f)\) where the strong buyer is connected to both sellers, while the weak buyer is only linked to seller \( S_2 \). As a consequence, both the weak buyer and seller \( S_1 \) are exclusively connected with, respectively, \( S_2 \) and \( B_1 \).\(^{17}\) This market configuration fits very well the case of most domestic gas markets, where the incumbent is usually endowed by a wider set of energetic sources than the smaller competitors.

It is immediately reckoned that the previous arguments in favour of the above discussed \( \text{Condition } k \) as a neutral and realistic simplifying restriction on buyers’ continuation payoffs can no longer keep their validity when we move to this asymmetric strong network. In fact, in the present case, \( B_1 \) reinforces his original strength due to the higher reservation value with the trading advantages conveyed by his central position in the graph. Thus, we should at the contrary argue that the strong buyer reasonably expects a surplus from the trade which may well be beyond any upper bound as implied by \( \text{Condition } k \). Therefore, all along the following analysis, we will separately consider both the case where \( 1 - \delta W(B_1) \geq \lambda - \delta W(B_2) \) and the one in which \( 1 - \delta W(B_1) \leq \lambda - \delta W(B_2) \).

We now provide a full description of the final equilibria. Details and proofs are in the Appendix.

**Proposition 8** For sufficiently high values of the discount rate \( \delta \in \left[ \frac{\lambda}{\lambda}, 1 \right] \) and medium-low values of the reservation price \( \frac{\lambda}{2} \leq \lambda \leq \frac{\lambda}{2} \), there exists a

\(^{17}\) Again the negotiation game in such a network is strategically equivalent to the one taking place in the homologous graph obtained re-labelling the nodes after having switched \( S_1 \) with \( S_2 \).
Proposition 9 For high values of the discount rate \( \delta \in \left[ \frac{5}{3}, 1 \right) \) and intermediate reservation price \( \frac{3}{5} \leq \lambda \leq \bar{\lambda} \), there exists a PSSPN equilibrium AS2 of the negotiation game in the asymmetric strong network such that

- \( B_1 \) proposes both sellers a price \( p^*_B_1 = \frac{4\lambda}{3} \) which is accepted by both \( S_1 \) and \( S_2 \)
- \( B_2 \) proposes \( S_2 \) a price \( p^*_B_2 = \delta V(S_2) \) which is accepted by \( S_2 \)
- \( S_1 \) proposes \( B_1 \) a price \( p^*_S_1 = 1 - \delta W(B_1) \), which is accepted by \( B_1 \)
- \( S_2 \) proposes both buyers a price \( p^*_S_2 = 1 - \delta W(B_1) \) which is accepted by \( B_1 \) only, while is rejected by \( B_2 \)

Traders’ expected continuation payoffs by entering a new stage of the negotiation game are

\[
\begin{align*}
W(B_1) &= \frac{1}{2(2-3\delta)} + \frac{\delta(1-\lambda)}{4(2-3\delta)} \\
W(B_2) &= \frac{\delta^2 - 6\lambda \delta - 6\lambda^3 - 22\lambda^2 - 85 + 32\lambda}{16(2-3\delta)(4-\delta)} \\
V(S_1) &= -\frac{3\delta^2 - 28 + 28\lambda + 8}{16(2-3\delta)(4-\delta)} \\
V(S_2) &= -\frac{\delta^2 - 3\delta^2 \lambda - 6\lambda + 8\lambda^3 + 8}{4(2-3\delta)(4-\delta)}
\end{align*}
\]

Note that, in the limit case as \( \delta \to 1 \), \( W(B_1) \to \frac{1}{3} + \frac{1-\lambda}{9} \), \( W(B_2) \to \frac{\lambda}{3} - \frac{1}{27} \), \( V(S_1) \to \frac{3}{16} + \frac{\lambda}{3} \) and \( V(S_2) \to \frac{1}{32} + \frac{\lambda}{9} \).

Proposition 10 For sufficiently low values of the discount rate \( \delta \in \left( 0, \frac{5}{3} \right] \) and for medium-high values of the reservation price \( \frac{3}{5} \leq \lambda \leq \bar{\lambda} \), there exists a PSSPN equilibrium AS3 of the negotiation game in the asymmetric strong network such that
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- $B_1$ proposes both sellers a price $p^*_{B_1} = \delta V(S_1)$ which is accepted only by $S_1$, while $S_2$ rejects it.
- $B_2$ proposes $S_2$ a price $p^*_{B_2} = \delta V(S_2)$ which is accepted by $S_2$.
- $S_1$ proposes $B_1$ a price $p^*_{S_1} = 1 - \delta W(B_1)$, which is accepted by $B_1$.
- $S_2$ proposes both buyers a price $p^*_{S_2} = 1 - \frac{\delta}{2}$ which is accepted by both buyers.

Traders’ expected continuation payoffs by entering a new stage of the negotiation game are:

\[
\begin{align*}
W(B_1) &= \frac{7\delta^2 + 8\delta + 16}{32(2 - \delta)} \\
W(B_2) &= \frac{7\delta^2 + 4\delta + 16}{32(2 - \delta)} \\
V(S_1) &= \frac{1 + 6\lambda}{4 - \delta} - \frac{\delta}{2(4 - \delta)} \\
V(S_2) &= \frac{1 + 6\lambda}{4 - \delta} - \frac{\delta}{2(4 - \delta)}
\end{align*}
\]

Note that, in the limit case as $\delta \rightarrow 1$, $W(B_1) \rightarrow \frac{17}{32}$, $W(B_2) \rightarrow \frac{13}{24}$, $V(S_1) \rightarrow \frac{13}{32}$ and $V(S_2) \rightarrow \frac{1}{6} + \frac{\lambda}{3}$.

**Proposition 11** For any $\delta \in (0, 1)$ and for high values of the reservation price $\lambda \geq \hat{\lambda}$, there exists a PSSPN equilibrium $AS_4$ of the negotiation game in the asymmetric strong network such that:

- $B_1$ proposes both sellers a price $p^*_{B_1} = \delta V(S_1)$ which is accepted only by $S_1$, while $S_2$ rejects it.
- $B_2$ proposes $S_2$ a price $p^*_{B_2} = \delta V(S_2)$ which is accepted by $S_2$.
- $S_1$ proposes $B_1$ a price $p^*_{S_1} = 1 - \delta W(B_1)$, which is accepted by $B_1$.
- $S_2$ proposes both buyers a price $p^*_{S_2} = \lambda - \delta W(B_2)$ which is accepted only by $B_2$, while $B_1$ rejects it.

Traders’ expected continuation payoffs by entering a new stage of the negotiation game are:

\[
\begin{align*}
W(B_1) &= \frac{1 + 4 \delta}{4 - \delta} \\
W(B_2) &= \frac{1 + 2 \lambda}{4 - \delta} \\
V(S_1) &= \frac{1 + 4 \delta}{4 - \delta} \\
V(S_2) &= \frac{1 + 2 \lambda}{4 - \delta}
\end{align*}
\]

In the limit case as $\delta \rightarrow 1$, $W(B_1) \rightarrow V(S_1) \rightarrow \frac{1}{4}$ and $W(B_2) \rightarrow V(S_2) \rightarrow \frac{1}{2}$.

**Proposition 12** For medium values of the reservation price $\tilde{\lambda} \leq \lambda \leq \hat{\lambda}$ and for sufficiently high values of the discount rate $\delta \in \left[\tilde{\delta}, 1\right)$, there exists a PSSPN equilibrium $AS_5$ of the negotiation game in the asymmetric strong network such that:

- $B_1$ proposes both sellers a price $p^*_{B_1} = \frac{\delta \lambda}{2}$ which is accepted by both sellers, while they both accept any other offer $p_{B_1} \in \left[\min\{\delta V(S_1), \delta V(S_2)\}, \frac{\delta \lambda}{2}\right)$.
– $B_2$ proposes $S_2$ a price $p^*_2 = \delta V(S_2)$ which is accepted by $S_2$
– $S_1$ proposes $B_1$ a price $p^*_1 = 1 - \delta W(B_1)$, which is accepted by $B_1$
– $S_2$ proposes both buyers a price $p^*_2 = 1 - \delta W(B_1)$ which is accepted by both buyers, while they both reject any other offer $p_{S2} \in [1 - \delta W(B_1), \min \{ \lambda, 1 - \frac{\delta}{2} \}]$.

Traders’ expected continuation payoffs by entering a new stage of the negotiation game are

$$W(B_2) = \frac{W(B_1) = \frac{-3\delta + 2\delta + 8}{3\delta - 20\delta + 32}}{3\delta + 4\delta + 11\delta + 86\delta - 32\delta - 248\delta + 210\delta - 80\delta + 80\delta - 384\delta + 128}{8(\delta^4 - 24\delta^3 + 112\delta^2 - 128)}$$
$$V(S_1) = \frac{-15\delta^2 + 16\delta + 32}{4\delta^2 - 20\delta + 32}$$
$$V(S_2) = \frac{-3\delta^4 + 4\delta^3 - 3\delta^2 + 20\delta^2 - 32\delta + 32}{\delta^4 - 24\delta^3 + 112\delta^2 - 128}$$

Notice that in the limit case as $\delta \to 1$, $W(B_1) \to \frac{7}{10} - \frac{1}{10}, V(S_1) \to \frac{27}{80} + \frac{9}{80} \lambda$ and $V(S_2) \to \frac{1}{10} + \frac{2}{10} \lambda$.

