Versioning when customers can buy both versions:

An application to intertemporal movie distribution

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Abstract

We re-consider the decision of a seller who can introduce two versions of a product of different quality to a population with differing valuations of quality. In contrast with the previous literature, we allow for the possibility that consumers buy both versions. This simple extension introduces novel results. It now becomes optimal to introduce both versions even when production costs are zero and preferences are uniformly distributed. The model fits particularly the movie industry, where consumers can both watch a movie in a theatre and a home video. The simultaneous introduction of both versions is also contrasted with their sequential release.

Keywords: Product segmentation; versioning; movie industry.

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1 Introduction

This paper is motivated by theatrical movie distribution. Films are typically first released in a theatre, then followed by its video release. If a video reduces the demand for theatrical exhibition, and vice versa, the content producer will want to release the movie following the principle of the 'second-best alternative'. First, it will distribute the movie in the channel that generates the highest revenues over the least amount of time. Then, the movie cascades in order of revenue contribution, down to markets with lower returns per unit of time (Waterman, 1985; Owen and Wildman, 1992; Vogel, 2001). Historically, this has resulted in theatrical release, followed by pay-TV programming, home video, network television, and finally local television syndication (Eliashberg et al., 2006). Staggered release schedule gives each distribution channel a “window” in which to profit from the movie.

This problem can be seen, in general terms, as one in which a firm offers different versions of the same good, where each version has a different level of quality, as this enables the firm to create factors that segment consumers, thereby inducing them to self-select their preferred product variant. Versioning is therefore a case of second-degree price discrimination (Mussa and Rosen, 1978; see also Sundararajan, 2004a). Examples of versioning abound in many industries, other than the movie industry. Product line pricing for PCs, or for cars, intertemporal discount practices, are possible applications of the idea of versioning. Many more examples can be found for information goods, where movies are just one possible application, alongside with software, music, etc. (Shapiro and Varian, 1998).

The literature has sharpened our understanding of how versioning should be conducted, and in particular whether versioning should be introduced at all. When a firm has a high-quality good, it faces a tension when thinking about the introduction of a lower-quality variant. The tension is between a market expansion effect thanks to the second version, which increases profits, and a cannibalization effect which negatively impacts on profits as some consumers who would have bought the high-quality good can now switch to the low-quality versions. In seminal works, Mussa and Rosen (1978) and Moorthy (1984) find that
versioning is optimal, while Stokey (1979) provides conditions under which (second-degree) price discrimination is not optimal. A key difference in these works stems from the marginal cost function. Salant (1989) reconciles these earlier studies and shows that price discrimination is not optimal if the marginal cost function of improving quality is linear. For example, if the costs of providing quality are of a fixed nature, while the variable production costs are constant or even zero, then under the model specifications of Mussa and Rosen (1978), it would be optimal to supply only the high-quality good. This has important implications especially for information goods, where indeed most of the costs are borne to produce the ‘first’ copy, while replication costs for further copies are negligible or at least not as important as the initial investment costs. Therefore, a single high-quality good should be offered and no versioning should occur (Bhargava and Choudhary, 2001).

The literature has generalized this analysis to different degrees. In recent related works, Bhargava and Choudhary (2008) show that versioning is optimal when the optimal market share of the lower quality version, offered alone, is greater than the optimal market share of the high quality version, offered alone. Anderson and Dana (forthcoming) find that an ‘increasing percentage differences condition’ is needed for versioning to be optimal, that is, the percentage change in total joint surplus (joint between the consumers and the firm) associated with a product upgrade is increasing in consumers’ willingness to pay.

Why, then, another paper on versioning? All the works mentioned above assume that consumers buy at most one unit of a good. This assumption fits well many examples of versioning. A consumer is interested in buying only one car, or one PC, one copy of a certain software, and may decide whether to buy a cheaper lower-quality version, or a more expensive higher-quality version according to her taste, if they are offered by the seller. However, this

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1This result can change in the presence of piracy. Wu and Chen (2008) show that versioning can be an effective and profitable instrument to fight piracy for digital information goods when the piracy costs are within a certain range.

2For a broad and comprehensive review of the literature on product development, see Krishnan and Ulrich (2001).
assumption does not really fit the movie industry we started from. While some consumers may watch a movie only at the theatre, or only at home, we should allow the possibility that some consumers, especially those with a high willingness to pay, may want to watch both versions.