**Proposition 13** Finally, either for any $\delta \in (0, 1)$ and for medium values of the reservation price $\bar{\lambda} \leq \lambda \leq \lambda$; or for medium-high values of the reservation price $\bar{\lambda} \leq \lambda \leq \hat{\lambda}$ and very high values of the discount rate $\delta \in \left[ \frac{\delta}{2}, 1 \right]$, there exists a PSSPN equilibrium $AS6$ of the negotiation game in the asymmetric strong network such that

– $B_1$ proposes both sellers a price $p^*_{B1} = \delta V(S_1)$ which, in the response phase, is accepted by $S_1$ only, while $S_2$ rejects it
– $B_2$ proposes $S_2$ a price $p^*_{B2} = \delta V(S_2)$ which is accepted by $S_2$
– $S_1$ proposes $B_1$ a price $p^*_1 = 1 - \delta W(B_1)$, which is accepted by $B_1$
– $S_2$ proposes both buyers a price $p^*_2 = 1 - \delta W(B_1)$ which is accepted by both buyers, while they both reject any other offer $p_{S2} \in [1 - \delta W(B_1), \min \{ \lambda, 1 - \frac{\delta}{2} \}]$.

Traders’ expected continuation payoffs by entering a new stage of the negotiation game are

$$W(B_2) = \frac{W(B_1) = \frac{-3\delta^2 + 28 + 8}{3\delta^2 - 20\delta + 32}}{3\delta^4 + 4\delta^3 + 11\delta^2 - 86\delta^3 + 32\delta^2 - 248\delta^2 + 210\delta^2 - 80\delta^2 + 80\delta^2 - 384\delta^2 + 128}{8(\delta^4 - 24\delta^3 + 112\delta^2 - 128)}$$
$$V(S_1) = \frac{-15\delta^2 + 16\delta + 32}{4\delta^2 - 20\delta + 32}$$
$$V(S_2) = \frac{-3\delta^4 + 4\delta^3 - 3\delta^2 + 20\delta^2 - 32\delta + 32}{\delta^4 - 24\delta^3 + 112\delta^2 - 128}$$

Notice that, in the limit case as $\delta \to 1$, $W(B_1) \to \frac{7}{10}, W(B_2) \to \frac{13}{22}, V(S_1) \to \frac{23}{22}$ and $V(S_2) \to \frac{1}{10} + \frac{8}{5} \lambda$.

Therefore, the negotiations within an asymmetric strong network show a rich multiplicity of equilibria. In fact, for low levels of $\lambda$ equilibrium $AS1$ is defined. Medium values of $\lambda$ are instead cover, in different ranges, by equilibria $AS5$, $AS2$ and $AS6$. For high levels of $\lambda$ equilibrium $AS3$ exists, while for extremely high values, equilibrium $AS4$ is defined.
Furthermore, it can be reckoned that, in equilibrium offers are often accepted by both traders on a side of the market, so that a random tie-break takes place. This is the case for all but one equilibria, the only exception being AS4. As a consequence, in some equilibria, trade occurs with delay, while in other equilibria, with half probability the least connected traders end up leaving the market without trading at all. In the former case, moreover, different prices usually form in the thin market. Therefore, inefficiency, both in terms of delay in trade and of impossibility to achieve full exploitation of all the potential surplus from trade, can not be ruled out in an asymmetric strong network.

4.6 The Complete Network

In the complete bipartite network \((g)\), each buyer is connected with both the sellers. The existence of links between all the possible buyer-seller pairs enables the exploitation of any potential trade in the thin market.

The case of a bilateral duopoly connected by a complete bipartite graph corresponds to the market structure studied by the public offers model in Chatterjee and Dutta (1998). In fact, our model of negotiations differs from the latter only in two, though crucial, aspects. First, while in Chatterjee and Dutta (1998) the bargaining procedure follows an alternating order of proposers between the supply and the demand side, in our model the negotiation entails a random order of proposers with identical odds for any trader. Second, in our model there is an explicit formalization of the strategic interaction occurring between traders of the same side of the market competing when responding to an offer. In fact, unlike Chatterjee and Dutta (1998), we explicitly model a simultaneous moves \(2 \times 2\) game between buyers (sellers) in the price response phase.

Furthermore, our model of negotiations in such a case may be seen as an extension of the model by Corominas-Bosch (2004) to the case of heterogeneous buyers and random selection of traders (rather than sides of the market).

Our model of negotiations among traders in such a complete bipartite graph implies that after a single buyer and a single seller have been matched to trade, have left the market and all their links have been removed, the two remaining traders have always the chance to stay in the market to carry on further negotiations. In fact they automatically access a standard bilateral bargaining with random order of proposers, whose PSSPN equilibrium payoffs are the one described by a standard Binmore-Rubinstein model.\(^{18}\)

Alike for the above networks where the buyers are in a symmetric position concerning the number of accessible connections, also in the complete

\(^{18}\) If the strong buyer is still in the market, each of the remaining traders expects from the following bargaining rounds a surplus of \(\frac{1}{2}\); alternatively, whenever the weak buyer is the one left, they expect a surplus of \(\frac{1}{2}\).
connected graph we will take advantage of Condition $k$,

$$
\delta [W(B_1) - W(B_1)] \leq 1 - \lambda,
$$

by which the discounted value of the difference in the buyers’ expected continuation payoffs can never exceed the relative distance between their primitive reservation prices.

As reported in the Appendix, only three equilibrium outcomes can emerge.\footnote{For a $I$-type equilibrium arising when $B_2$ has been selected to make offers in which continuation payoffs are such that $V(S_1) \leq V(S_2)$.}

**Proposition 14** For any $\delta \in (0,1)$ and for high reservation price $\lambda \geq \bar{\lambda}$, there exists a PSSPN equilibrium $C_1$ of the negotiation game in the complete network such that

- $B_1$ proposes both sellers a price $p^*_B = \delta V(S_1)$ which is accepted by $S_1$
- $B_2$ proposes both sellers a price $p^*_B = \delta V(S_1)$ which is accepted by $S_1$
- $S_1$ proposes both buyers a price $p^*_S = 1 - \delta W(B_1)$, which is accepted by $B_1$
- $S_2$ proposes both buyers a price $p^*_S = 1 - \delta W(B_1)$ which is accepted by $B_1$.

Traders’ expected continuation payoffs by entering a new stage of the negotiation game are

$$
\begin{align*}
W(B_1) &= \frac{-2\delta^2 - \delta^3 \lambda - 2\delta + 8}{2(35\delta^2 - 166\delta + 16)} \\
W(B_2) &= \frac{\delta^3 + \delta^2 \lambda + \delta^2 \lambda - 8\delta - 8\delta \lambda + 32\lambda}{8(35\delta^2 - 166\delta + 16)} \\
V(S_1) &= \frac{-\delta^2 - 3\delta^2 \lambda - 6\delta + 4\lambda + 8}{2(35\delta^2 - 166\delta + 16)} \\
V(S_2) &= \frac{5\delta^3 + 7\delta^2 \lambda - 8\delta^2 - 32\delta \lambda - 24\delta + 32\delta \lambda + 32}{8(35\delta^2 - 166\delta + 16)}
\end{align*}
$$

Notice that in the limit case as $\delta \to 1$, $W(B_1) \to \frac{2}{11} - \frac{\lambda}{11}$, $W(B_2) \to \frac{1}{11} \lambda - \frac{1}{11}$, $V(S_1) \to \frac{1}{11} + \frac{\lambda}{11}$ and $V(S_2) \to \frac{2}{11} \lambda + \frac{7}{11} \lambda$.

**Proposition 15** For any $\delta \in (0,1)$ and for medium-high levels of the reservation price $\lambda \leq \lambda \leq \bar{\lambda}$, there exists a PSSPN equilibrium $C_2$ of the negotiation game in the complete network such that

- $B_1$ proposes both sellers a price $p^*_B = \frac{\delta \lambda}{2}$ which is accepted by both $S_1$ and $S_2$, who would accept any offer in the range $[\frac{\delta \lambda}{2}, \delta V(S_1)]$
- $B_2$ proposes both sellers a price $p^*_B = \delta V(S_1)$ which is accepted by $S_1$
- $S_1$ proposes both buyers a price $p^*_S = 1 - \delta W(B_1)$, which is accepted by $B_1$
- $S_2$ proposes both buyers a price $p^*_S = 1 - \delta W(B_1)$ which is accepted by $B_1$.\footnote{For a $I$-type equilibrium arising when $B_2$ has been selected to make offers in which continuation payoffs are such that $V(S_1) \leq V(S_2)$.}
Traders’ expected continuation payoffs by entering a new stage of the negotiation game are

\[
\begin{align*}
W(B_1) &= \frac{2+\delta(1-\lambda)}{4(\delta\lambda-2h)h} \\
W(B_2) &= \frac{\delta^2\lambda-\lambda h^2-6\delta\lambda-20h\lambda+3\delta^2+3\delta}{16(\delta^3-7h^2+8)} \\
V(S_1) &= \frac{-\delta^2+\delta\lambda-\lambda h^2-6\delta\lambda-20h\lambda+3\delta^2+3\delta}{4(\delta^3-7h^2+8)} \\
V(S_2) &= \frac{\delta^2(1+\lambda)-12\delta+10h\lambda+8}{16(\delta^3-7h^2+8)}
\end{align*}
\]

Notice that, in the limit case as \( \delta \to 1 \), \( W(B_1) \to \frac{3}{2} - \frac{2}{3} \lambda \), \( W(B_2) \to \frac{20}{45} \lambda - \frac{1}{45} \), \( V(S_1) \to \frac{1}{12} + \frac{3}{32} \lambda \) and \( V(S_2) \to \frac{7}{12} + \frac{9}{32} \lambda \).

**Proposition 16** For high values of the discount rate \( \delta \in [\bar{\delta}, 1) \) and intermediate values of the reservation price \( \bar{\lambda} \leq \lambda \leq \bar{\lambda} \), there exists a PSSPN equilibrium C3 of the negotiation game in the complete network such that

- \( B_1 \) proposes both sellers a price \( p_{B_1}^* = \delta V(S_1) \) which is accepted by both \( S_1 \) and \( S_2 \), who would reject any offer in the range \( [\frac{\lambda}{2}, \delta V(S_1)] \)
- \( B_2 \) proposes both sellers a price \( p_{B_2}^* = \delta V(S_1) \) which is accepted by \( S_1 \)
- \( S_1 \) proposes both buyers a price \( p_{S_1}^* = 1 - \delta W(B_1) \), which is accepted by \( B_1 \)
- \( S_2 \) proposes both buyers a price \( p_{S_2}^* = 1 - \delta W(B_1) \) which is accepted by \( B_1 \).

Traders’ expected continuation payoffs by entering a new stage of the negotiations game are

\[
\begin{align*}
W(B_1) &= \frac{-3\delta^2(1+\lambda)-25+16h}{8(\delta^3-7h^2+8)} \\
W(B_2) &= \frac{\delta^2\lambda-\lambda h^2-6\delta\lambda-20h\lambda+3\delta^2+3\delta}{16(\delta^3-7h^2+8)} \\
V(S_1) &= \frac{-\delta^2+3\delta\lambda-\lambda h^2-6\delta\lambda-20h\lambda+3\delta^2+3\delta}{4(\delta^3-7h^2+8)} \\
V(S_2) &= \frac{\delta^2(1+\lambda)-12\delta+10h\lambda+8}{16(\delta^3-7h^2+8)}
\end{align*}
\]

Notice that, in the limit case as \( \delta \to 1 \), \( W(B_1) \to \frac{11}{10} - \frac{2}{3} \lambda \), \( W(B_2) \to \frac{17}{32} \lambda - \frac{1}{32} \), \( V(S_1) \to \frac{1}{8} + \frac{3}{32} \lambda \) and \( V(S_2) \to \frac{7}{32} + \frac{9}{32} \lambda \).

### 5 Comparisons across different networks

In this section, we draw some considerations on the impact of network structures on the bargaining process among traders in a gas bilateral duopoly. Here we conduct our analysis taking as given a particular network architecture and we compare traders’ equilibrium payoffs across network configuration.

---

There exist three other equilibria corresponding to the case a II-equilibrium arises when the weak buyer has been selected to make offers in which continuation payoffs are such that \( V(S_2) \leq V(S_1) \). Such perfectly symmetric equilibria are immediately obtained by switching the labelling for the two sellers. Note that the payoffs for the two buyers remain unaltered.
5.1 Equilibria

In order to carry on some simple comparisons we first need to be able to rank the different equilibria for any network in some sensible way. In fact, we can order all the possible equilibria across different networks in a \((0, 1)\) square box having the weak buyer’s reservation price \(\lambda\) on its horizontal axis and the common discount factor \(\delta\) on its vertical one. Is then possible to draw all the \((\lambda, \delta)\) regions where any equilibrium for a given network is defined and thus see which equilibria are indeed comparable across different architectures. The only drawback of such procedure is that the final graphic representation turns out to be truly cumbersome. However, for providing a hint of our main qualitative findings here we draw an overall picture of how the equilibria can be ranked across networks just according to the values of the weak buyer’s reservation price \(\lambda\), for a fixed level of the discount factor \(\delta = 0.85\). Qualitative results, however, remain unaltered for any other realistical value of the impatience rate.

Clearly, some network configurations only present a single equilibrium for all values of \(\lambda\). This is the case not only for the exclusive trade network - and the nested strong and weak couple architectures - but also for the asymmetric weak architecture. On the other hand, while both the \(B_1\) and \(B_2\)-short side structures show two coexisting equilibria, the supply-short-side network presents one equilibrium for high values of \(\lambda\), one for intermediate levels and one for relatively low values of \(\lambda\).

Similar is the case of the complete network where only equilibrium \(C1\) is defined for high values of \(\lambda\), equilibrium \(C2\) for medium-high levels, and both equilibria \(C2\) and \(C3\) exist for medium-low values of \(\lambda\).

Even richer is the asymmetric strong network. In fact, for relatively low levels of \(\lambda\) equilibrium \(AS1\) is defined. Medium values of \(\lambda\) are instead cover, in different ranges, by equilibria \(AS5\), \(AS2\) and \(AS6\). For high levels of \(\lambda\) equilibrium \(AS3\) exists, while for extremely high values, equilibrium \(AS4\) is defined.

Some general considerations are in order. In fact, by looking at the most salient features of the above equilibria, it can be reckoned that, offers are often accepted by both traders on a side of the market, so that a random tie-break takes place. This is the case for both equilibria in the \(B_1\) and \(B_2\)-short side architectures, for two out of three equilibria in the supply-short-side network, for all but one equilibrium in the asymmetric strong network (excluding \(AS4\)) and even in two of the three equilibria in the complete network. In some of such equilibria, then, trade occurs with delay, while in other equilibria, with half probability the least connected traders end up leaving the market with no trading at all. In the former case, moreover, different prices usually form in the thin market. Therefore, inefficiency, both in terms of delay in trade and of impossibility to achieve full exploitation of all the potential surplus from trade, can not be ruled out from the above described equilibria.
Moreover, comparisons are possible across equilibria for different network structures which are defined within compatible values of the primitive parameters $\delta$ and $\lambda$. In the following, we discuss some of the main results we have found out by comparing, by means of direct computations and simulations, the equilibrium payoffs of the traders across compatible equilibria in different networks. As the primary interest of the paper lies in the investigation of buyers’ bargaining power and the model itself is in fact symmetric between sellers, we have limited our comparisons to the payoffs of strong and the weak buyer. Analogous simulations and comparing procedures, however, can be easily extended to sellers too.

In fact, there are two main conjectures one can be interested in confirming or rejecting in view of direct comparisons. First, one can guess that the weaker gas purchaser $B_2$, who is clearly in weaker original conditions to start...
negotiations, if embedded in favourable network configurations, should be in theory able to counterbalance, at some extent, the overwhelming natural advantage of the strong buyer. To seek confirmation of such a guess, one should look at the expected equilibrium payoffs in a given network structure to compare the surplus experienced by the two buyers.

It is immediate to check, however, that such intuitive guess is rejected by the model’s predicted payoffs. In fact, the only network architectures where $B_2$ is unambiguously better off than the strong buyer are the obvious cases of the weak-couple and the $B_2$-short side network. While it cannot surprise that the strong buyer indeed experiences systematically higher surplus in an asymmetric strong network, this does sound less obvious for the other two salient connected networks. However, direct comparisons clearly show that the weak buyer is always worse off than $B_1$ even in the unique equilibrium of the asymmetric weak structure. Moreover, it turns out that also in a complete network the strong buyer is always strictly better off than the weak, except in the extraordinary case of values of $\lambda$ so extremely high to approach the limit case $\lambda \to 1$ of symmetric buyers. In such a case, the relative counterbalance of $B_1$’s surplus seems to be due not only to the closeness of the reservation prices, but also to the fact that in the $C1$ equilibrium the weak buyer accesses bilateral negotiations more often than $B_1$.

The second conjecture, instead, is related to the surplus of a given buyer across different networks. In fact, one can intuitively argue that any buyer should always be in a better trading position toward the sellers whenever he is located in a more connected node than the competing buyer. In other words, intuition may suggest that the strong buyer would manage to extract better trading opportunities from being not only in a complete or asymmetric strong network rather than in an asymmetric weak, but also in a asymmetric strong rather than a complete structure. The idea, in fact, is that being connected with more potential partners than the competitor enables a player to enjoy better trading conditions.

To confirm or discard such a conjecture we need to compare the payoffs for each buyers across different network architectures.

5.2 Strong buyer

We start with the strong buyer. By direct computations and numerical simulations we find out several results of interest. First, clearly, the highest surplus experienced by the strong buyer is the one attainable in a $B_1$-short-side network, while the worst is the equilibrium payoff in a $B_2$-short-side network. Secondly, the surplus faced within a strong couple network is equivalent, for very low values of $\lambda$, to the equilibrium payoff from the $SS3$ equilibrium payoff in a supply-short-side network. Third, interestingly, the equilibrium surplus earned by the strong buyer within the exclusive trade exactly corresponds to the one gained in equilibrium within the asymmetric weak network. Moreover, by direct comparisons, it turns out that such
surplus is always strictly lower than what the strong buyer can obtain in equilibrium from bargaining either in an asymmetric strong, or in a complete, so that the following holds:

$$\Pi(B_1)_{ET} \equiv \Pi(B_1)_{AW} < \{\Pi(B_1)_{AS}, \Pi(B_1)_{C}\} \leq \Pi(B_1)_{B_1-Short}$$

This confirms, therefore, that any connected bipartite graph makes \(B_1\) strictly better off than within an exclusive bilateral negotiation, thus providing a natural incentive to the strong buyer to avoid locking in an exclusive partnership and to rather prefer to be embedded into more connected architectures.

Moreover, as intuition would suggest, it turns out that \(B_1\) always enjoy strictly higher surplus in an exclusive trade network than in any equilibrium of the supply-short-side architecture, unless for low values of \(\delta\) when \(\lambda\) is extremely low (SS2). Thus, it can be checked that, a fortiori, any from the complete, the asymmetric strong and the asymmetric weak network ensures to the strong buyer at least as large equilibrium surplus than the supply-short-side structure:

$$\Pi(B_1)_{Supply-Short} \leq \Pi(B_1)_{ET} \equiv \Pi(B_1)_{AW} < \{\Pi(B_1)_{AS}, \Pi(B_1)_{C}\}$$

Finally, it is possible to directly compare and rank the equilibrium payoffs which the completely connected graphs convey to the strong buyer. Our findings from computations and simulations are rather interesting.