Our contribution to the literature is therefore to analyze a simple problem of versioning, where a seller can sell two different variants to a continuum of consumers with different willingness to pay. Both variants can be produced at the same (constant) marginal cost. Our innovation is that we allow for consumers to buy both variants. This simple change has profound implications. In particular, we employ a framework where, if the single unit purchase assumption is imposed, versioning is never optimal. Instead, when consumers are allowed eventually to buy both versions, we show how versioning becomes optimal, with some consumers buying only the high-quality good, some buying the low-quality good, and some consumers buying both. For this result to hold, the two versions must be not too substitute for each other (in the case of perfect substitution between variants, our model boils down to standard results). Hence, our model predicts that versioning is more profitable than otherwise found by the literature, and therefore should also be observed empirically quite often in practice when consumers can buy both variants, instead of only one.

We then apply our model on versioning to analyze the movie industry, which is characterized by the vertical separation between movie exhibitors and distributors. As copyright holders of movies, distributors sell the video version directly to consumers or commercialize them through video stores, while they have to reach agreements with theatres to exhibit the theatrical version. Under this market configuration, we show that even when the two versions are perfect substitutes distributors may accept to supply the two versions if the quality differential between versions is sufficiently low. The reason is that in this case the vertical separation of the industry prevents the distributor from fully internalizing the profits by selling just one version. Finally, we discuss the conditions under which the distributor and the exhibitor will supply the two versions sequentially.
The rest of the paper is organized as follows. Section 2 reviews the literature and presents relevant stylized facts about the movie industry. Section 3 sets up the model and its main assumptions. Section 4 analyzes the optimal versioning strategy when both versions are introduced by a monopolist, both simultaneously and sequentially. Section 5 re-assesses the main results when the two versions are sold by a distributor and by an exhibitor. Section 6 concludes and offers directions for future research.

2 Literature review and the movie industry

2.1 Literature review

The option of a joint purchase of two versions of a product that we consider in this paper is realistic in the movie industry, but has not been contemplated by the existing theoretical literature.

The extant literature has nevertheless tackled other fundamental questions in the movie industry, such as the decision of sequentially or simultaneously introducing the different versions. Moorthy and Png (1992) use the framework of Mussa and Rosen (1978) to analyze the optimal introduction of a product for a monopoly seller. They demonstrate that with a simultaneous introduction of two products, the lower quality product would cannibalize demand for the higher quality. The authors show that an alternative strategy of the seller is to delay the introduction of the low quality product, although this implies the postponement of profits. The sequential introduction of the products wherein the monopoly first serves the consumers with high preferences and afterwards the consumers with lower preferences might be profitable for the firm when consumers are relatively more impatient than the seller (i.e., when consumers have a higher discount rate). Notice that Moorthy and Png (1992) employ a model with increasing marginal costs of producing each unit of a certain quality, which possibly does not fit particularly well information goods. If marginal costs were constant (or zero in the limit) versioning, and even more so sequential versioning, would not be optimal
in their framework.

Riggins (2004) extends the model of Moorthy and Png (1992) to consider the case where the seller markets its products in two channels simultaneously, the online (Internet) channel and the offline (bricks-and-mortar) channel. He assumes that there is a digital device and, as a result, a different fraction of the low and the high-type consumers migrate to the online channel. Taking this into account, the seller can potentially sell simultaneously high and low-quality versions of the good in both channels, resulting in, at most, four quality versions. The author shows that, even when cannibalization is low, if the digital device is important low types consumers are served only in the offline market, since there will not be enough low-type consumers in the online market to make it profitable for the seller to offer a low-quality good online.

Padmanabhan et al. (1997) analyze the monopoly’s marketing strategy in the presence of demand externalities. They assume that the firm is exogenously endowed with some demand externality and that consumers are uncertain about it. Their analysis demonstrates the relevance of the consumers information about network externalities. Firms earn a higher profit with a one-shot new product introduction strategy when consumers are informed about demand externality. However, sequential introduction of the products is optimal when consumers are not informed about demand externalities. The firm can introduce first a product with less than full quality and afterwards an upgrade to offer a credible signal of high network externalities. For example, a part of a software product can be given away free to signal the attractiveness of the commercially sold version. Underprovision of introductory quality serves as a signal of high externality, and upgrades serves as the mechanism for implementation of the signaling strategy. In this context, when the firm’s product enjoys high potential demand it follows a sequential introduction. But when network externalities are low, the firm offers a product with efficient quality in the first period and does not offer any upgrades in the