First, as already seen, it clearly turns out that both in an asymmetric strong and in a complete network, \(B_1\) gets equilibrium payoffs never lower than in asymmetric weak. More precisely, the strong buyer is always strictly better off in an asymmetric strong network, unless when \(\lambda\) is extremely high, in which case he earns the same surplus in both architectures.

Furthermore, we can carry on a direct check of our second conjecture. In fact, direct computations reject the hypothesis that the strong buyer would always be better off in an asymmetric strong network in which he would enjoy more trading links than the weak competitor. Indeed, while for \(\lambda\) high enough \(B_1\) is unambiguously better off within an asymmetric strong network, this is no longer true for lower values of the weak buyer’s reservation price. At the contrary, while, for low values of \(\lambda\), comparisons among payoffs are not possible since there are no defined equilibria for the asymmetric strong and the complete networks, for intermediate levels of \(\lambda\), the strong buyer is strictly better off within a complete network.

To shed some light on this, perhaps surprising, result, we provide a tentative explanation. In fact, consider an asymmetric strong network where the weak buyer, characterized by a low reservation price, is exclusively linked with seller \(S_2\). Intuitively, the fact that is linked with an exclusive relationship with the weak buyer provides seller \(S_2\) a safe outside option she can always rely on, in the sense that, whenever the weak buyer is selected to make offers, \(S_2\) benefits from having an exclusive partnership with \(B_2\).
in terms of high trading prices. Hence, the existence of such alternative trading opportunity implies that, when bargaining with $B_1$, seller $S_2$ would never accept any proposal making her worse off with respect to such outside option.

In other words, the possibility of exclusive dealing with $B_2$ indirectly provides a lower bound for competition between the two sellers when fighting for serving the strong buyer. In fact, even $S_1$ expects that $S_2$ would never accept from the strong buyer any price below a proposal making her indifferent to what she can get from the weak buyer. Therefore, is common knowledge that $S_2$ would never exert any competitive pressure below that threshold. However, even $S_1$ has no interest in proposing the strong buyer something more favourable than $S_2$’s outside option. Thus, both sellers have no incentives to compete too fiercely for the strong buyer, by proposing prices below what $S_2$ can get from the weak buyer. The existence of such implicit lower bound for sellers’ competition clearly hurts the strong buyer, as he is not able to extract larger trading surplus from negotiations. This is because, when making offers to $B_1$, both sellers are likely to ask something comparable to what $S_2$ can get from the weak buyer.

Therefore, to avoid being hurt by such price floor limit to competition, the strong buyer may be better off in a complete network. In fact, as long as $B_2$’s reservation price is kept on low or medium levels, $B_1$ prefers that the weak buyer takes part into negotiations from a fully connected, rather than in a less central node. From this point of view, it seems that the strongest competing purchaser genuinely prefers a market structure where communication and trading opportunities are easier and less constrained to one with protected exclusive partnerships. Asymmetry across reservation prices is sharp enough to guarantee the strong buyer getting larger surplus than in an asymmetric strong network anyway. This result seems counterintuitive, though, and is susceptible of interesting regulation policy implications.\footnote{Can we imagine the italian Antitrust Authority trying to persuade the chairman of ENI that his company would make higher profits allowing a small competitor with a more limited portfolio of gas suppliers to freely negotiate with all ENI suppliers?}

There is a limit, however, to such $B_1$’s preference towards the complete network. In fact, as $\lambda$ approaches high levels, buyers become more similar in terms of attractiveness for the sellers. Thus, while in a complete network, competition to serve the strong buyer becomes less fierce as both sellers can sustain high prices selling to the weak buyer, in the asymmetric strong network, $B_1$ is able to take advantage of the possibility that $S_2$ exclusively deals with $B_2$, by obtaining from $S_1$ prices similar to the one emerging in bilateral negotiations, which, in turn, are now significantly lower than $\lambda$.\footnote{Can we imagine the italian Antitrust Authority trying to persuade the chairman of ENI that his company would make higher profits allowing a small competitor with a more limited portfolio of gas suppliers to freely negotiate with all ENI suppliers?}
5.3 Weak buyer

Such a preference for bargaining in a complete architecture is partially common to the weak buyer too. Clearly, it immediately turns out that the weak buyer is always strictly better off within a complete rather than in an asymmetric strong network. Of course, also all the other intuitive results are confirmed for the weak buyer too.\footnote{In particular, $B_2$ gets its worse equilibrium payoff in a $B_1$-short-side and its best in a $B_2$-short-side network. Again, it turns out that bargaining in an exclusive trade network delivers $B_2$ exactly the same equilibrium payoffs than negotiations in an asymmetric weak architecture. Such a positive externality from being better connected than in an exclusive partnership arises, again rather intuitively, also within a complete network, but only as $\lambda$ is high enough.}

Moreover, interestingly, from direct comparisons it also turns out that $B_2$ prefers to bargain in a complete network only when $\lambda$ is high enough, while he is better off in an asymmetric weak architecture for lower levels of $\lambda$. These findings are intuitive too. Infact, better connections can help $B_2$ to overcome significant disadvantages in the original trading capability of the weak buyer. However, a line of arguments which are the mirror image of the ones discussed for the strong buyer, implies that the protection of a more central node from the competitive pressure of $B_1$’s outside option is no longer a sufficient trading guarantee when this weakness is less pronounced. Therefore, the weak buyer would prefer negotiating in a complete network exactly for levels of $\lambda$ for which the strong buyer would not.

Moreover, a tension between buyers’ interests can be easily reckoned. In fact, the strong buyer prefers to be embedded within a complete network when $\lambda$ takes medium-low values, while within an asymmetric strong for high levels of $\lambda$. On the contrary, the weak buyer prefers to negotiate within an asymmetric weak architecture when $\lambda$ is medium-low and within a complete network when his reservation price is relatively high.

The emergence of such a prominent conflict of interests among buyers can be regarded as a fascinating prelude to the the investigation of the endogenous strategies of link formation by the traders. As already mentioned, this goal is left for a companion paper.

5.4 Extensions and concluding remarks

We have analyzed the interaction between strategic negotiations and network structures in a bilateral oligopoly with identical sellers and heterogeneous buyers. We have provided a full characterization of all the subgame perfect Nash equilibria in pure and stationary strategies possibly emerging in the negotiations stage in any fixed network architecture. We have then described some salient features of such bargaining equilibria and compare traders’ payoffs within and across networks.

We find that, depending on the inter-temporal discount factor and the dispersion of reservation values across buyers, negotiations may lead, even
in a completely connected buyers-sellers network, to multiple equilibria, coexistence of different prices, delays in trade and inefficient allocations. By comparing traders’ payoffs across networks, we then derive the endogenous bargaining power of each trader as a function of her position in the communication network. We show that, for intermediate levels of the buyers’ heterogeneity, the strongest competing purchaser may prefer to bargain within a market structure where communication and trading opportunities are easier and less constrained rather than in one with protected exclusive partnerships.

Since multiplicity of equilibria is inherently related to the behavioural strategic interaction of traders in thin markets, we believe that the analysis will be enriched if the findings would be verified experimentally\textsuperscript{23}. An experimental test of this model, in fact, would not only overcome the difficulty of obtaining individual data on strategic behaviour in gas thin markets but, perhaps more importantly, allow for testing the theoretical results under the same controlled conditions as the theory itself. Indeed, experimental analysis would allow us to test whether the model (quoting Hey, 1991) “…survives the transition from the world of the theory to the… real world – the world in which data is gathered”. This remains indeed the final goal of our current research.

6 Technical Appendix

In the following, we report a sketch of the proofs for the two first Propositions only. The remaining proofs follow analogous arguments and are omitted. They are available on request from the author.

6.1 Proof of Proposition 1 (Exclusive trade network)

By an usual argument in bargaining theory, the only possible equilibrium offer by any trader is to propose her (his) exclusive potential partner just his (her) continuation payoff. For instance, imagine the strong buyer has been selected to make offers. In a PSSPN equilibrium, $B_1$ should propose $S_1$ a price exactly identical to her own continuation value $V(S_1)$, which, is clearly accepted by $S_1$.

In fact, higher price from $B_1$ would still be accepted but would be strictly dominated strategies. On the other hand, lower offers from the strong buyer will certainly be rejected by $S_1$, thus delivering the former just his own continuation value $\delta W(B_1)$. However, the proposal $p_{B_1} = \delta V(S_1)$ gives the strong buyer a surplus $1 - \delta V(S_1)$ as good as the payoff $\delta W(B_1)$ guaranteed by any, alternative, unacceptable offer: in fact, $W(B_1) \leq 1 - \delta V(S_1)$ always

\textsuperscript{23} Except our current project, in fact, as far as we know, the only previous experiment on bargaining in buyers-sellers networks is the one by Charness, Corominas-Bosch and Frechette (2005).
holds as the maximum surplus the strong buyer can manage to extract from negotiations in an exclusive trade network is exactly what left from his own reservation price once he has paid to his only potential partner \( S_1 \) her continuation payoff. Hence \( \delta W(B_1) < W(B_1) \leq 1 – \delta V(S_1) \) must a fortiori be holding. Therefore, the proposal \( p^*_B = \delta V(S_1) \) is the optimal strategy by the strong buyer among all, acceptable and unacceptable, offers.

Analogously, when selected seller \( S_1 \) makes offer \( p^*_S = 1 – \delta W(B_1) \). In both cases, once either \( S_1 \) or \( B_1 \) has proposed the above offer and the relative counterpart has accepted it, the pair trade at the agreed price and leave the market with the corresponding payoffs.

The remaining traders \( S_2 \) and \( B_2 \) are still in the market and, being connected each other, are allowed to carry on further negotiations. After \( S_1 \) and \( B_1 \) have traded and left the market, these negotiations are equivalent to a standard bilateral bargaining with random order of proposers. By accessing bilateral negotiations after \( S_1 \) and \( B_1 \) have traded and left the market, the weak buyer and \( S_2 \) just expect to get a surplus \( \frac{\delta}{2} \) each.

Denote \( \delta V(S_1) \) is the discounted value of the expected continuation payoff by seller \( S_1 \) from entering a new negotiation stage, and \( \delta W(B_1) \) the analogous discounted value of the expected continuation payoff by buyer \( B_1 \) from entering a new negotiation stage.

Thus, the unique PSSPN of the negotiations game within an exclusive-trade network following

- a random selection of the strong buyer \( B_1 \) implies, in the price-offer phase, \( B_1 \) always proposing \( S_1 \) a price \( p^*_B = \delta V(S_1) \), and, in the response phase, \( S_1 \) accepting \( p^*_B \). After \( B_1 \) and \( S_1 \) have traded at \( p^*_B \) and left the market, \( B_2 \) and \( S_2 \) enter further bilateral negotiations. The equilibrium payoffs for the traders are then

\[
\begin{align*}
\Pi(B_1) &= 1 – \delta V(S_1) \\
\Pi(B_2) &= \frac{\delta}{2} \\
\Pi(S_1) &= \delta V(S_1) \\
\Pi(S_2) &= \frac{\delta}{2}
\end{align*}
\]

Analogous results hold for a random selection of seller \( S_1 \).

- a random selection of the weak buyer \( B_2 \) implies, in the price-offer phase, \( B_2 \) always proposing \( S_2 \) a price \( p^*_B = \delta V(S_2) \), and, in the response phase, \( S_2 \) accepting \( p^*_B \). After \( B_2 \) and \( S_2 \) have traded at \( p^*_B \) and left the market, \( B_1 \) and \( S_1 \) enter further bilateral negotiations. The equilibrium payoffs for the traders are then

\[
\begin{align*}
\Pi(B_1) &= \frac{\delta}{7} \\
\Pi(B_2) &= \lambda – \delta V(S_2) \\
\Pi(S_1) &= \frac{\delta}{7} \\
\Pi(S_2) &= \delta V(S_2)
\end{align*}
\]
Analogous results hold for a random selection of seller \( S_2 \).

By taking each above equilibrium payoff as weighted with \( \frac{1}{4} \) probability, we can write the expected continuation payoffs of the traders in such PSSPN equilibrium as

\[
\begin{align*}
W(B_1) &= \frac{1}{4} (1 - \delta V(S_1)) + \frac{1}{4} \delta W(B_1) + \frac{1}{2} (\frac{\lambda}{2}) \\
W(B_2) &= \frac{1}{2} (\frac{\delta \lambda}{\sqrt{2} - \delta}) + \frac{1}{4} (\lambda - \delta V(S_2)) + \frac{3}{4} W(B_2) \\
V(S_1) &= \frac{1}{4} \delta V(S_1) + \frac{1}{4} (1 - \delta W(B_1)) + \frac{1}{2} (\frac{\lambda}{2}) \\
V(S_2) &= \frac{1}{2} (\frac{\delta \lambda}{\sqrt{2} - \delta}) + \frac{\delta}{4} V(S_2) + \frac{1}{4} (\lambda - \delta W(B_2))
\end{align*}
\]

which can be solved as as system, returning the final expressions

\[
\begin{align*}
W(B_1) &= \frac{1 + \delta}{4} \\
W(B_2) &= \frac{(1 + \delta) \lambda}{2(\sqrt{2} - \delta)} \\
V(S_1) &= \frac{1 + \delta}{4} \\
V(S_2) &= \frac{(1 + \delta) \lambda}{2(\sqrt{2} - \delta)}
\end{align*}
\]

that, in the limit case as \( \delta \to 1 \), clearly approach the values \( W(B_1) \to V(S_1) \to \frac{\lambda}{2} \) and \( W(B_2) \to V(S_2) \to \frac{\lambda}{2} \).

We can then summarize the previous results stating Proposition 1. Analogous results hold for the weak couple network for which the expected payoffs are

\[
\begin{align*}
W(B_1) &= 0 \\
W(B_2) &= \frac{\lambda}{2(\sqrt{2} - \delta)} \\
V(S_1) &= 0 \\
V(S_2) &= \frac{\lambda}{2(\sqrt{2} - \delta)}
\end{align*}
\]

6.2 Proof of Proposition 2 (Supply-short-side network)

We describe one by one each subgame of the negotiations game, starting whenever any from the four traders is randomly selected to make offers.

6.2.1 \( B_1 \) proposes offers  The shape of the supply-short-side network implies that, whenever in a round either buyer has been selected to make proposals, which occurs with identical \( \frac{1}{4} \) probability each, they both face no other alternatives but making offers to the only linked seller \( S_2 \).

In particular, by an usual argument the only possible equilibrium offer by the strong buyer is to propose \( S_2 \) a price exactly identical to the discounted value of her own continuation value \( \delta V(S_2) \), which, in equilibrium is clearly accepted by \( S_2 \). Thus, within a supply-short-side network, the unique PSSPNE of the game following a random selection of the strong buyer \( B_1 \) implies, in the price-offer phase, \( B_1 \) always proposing \( S_2 \) a price \( p_{B_1} = \delta V(S_2) \), and, in the response phase, \( S_2 \) accepting \( p_{B_1} \). The equilibrium payoffs for the traders whenever \( B_1 \) has been selected as proposer, are then \( \Pi(B_1) = 1 - \delta V(S_2) \), \( \Pi(B_2) = \Pi(S_1) = 0 \) and \( \Pi(S_2) = \delta V(S_2) \).
6.2.2 Seller $S_2$ proposes offers  Consider now the $\frac{1}{4}$ probability that $S_2$ has been selected to propose offers to both connected buyers. By using backward induction, we first describe, for any given proposed price $p_{S_2}$, the set of all the possible Nash Equilibria in the response game played by the two linked buyers, and we then look for the optimal pricing strategy by $S_2$ in the offer phase.

It can be immediately seen that the payoff matrix of the buyers response game, for a given proposal $p_{S_2}$, is the one reported in Figure 2.2.

![Fig. 3 Buyers’ response game in a supply-short-side network](image)

The set of strategies available to each buyer has just two elements: either strategy *Accept $p_{S_2}$* or strategy *Reject $p_{S_2}$*. For instance, given that $B_2$ chooses *Reject $p_{S_2}$*, it is better for the strong buyer to *Accept $p_{S_2}$* if and only if $1 - p_{S_2} \geq \delta W(B_1)$, while, given that $B_2$ plays *Accept $p_{S_2}$*, it is better for $B_1$ to also accept it if and only if $\frac{1}{2} (1 - p_{S_2}) \geq 0$, that is, if $p_{S_2} \leq 1$. On the other hand, given that $B_1$ accepts $p_{S_2}$, it is optimal for $B_2$ to also accept it if and only if $\frac{1}{2} (\lambda - p_{S_2}) \geq 0$, that is, if $p_{S_2} \leq \lambda$. Finally, given that $B_1$ rejects $p_{S_2}$, it is optimal for $B_2$ to accept it if and only if $\lambda - p_{S_2} \geq \delta W(B_2)$.

The combination of $[B_1, B_2]$ pure strategies $[\text{Accept } p_{S_2}, \text{Accept } p_{S_2}]$ is a Nash equilibrium of the response game if and only if $\left\{ \begin{array}{l} p_{S_2} \leq 1 \\ \text{or} \\ p_{S_2} \leq \lambda \end{array} \right.$, that is, whenever $p_{S_2} \leq \lambda$.

---

24 Hereafter, we just take advantage of standard tie-breaking assumptions by which if a trader is perfectly indifferent between accepting or rejecting an offer she accepts it, while, if a trader is ever indifferent between proposing acceptable or unacceptable offers, she makes the acceptable one.
On the other hand, pure strategies \( \{\text{Accept } p_{S_2}, \text{Reject } p_{S_2}\} \) are a Nash equilibrium of the response game if \( \{p_{S_2} \leq 1 - \delta W(B_1), \ p_{S_2} > \lambda\} \), while \( \{\text{Reject } p_{S_2}, \text{Accept } p_{S_2}\} \) are a pure-strategies Nash equilibrium if \( \{p_{S_2} > 1, \ p_{S_2} \leq \lambda - \delta W(B_2)\} \).

Finally, \( \{\text{Reject } p_{S_2}, \text{Reject } p_{S_2}\} \) are a pure-strategies Nash equilibrium if \( \{p_{S_2} > 1 - \delta W(B_1), \ p_{S_2} > \lambda - \delta W(B_2)\} \), that is if \( p_{S_2} > \max\{1 - \delta W(B_1), \ \lambda - \delta W(B_2)\} \). By the above discussed Condition k, \( \delta [W(B_1) - W(B_1)] \leq 1 - \lambda \), the latter condition reduces to \( p_{S_2} > 1 - \delta W(B_1) \).

Furthermore, it is easily reckoned that offer \( p_{S_2} \) can never verify the set of restrictions \( \{p_{S_2} > 1 - \delta W(B_1), \ p_{S_2} \leq \lambda - \delta W(B_2)\} \), as, by definition, \( \lambda \leq 1 \) and \( B_2 \)'s continuation payoff is surely non-negative, \( \delta W(B_2) \geq 0 \). Hence, there is no offer \( p_{S_2} \) verifying \( \lambda - \delta W(B_2) \geq p_{S_2} > 1 \).