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3 In the last decade, online distribution and piracy have affected the strategy of firms for introducing movies and television programs. Wildman (2008) discusses the optimal strategy of content producers for combining distribution through Internet and traditional channels.
The basic model that we propose does not rely on network externalities, signaling, piracy, or differences between the discount factors of the firm and its consumers in order to explain versioning and sequential introduction. In our model with joint purchase, the optimal versioning strategy is always to introduce either one or two products simultaneously, according to the degree of substitution between the two versions. Although this basic model already offers a richer set of circumstances under which versioning will occur than the standard literature on information goods, we also find that sequential introduction is never optimal. This latter result contrasts with the present practice in the movie industry of exhibiting the movies in theatres in advance of their DVD release. In order to explain this stylized fact, we adapt our basic model to take into account the vertical structure of the movie industry, where there is a clear separation between distributors and theater exhibitors.

2.2 The movie industry

Studios hold all rights about movies and control the distribution of theatrical and video versions. The Hollywood major studios (Universal Pictures, Paramount Pictures, MGM, Fox Film Corporation, Columbia Pictures, Disney, and Warner Brothers) usually produce and distribute their movies. They account for 80 to 90 percent of the total receipts from the distribution of movies to theatres and other media in the United States (Waterman, 2005, p. 15). Independent studios perform the same basic functions, although they can subcontract distribution to foreign markets, or certain media to other distributors.

The vertical structure of the movie industry has changed over the last century. Before the late 1940's, all the major owned chains of movie theatres. In the 1940's the five largest studios were Fox, MGM, Paramount, RKO and Warner. Other important studios were Columbia, United Artists and Universal.

4 The literature reviewed above typically considers model with only two periods, and analyzes whether to introduce variants in the first or in the second period. These two periods are however pre-determined. Prasad et al. (2004) are an exception in that they examine the issue of when to introduce a new variant.

5 In the 1940's the five largest studios were Fox, MGM, Paramount, RKO and Warner. Other important studios were Columbia, United Artists and Universal.
(2005) explain that in the 1920’s and 1930’s, some representatives of the majors and theatre owners constituted cartels to control local markets. These associations assigned the run stage, run length intervals, minimum admission price, and geographic and temporal clearances to each theatre in urban areas. In 1948, the US Supreme Court in the *United States v. Paramount* considered that these cartels were violating the Sherman Act and required majors to divest themselves of their theatre chains. The Courts established a number of regulations to allow the entry of independent exhibitors and distributors and to prevent majors intervention in setting admission prices and exclusivity contracts with theatre chains.\(^6\)

The vertical separation between distributors and theatre exhibitors has persisted to the present days. When theatres are interested in a movie they can lease it through two different ways (De Vany, 2004, p. 12). Theatres can bid an amount for the right to show the movie. Distributors send out letters announcing when the film will be available and their requirements for exhibition. Theatres make their bids for each movie and one or several of them are selected. More frequently, however, distributors and theatres can agree to share a percentage of the theatre box-office receipts. Under this alternative arrangement, theatres pay the distributor a fixed fee per week and keep a “house nut” (approximately the exhibitor’s weekly cost of operating the theatre). In addition, contracts include a sliding scale for sharing box office receipts that exceed the house nut. A typical contract for a four-week run might offer distributors a minimum box-office percentage of 70-90 percent in the first two weeks and thereafter distributor’s shares may decrease to 60 per cent. Contracts also establish a minimum number of time for exhibiting the film (usually 4 weeks).

The interests of distributors and exhibitors are not perfectly aligned, and the contractual forms between them are incomplete and give room to tensions. For instance, McKenzie (2008, p. 91) argues that as distributors get an important percentage of the revenues generated by theatrical versions, exhibitors have incentives to keep their tickets prices low in order to raise

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\(^6\)Interestingly, these restrictions only apply to the signatories of the Decree, but Sony and other new entrants are free to own theatres.
their popcorn and other concession prices. This situation creates a constant struggle between
distributors and exhibitors over admission prices, with distributors wanting higher admission
prices than theatres.

After the Paramount decision, distributors have less than perfect control of exhibitors.
Exhibitors enjoy some market power because they may be the only theater in town or because
the movies they show are licensed exclusively to them in their geographic area. As a result,
two mark-ups appear in the industry, the first imposed by a distributor because of its exclusive
rights over the movie and the second by exhibitors because of their monopolistic condition.
Distributors cannot avoid this with resale price maintenance or with vertical integration.