Therefore, only three possible combinations of strategies can represent Nash equilibria of the response game under some range of restrictions on the price offer \( p_{S_2} \): either \( \{\text{Accept } p_{S_2}, \text{Accept } p_{S_2}\} \) if \( p_{S_2} \leq \lambda \), or \( \{\text{Accept } p_{S_2}, \text{Reject } p_{S_2}\} \) are a Nash equilibrium of the response game if \( \{p_{S_2} \leq 1 - \delta W(B_1), \ p_{S_2} > \lambda\} \), or, finally, \( \{\text{Reject } p_{S_2}, \text{Reject } p_{S_2}\} \) if \( p_{S_2} > 1 - \delta W(B_1) \). This leads to one consideration.

Multiple Nash equilibria in the response game can arise whenever \( S_2 \) proposes an offer \( p_{S_2} \) such that both \( p_{S_2} \leq \lambda \) and \( p_{S_2} > 1 - \delta W(B_1) \) hold. In such a case two alternative Nash equilibria coexist: one where both buyers accept \( p_{S_2} \), the other where both buyers reject it. We will provide a full characterization of all the resulting multiple equilibria in the response game and we will look for the \( \text{PSSPN} \) equilibria, if any, inducing each corresponding alternative equilibrium in the response game. Denote \( \delta V(S_i) \) and \( \delta W(B_i) \) the discounted value of the expected continuation payoff by \( S_i \) and \( B_i \), respectively, from entering a new negotiation stage.

The set of potential pure-strategies Nash equilibria turns out to be even narrower under specific combinations of parameters. In order to characterize it in greater detail, we need to consider all the possible, mutually exclusive, ranking of the thresholds.

In fact, all the above sets of conditions just depend upon the relative size of two levels: \( \lambda \) and \( 1 - \delta W(B_1) \). Consider either ranking, henceforth called cases I and II:

\[
I: 1 - \delta W(B_1) > \lambda \\
II: \lambda \geq 1 - \delta W(B_1)
\]

which correspond on imposing, respectively

\[
I: \delta W(B_1) < 1 - \lambda \\
II: \delta W(B_1) \geq 1 - \lambda
\]

We now look at them in greater detail.
Case I Under case I, the buyers’ response game can show any of the three potential pure strategies Nash equilibria within some range of the offered price \(p_{S_2}\): either \([Accept \ p_{S_2}, \ Accept \ p_{S_2}]\) if \(p_{S_2} \leq \lambda\), or \([Accept \ p_{S_2}, \ Reject \ p_{S_2}]\) if \(p_{S_2} \leq 1 - \delta W(B_1)\), or, finally, \([Reject \ p_{S_2}, \ Reject \ p_{S_2}]\) if \(p_{S_2} > 1 - \delta W(B_1)\).

The latter conditions are not only such that the whole range of parameters is covered by some equilibrium, but are also mutually exclusive, therefore ruling out any multiplicity of equilibria.

The seller \(S_2\)'s choice in case I is straight. Whenever she charges more than \(1 - \delta W(B_1)\) both buyers are going to reject her offer: all the traders would enter further negotiations, and she would get nothing but her own continuation payoff \(\delta V(S_2)\).

On the other hand, whenever she proposes any price \(p_{S_2}\) not higher than \(1 - \delta W(B_1)\), that offer would immediately be accepted either by the strong buyer only, if \(1 - \delta W(B_1) \geq p_{S_2} > \lambda\), or by both buyers, if \(p_{S_2} \leq \lambda\), thus delivering her a payoff of \(p_{S_2}\). Among all such possible acceptable offers, \(p_{S_2} = 1 - \delta W(B_1)\) is clearly a dominant strategy by seller \(S_2\), as any lower price, still accepted by some buyer, would return her a strictly lower surplus.

Therefore, the optimal decision rule by seller \(S_2\) is that, as long as \(\delta V(S_2) \leq 1 - \delta W(B_1)\), the best strategy for \(S_2\) is to propose a price offer \(p_{S_2}^* = 1 - \delta W(B_1)\), and, otherwise to make any highest, unaccept-able, offer. However condition \(\delta V(S_2) \leq 1 - \delta W(B_1)\) is always verified as \(\delta W(B_1) \leq W(B_1) \leq 1 - \delta V(S_2)\) comes from the fact that the most the strong buyer can get from negotiations within such a network is what is left out of his surplus once seller \(S_2\) has been paid her continuation payoff. In fact, from Condition k, what she can earn from negotiating with the weak buyer is certainly lower as \(\lambda - \delta W(B_2) \leq 1 - \delta W(B_1)\).

Thus, as long as condition \(\delta W(B_1) < 1 - \lambda\) holds, in the price offer phase, \(S_2\) offers a price \(p_{S_2}^* = 1 - \delta W(B_1)\), which in the response game is accepted in equilibrium by the strong buyer only, while the weak buyer rejects it and leaves consequently the market with a zero payoff. Whenever seller \(S_2\) is selected to make offers, traders’ expected payoffs from case I equilibrium are

\[
\begin{align*}
\Pi(B_1) &= \delta W(B_1) \\
\Pi(B_2) &= 0 \\
\Pi(S_1) &= 0 \\
\Pi(S_2) &= 1 - \delta W(B_1)
\end{align*}
\]

The described pure strategies indeed constitute a subgame perfect equilib-rium. First, we can check that \([Reject \ p_{S_2}^*, \ Accept \ p_{S_2}^*]\) is in fact a pure strategies equilibrium in the response game. Given that \(B_1\) accepts \(p_{S_2}^* = 1 - \delta W(B_1)\), \(B_2\) cannot profitably deviate by also accepting \(p_{S_2}^*\) as it would give him a payoff \(\frac{1}{2}(\lambda - 1 + \delta W(B_1))\) which is never higher than the zero payoff associated to leaving the market, as condition \(\delta W(B_1) < 1 - \lambda\) is always satisfied under case I. On the other hand, given that \(B_2\) rejects, the strong buyer would get exactly the same payoff \(\delta W(B_1)\) if he rejects.
Finally, given the buyers’ behavior in the response game, seller $S_2$ has no way to profitably deviate: in fact, if she propose any lower price $p'_{S2} = p^*_{S2} - \varepsilon$, still accepted at least by the strong buyer, she would gain a lower surplus, while for any higher, unaccepted, price $p''_{S2} = p^*_{S2} + \varepsilon$, she would just get her own continuation value, which is never better as $\delta V(S_2) \leq 1 - \delta W(B_1)$ is always holding.

Case II Under case II, the ranking $\lambda \geq 1 - \delta W(B_1)$ implies that the set of conditions \[ p_{S2} \leq 1 - \delta W(B_1) \] never identify any non-empty range: in fact, the lower bound on the strong buyer’s continuation payoff is clearly incompatible with the conditions necessary to meet a \{Accept $p_{S2}$, Reject $p_{S2}$\} pure strategies Nash equilibrium in the response game.

Even in this case the whole range of parameters is covered by some equilibrium. However, in case II the conditions describing the Nash equilibria in the response game are no longer mutually exclusive. In fact, whenever $S_2$ proposes any offer $\tilde{p}_{S2}$ such that $\lambda \geq \tilde{p}_{S2} > 1 - \delta W(B_1)$ two alternative Nash equilibria co-exist in the response game: either both buyers accept $\tilde{p}_{S2}$, or they both reject it.

Therefore, the overall game can be solved by separately considering each of the equilibria in the response subgame following any offer $\lambda \geq \tilde{p}_{S2} > 1 - \delta W(B_1)$.

In fact, consider the equilibrium in the response game where both buyers accept any offer $\lambda \geq \tilde{p}_{S2} > 1 - \delta W(B_1)$, henceforth called Ha. The subsequent random tie-break determines which buyer is entitled to trade with $S_2$ at $\tilde{p}_{S2}$ and which one leaves the market with zero payoff. Following an offer $\lambda \geq \tilde{p}_{S2} > 1 - \delta W(B_1)$, the overall game can thus be solved by plugging the corresponding traders’ equilibrium payoffs into the final nodes at the response phase induced by such a proposal:

\[
\begin{align*}
I_i (B_1) &= \frac{1}{2} (1 - \tilde{p}_{S2}) \\
I_i (B_2) &= \frac{1}{2} (\lambda - \tilde{p}_{S2}) \\
I_i (S_1) &= 0 \\
I_i (S_2) &= \tilde{p}_{S2}
\end{align*}
\]

Moving back to the offer phase, consider now $S_2$’s choice in case Ha. Whenever she charges any price $\tilde{p}_{S2} \leq \lambda$, her proposal is immediately accepted by both buyers, delivering her a payoff of $\tilde{p}_{S2}$. Among all such acceptable offers, $\tilde{p}_{S2} = \lambda$ is clearly a dominant strategy for seller $S_2$, as any lower price, still accepted, would return her a strictly lower surplus. For any other possible offer above $\lambda$, the response game shows a unique equilibrium where both buyers reject that offer, all traders enter a new round of negotiations so that her final payoff is just her own continuation value $\delta V(S_2)$.

Therefore, the optimal decision rule by seller $S_2$ is to propose an acceptable offer $p^*_{S2} = \lambda$, as long as $\delta V(S_2) \leq \lambda$ and, otherwise to make any highest, unacceptable, offer. In order to ensure acceptable offers in equi-
librium, we need to impose explicit restrictions on seller $S_2$’s continuation payoff. As long as one of the following sets of conditions holds

\[
\begin{aligned}
&\delta W(B_1) \geq 1 - \lambda \\
&\delta V(S_2) \leq \lambda
\end{aligned}
\]

we can characterize a PSSP equilibrium within case IIa, whenever seller $S_2$ is selected to make a proposal.

The equilibrium in case IIa is as follows. In the price offer phase, $S_2$ offers a price $p_{S2} = \lambda$, which, in the response game, is accepted by both buyers. The subsequent random tie-break selects which buyer is going to trade with seller $S_2$ at $p_{S2} = \lambda$, and which, instead, leaves the market with no trade and surplus. Hence, whenever $S_2$ is selected to make offers, traders’ expected payoffs from case IIa equilibrium are

\[
\begin{aligned}
&\Pi(B_1) = \frac{1-\lambda}{2} \\
&\Pi(B_2) = 0 \\
&\Pi(S_1) = 0 \\
&\Pi(S_2) = \lambda.
\end{aligned}
\]

Again, it is quickly checked that the described pure strategies constitute a subgame perfect equilibrium. First, we can check that $[\text{Accept } p_{S2}^*, \text{Accept } p_{S2}^*]$ is in fact a pure strategies equilibrium in the response game. Given that $B_2$ accepts $p_{S2}^* = \lambda$, $B_1$ cannot profitably deviate by rejecting $p_{S2}^*$ as it would return him a zero payoff, of course lower as $\lambda < 1$. On the other hand, given that $B_1$ accepts $p_{S2}^* = \lambda$, if $B_2$ deviates by rejecting $p_{S2}^*$ he would get exactly the same zero surplus. Finally, given the buyers’ behavior in the response game, seller $S_2$ has no way to profitably deviate: in fact, if she proposes any lower price $p_{S2}^* = \lambda - \epsilon$, still accepted by the buyers even if such that $p_{S2}^* > 1 - \delta W(B_1)$, she would gain a lower payoff, while for any, rejected, higher price $p_{S2}^* = \lambda + \epsilon$, she would just get her own continuation value, which is never better as condition $\delta V(S_2) \leq \lambda$ holds. Therefore, the above described is indeed a pure strategies subgame perfect equilibrium.

On the other hand, consider the alternative case where, following any offer $\lambda \geq \tilde{p}_{S2} > 1 - \delta W(B_1)$ from $S_2$, the unique equilibrium in the response phase is such that both buyers reject $\tilde{p}_{S2}$, henceforth called case IIr. That implies that, whenever $S_2$ proposes an offer $\lambda \geq \tilde{p}_{S2} > 1 - \delta W(B_1)$, all the traders enter a further round of negotiations. After a proposal $\lambda \geq \tilde{p}_{S2} > 1 - \delta W(B_1)$, the overall game can thus be solved by plugging the corresponding traders’ continuation payoffs into the final nodes at the response phase induced by such an offer.

Moving back to the offer phase, in fact, consider $S_2$’s choice in case IIr. Whenever she charges any price $\lambda \geq \tilde{p}_{S2} > 1 - \delta W(B_1)$, her proposal is immediately rejected by both buyers, delivering her a payoff of $\delta V(S_2)$. For any other possible proposal above $\lambda$, the response game shows a unique equilibrium where both buyers reject that offer, all traders enter a new round of negotiations so that the final payoff is again her own continuation value $\delta V(S_2)$. Therefore, any price $\tilde{p}_{S2} > 1 - \delta W(B_1)$ would return her
nothing but her continuation value. On the other hand, any price at most as high as \(1 - \delta W(B_1)\) would be certainly accepted by both buyers. Among all such possible acceptable offers, \(p_{S_2} = 1 - \delta W(B_1)\) is clearly a dominant strategy for \(S_2\), as any lower price, still accepted, would return her a strictly lower earning.

Therefore, the best strategy for \(B_1\) is to propose a price offer \(p_{B_1}^* = 1 - \delta W(B_1)\), as long as \(\delta V(S_2) \leq 1 - \delta W(B_1)\) and, otherwise to make any unacceptable offer. Notice that the latter condition is always verified: in fact \(\delta W(B_1) \leq W(B_1) \leq 1 - \delta V(S_2)\) holds from the fact that the most the strong buyer can ever get from negotiations is what remains from his potential surplus once he has paid \(S_2\) her own continuation payoff. Hence, as long as condition \(\delta W(B_1) \geq 1 - \lambda\) holds, we can characterize a PSSP equilibrium within case IIr, for the case \(S_2\) is selected to make a proposal.

Case IIr equilibrium is as follows. \(S_2\) offers a price \(p_{S_2}^* = 1 - \delta W(B_1)\), which, in the response game’s equilibrium is accepted by both buyers. The subsequent random tie-break selects which buyer is going to trade with \(S_2\) at \(p_{S_2}^* = 1 - \delta W(B_1)\), and which, instead, can, eventually, access further bilateral negotiations with \(S_1\), if linked together. Hence, whenever \(S_2\) is selected to make offers, traders’ expected payoffs from case IIr equilibrium are:

\[
\begin{align*}
\Pi(B_1) &= \frac{1}{2} (1 - (1 - \delta W(B_1))) = \frac{\delta W(B_1)}{2} \\
\Pi(B_2) &= \frac{1}{2} (\lambda - (1 - \delta W(B_1))) = \frac{\delta W(B_1)}{2} + \frac{\lambda - 1}{2} \\
\Pi(S_1) &= 0 \\
\Pi(S_2) &= 1 - \delta W(B_1)
\end{align*}
\]

The above described strategies are a pure strategies subgame perfect equilibrium. First, one can check that \([\text{Accept } p_{S_2}^*, \text{Accept } p_{S_2}^*]\) is a pure strategies equilibrium in the response game. Given that \(B_2\) accepts \(p_{S_2}^*\), \(B_1\) cannot profitably deviate by rejecting \(p_{S_2}^*\), as it would only give him a zero payoff, which is clearly lower than any positive \(\frac{\delta W(B_1)}{2}\). On the other hand, given that \(B_1\) accepts \(p_{S_2}^* = 1 - \delta W(B_1)\), if \(B_2\) deviates by rejecting \(p_{S_2}^*\) he would take a zero payoff which is clearly never better than \(\frac{\delta W(B_1)}{2} + \frac{\lambda - 1}{2}\), as in case IIr \(\delta W(B_1) \geq 1 - \lambda\) always holds. Then, given such equilibrium behavior in the response game, neither seller \(S_2\) has any way to profitably deviate: in fact, if she deviates by proposing any lower price \(p_{S_2}^* = p_{S_2}^* - \varepsilon\), still accepted, he would clearly get a smaller earning, while if she deviates by any price \(p_{S_2}^*\) strictly above \(p_{S_2}^*\), rejected by both buyers, she would earn her continuation value, which is never better as \(\delta V(S_2) \leq 1 - \delta W(B_1)\) is always true. Therefore, the above described is indeed a subgame perfect equilibrium in pure and stationary strategies of the negotiations game within a supply-short-side network.

We can then summarize the above findings in the following characterization of the equilibrium offers by the seller. Whenever seller \(S_2\) is selected to make offers within a supply-short-side network, under Condition \(k\), three PSSPN equilibria can arise, according to the levels of the continuation payoffs:
\[ - \text{If } \delta W(B_1) < 1 - \lambda \text{ is satisfied, then there exists a PSSPN equilibrium where, in the price offer phase, } S_2 \text{ offers a price } p^*_{S_2} = 1 - \delta W(B_1) \text{ and in the response phase, only } B_1 \text{ accepts } p^*_{S_2} = 1 - \delta W(B_1), \text{ while } B_2 \text{ rejects it and then leaves the market without trading. Hence, traders’ expected payoffs from such PSSPN } I\text{-equilibrium are}
\]
\[
\begin{align*}
\Pi (B_1) &= \delta W(B_1) \\
\Pi (B_2) &= 0 \\
\Pi (S_1) &= 0 \\
\Pi (S_2) &= 1 - \delta W(B_1)
\end{align*}
\]

\[ - \text{If, instead, the following set of conditions holds}
\]
\[
\begin{align*}
\delta W(B_1) &\geq 1 - \lambda \\
\delta V(S_2) &\leq \lambda
\end{align*}
\]

\[ \text{then there exists a PSSPN equilibrium where, in the price offer phase, } S_2 \text{ offers a price } p^*_{S_2} = \lambda \text{ and in the response phase, both } B_1 \text{ and } B_2 \text{ accept } p^*_{S_2} = \lambda \text{ as well as any offer within the range } (1 - \delta W(B_1), \lambda]. \text{ In such equilibrium, thus, buyers enter a random selection to determine which one trades with seller } S_2 \text{ at } p^*_{S_2} \text{ and which, instead, leaves the market. Hence, traders’ expected payoffs from such PSSPN } IIa\text{-equilibrium are}
\]
\[
\begin{align*}
\Pi (B_1) &= \frac{1 - \lambda}{2} \\
\Pi (B_2) &= 0 \\
\Pi (S_1) &= 0 \\
\Pi (S_2) &= \lambda.
\end{align*}
\]

\[ - \text{Finally, if just } W(B_1) \geq 1 - \lambda \text{ holds, then there exists a PSSPN equilibrium where, in the price offer phase, } S_2 \text{ offers a price } p^*_{S_2} = 1 - \delta W(B_1) \text{ and in the response phase, both } B_1 \text{ and } B_2 \text{ accept } p^*_{S_2} = 1 - \delta W(B_1) \text{ while they both reject any offer within the range } (1 - \delta W(B_1), \lambda]. \text{ In such equilibrium, thus, buyers enter a random selection to determine which one trades with seller } S_2 \text{ at } p^*_{S_2} \text{ and which, instead, leaves the market. Hence, traders’ expected payoffs from such PSSPN } IIr\text{-equilibrium are}
\]
\[
\begin{align*}
\Pi (B_1) &= \frac{\delta W(B_1)}{2} \\
\Pi (B_2) &= \frac{\delta W(B_1)}{2} + \frac{\lambda - 1}{2} \\
\Pi (S_1) &= 0 \\
\Pi (S_2) &= 1 - \delta W(B_1)
\end{align*}
\]

6.2.3 \( B_2 \) proposes offers  By analogous arguments it can be seen that, if condition \( \delta W(B_2) \leq \lambda - \delta V(S_2) \) is satisfied, the optimal behaviour by \( B_2 \) is to make the lowest possible acceptable offer, that is a proposal \( \hat{p}_{B_2} = \delta V(S_2) \). Otherwise, if \( \delta W(B_2) > \lambda - \delta V(S_2) \), his optimal strategy is at the contrary to propose any offer strictly below \( \hat{p}_{B_2} = \delta V(S_2) \) such that is going to be rejected. In such a case all the traders access a new round of negotiations, which guarantees to the weak buyer higher payoffs than any acceptable offer.
Whenever $\lambda$ is too low, the weak buyer strictly prefers to stay out of negotiations as paying the seller’s continuation payoff is excessively costly to him. Within a supply-short-side network, if condition $\delta W(B_2) \leq \lambda - \delta V(S_2)$ holds, the unique Acc-PSSPN of the game following a random selection of the weak buyer $B_2$ implies

- in the price-offer phase, $B_2$ always proposing $S_2$ a price $p'_{B_2} = \delta V(S_2)$, and,
- in the response phase, $S_2$ accepting $p'_{B_2}$.

The equilibrium payoffs for the traders are then $\Pi(B_1) = \Pi(S_1) = 0$, $\Pi(B_2) = \lambda - \delta V(S_2)$ and $\Pi(S_2) = \delta V(S_2)$.

Within a supply-short-side network, if condition $\delta W(B_2) > \lambda - \delta V(S_2)$ holds, instead, the unique Unacc-PSSPN of the game following a random selection of the weak buyer $B_2$ implies

- in the price-offer phase, $B_2$ always proposing $S_2$ any unacceptable price $p'_{B_2} < \delta V(S_2)$, and,
- in the response phase, $S_2$ rejecting any such $p'_{B_2}$.

The equilibrium payoffs for the traders are then just their discounted continuation values.

6.2.4 Seller $S_1$ proposes offers As long as some not isolated agents still remain in the market, any trader is selected at any round with an identical probability to make offers. Therefore, in the present supply-short-side market, even the isolated seller $S_1$ has the chance to ask for some price at which, however, she would never be able to trade at. In such a case all agents incur delays in trade with $\frac{1}{4}$ probability, at least any time $S_1$ is selected.

However, for whatever price would be actually announced by seller $S_1$, the final outcome of any $\frac{1}{4}$ selection of $S_1$ can never be anything else but a further round of negotiations all the traders are forced to access. Therefore, whenever the isolated seller $S_1$ is selected to make offers, all the traders just expect their continuation values.

6.2.5 Description of the equilibria Insofar we have provided a full characterization of all the possible subgame-perfect Nash equilibria in pure and stationary strategies which may arise in the negotiation game for any random selection of the trader entitled to make offers. Each equilibrium has been described with a companion set of conditions on the traders’ expected continuation payoffs that restricts the compatible range of the primitive parameters in which such equilibrium is possible.

However, we should now characterize the expected continuation values, by combining each possible equilibrium outcome for any $\frac{1}{4}$ probability selection of the trader making offers. Clearly, each set of expected continuation payoffs is only possible within a particular set of restrictions, namely the ones resulting from the conditions characterizing the equilibrium outcome.
under the selection of $B_2$ and $S_2$ as proposers. As these restrictions hold simultaneously, whenever an expression for a trader’s expected continuation payoff violates any of them, the corresponding combination of equilibrium outcomes following each $\frac{1}{4}$ probability selection can not be viewed as a candidate equilibrium, and should instead be discarded.

Such a repeated process will finally eliminate all the possible equilibria whose expected payoffs are not consistent with any from the compatibility restrictions, thus providing a full characterization of all the possible PSSPN equilibria of the negotiation game within the supply-short-side network as the ones which survive such a feasibility check.

For instance, consider a new round of the negotiation game in which all the traders expect that

- with $\frac{1}{4}$ probability $B_1$ is selected to make offers, in a PSSPN equilibrium, $B_1$ always offers $S_2$ a price $p_{B_1}^* = \delta V(S_2)$, which in the response phase is accepted by $S_2$, thus delivering the following equilibrium payoffs $\Pi(B_1) = 1 - \delta V(S_2)$, $\Pi(S_2) = \delta V(S_2)$, $\Pi(B_2) = \Pi(S_1) = 0$.

- with $\frac{1}{4}$ probability $B_2$ is selected to make offers, and, as condition $\delta W(B_2) < \lambda - \delta V(S_2)$ holds, in the unique Acc-PSSPN equilibrium $B_2$ always offers a price $p_{B_2}^* = \delta V(S_2)$ which, in the response phase is accepted by seller $S_2$. Hence the traders’ equilibrium payoffs are $\Pi(B_1) = \Pi(S_1) = 0$, $\Pi(B_2) = \lambda - \delta V(S_2)$ and $\Pi(S_2) = \delta V(S_2)$.

- with $\frac{1}{4}$ probability $S_1$ is selected to make offers and in a PSSPN equilibrium, for any offer proposed by $S_1$, the expected payoffs for the traders are just their continuation payoffs.

- with $\frac{1}{4}$ probability $S_2$ is selected to make offers, and, as condition $\delta W(B_1) < 1 - \lambda$ is satisfied, then in the PSSPN I-equilibrium $S_2$ offers a price $p_{S_2}^* = 1 - \delta W(B_1)$ which in the response phase is accepted only by $B_1$, while $B_2$ rejects it and then leaves the market without trading. Thus, expected payoffs by the traders from such PSSPN I-equilibrium are $\Pi(B_1) = \delta W(B_1)$, $\Pi(S_2) = 1 - \delta W(B_1)$, $\Pi(B_2) = \Pi(S_1) = 0$.

Therefore, under the conditions obtained from the restrictions above, namely the ones holding when $S_2$ or $B_2$ have been selected to make offers, we can compute the exact expressions for the expected continuation payoffs in a PSSPN equilibrium characterized by the above strategies for any possible random selection of the proposer.

In fact, by taking each above equilibrium payoff as weighted with $\frac{1}{4}$ probability, we can write the expected continuation payoffs of the traders in such PSSPN equilibrium as

\[
\begin{align*}
W(B_1) &= \frac{1}{4} (1 - \delta V(S_2)) + \frac{3}{4} W(B_1) \\
W(B_2) &= \frac{1}{4} (\lambda - \delta V(S_2)) + \frac{3}{4} W(B_2) \\
V(S_1) &= 0 \\
V(S_2) &= \frac{3}{4} \delta V(S_1) + \frac{1}{4} (1 - \delta W(B_1))
\end{align*}
\]
which can be solved as a system, returning the final expressions

\[
\begin{align*}
W(B_1) &= \frac{4(1-\delta)}{5\delta^2-20\delta+16} \\
W(B_2) &= \frac{-35^2-55^2\lambda+14+205\lambda-16\lambda}{5\delta^2-40\delta+96\delta-64} \\
V(S_1) &= 0 \\
V(S_2) &= \frac{4-35}{5\delta^2-20\delta+16}
\end{align*}
\]

that, in the limit case as \( \delta \to 1 \) approach the values \( W(B_1) \to V(S_1) \to 0 \), \( V(S_2) \to 1 \) and \( W(B_2) \to \frac{1}{4} (\lambda - 1) \).

We now need to check whether all the found expressions for the expected continuation payoffs are fully compatible with Condition \( k \) and the above restrictions \( \delta W(B_2) \leq \lambda - \delta V(S_2) \) and \( \delta W(B_1) < 1 - \lambda \). We help our analysis by means of numerical simulations over the primitive parameters of the model, namely, the intertemporal discount rate \( \delta \) and the reservation price of the weak buyer \( \lambda \), both contained by definition within a range \((0, 1)\).

First of all, simulations show that Condition \( k \) is indeed always verified for \( \lambda \) sufficiently low, namely, if, for \( \delta=0.85 \), \( \lambda \) is approximately such that \( \lambda \approx 0.9 \). Simulations also suggest that the strong buyer’s expected continuation payoff is such that also condition \( \delta W(B_1) < 1 - \lambda \) is verified for sufficiently low values of the \( \lambda \) parameter, say, when \( \delta=0.85 \), for approximately any \( \lambda < \frac{\lambda}{0.825} \).

Moreover, the weak buyer’s and seller \( S_2 \)’s payoffs are such that condition \( \delta W(B_2) \leq \lambda - \delta V(S_2) \) is satisfied for values of \( \lambda \) high enough, in particular, for \( \lambda \approx 0.5 \), which is compatible with the previous constraint.

Therefore, for any value of the discount rate \( \delta \in (0, 1) \) and for intermediate values of the weak buyer’s reservation price \( \lambda \), the expected continuation payoffs in the above characterized equilibrium are perfectly consistent with the restriction holding for a PSSPN equilibrium raising whenever \( S_2 \) or \( B_2 \) are selected to make offers. This, in turn, allows us to state Proposition 2.

An analogous line of reasoning lies behind the process of elimination of any combination of strategies inconsistent with the restrictions over the \((\delta, \lambda)\) parameters necessary for their characterization. Indeed, direct calculations supported by numerical simulations show that another equilibrium exists for \( \lambda \) very high, namely, for the complementary case \( \lambda \geq \frac{\lambda}{0.825} \) for \( \delta=0.85 \), as stated in Proposition 3; and that a final equilibrium exists for relatively low values of \( \lambda \), namely, for the complementary case of \( \lambda < \frac{\lambda}{0.5} \) for \( \delta=0.85 \), as stated in Proposition 4.

References