Another important channel for distributors to exhibit movies is the home video. In the
last decade, the home video market has experienced an extraordinary growth with the intro-
duction of DVDs and nowadays video rentals and DVD sales are by far the largest source of
domestic revenue for studios. Mortimer (2007) and Ho et al. (2008) analyze different pricing
mechanisms that distributors use in their contracts with video stores. They show that Block-
busters Video adopted revenue-sharing agreements with several studios in 1998, and quickly
other retailers adopted the same mechanism. These arrangements are not regulated and can
be quite sophisticated. In our model, we abstract from these contractual arrangements be-
tween studios and video stores and we consider that videos are directly commercialized to
consumers.

A key feature of the movie industry is the sequence in which distributors release the
movie in each media. The period of time between the release theatrical and video versions is
called “video window”. Distributors may benefit with quicker video release of movies because
potential consumers are still influenced by the publicity from the theatrical release and because
they delay less their video revenues. However, if consumers expect a short window they may
decide not to go to the theatre and wait for the video version.

Waterman (2005, p. 124) explains that, in the United States, theatre exhibition contracts
never specify a video window explicitly, but that in the past the industry may have cooperated
to control the release sequence of movies. In European countries, the video window coordination problem has been addressed by industry-wide agreements or has even being set by law. Waterman et al. (2007) show that in the mid-1990’s the window in most European countries was set at either 6 or 8 months but it was 12 months in France and Portugal, which have statutory windows. In recent years, the extent of the video window has abruptly decreased, coinciding with the growth of the video market.7

Several authors have empirically analyzed the factors affecting the video window.8 Frank (1999) and Lehmann and Weinberg (2000) show that large windows reduce the cannibalization of the first version, but theatrical marketing and word-of-mouth effects from cinema are not used to increase video sales. Prasad et al. (2004), Waterman et al. (2007) and Luan and Sudhir (2007) consider that movie viewers form expectations about the extent of the video window, and that this leads distributors and exhibitors to coordinate around longer windows. In particular, Waterman et al. (2007) show that distributors that belong to the Motion Picture Association of America have had significantly longer windows on average than non-members of this association. Henning-Thuran et al. (2007) analyze the optimal determination of windows in a market with three or more channels to exhibit movies. They also consider how order changes will affect studios revenues and account for regional differences.

3 The model

We consider a single firm that offers two versions of a product, a high-quality version denoted as $H$ and a low-quality version denoted as $L$. Our departure from the extant literature is that we allow consumers to buy both versions, if they wish to do so. Thus consumers can buy $H$ alone, or $L$ alone, or both versions (we denoted this case as $B$, a mnemonic for ‘both’), or  

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7Between 1970 and 1980 the average window between theatrical exhibition and broadcasting television was nearly 4 years, 5 years at the beginning of the period, and 3 at the end Waterman (2005, p. 54). Bakhshi (2007) shows that the theatrical window has fallen in the UK from around 190 days in 1999 to 125 in 2006.

they can also decide to buy nothing, denoted as 0.

Let $u_i$ denote the quality of product $i = H, L, B$. When both $H$ and $L$ are bought the resulting quality of both versions consumed jointly is

$$u_B = u_H + u_L (1 - s),$$

where $s$ represents the level of substitutability between $H$ and $L$. When $s = 0$ goods are independent, when $0 < s < 1$ the two products are partial substitutes, and when $s = 1$ they are perfect substitutes. Notice that this specification includes the standard case when consumers are limited to a single-unit purchase. In fact, the limiting case $s = 1$ corresponds to the received literature: If a consumer has already bought $H$, buying $L$ confers no additional utility on top, and therefore two versions will never be bought together as $L$ is useless (a fortiori, this would also be the case for $s > 1$, which we do not further consider here as it does not make much economic sense in our context).

The more interesting case is when $0 < s < 1$. Consider, for example, the possibility of watching a movie in a theater (the high-quality version) and/or at home (the low-quality version). The case where $0 < s < 1$ represents the situation where consumers are willing to watch at home a movie that they have already watched in a theater, though the additional benefit they enjoy is not as high as if they were watching the movie at home for the very first time. When $s < 0$, the two products are complements. This situation reflects, for example, the case where consumers obtain more utility from a concert if they have previously listened to the same music in a CD.\(^9\)

Preferences of consumers for the products are heterogeneous. Each consumer is represented by her type $\theta$, which is uniformly distributed over the segment $[0, 1]$. The surplus of a consumers that buys a product of quality $u_i$ at price $p_i$ is given by $\theta u_i - p_i$.

The two products are sold separately. Thus, when a consumer buys both versions, her net surplus is $\theta u_B - p_H - p_L$. This is the more interesting and realistic case for the movie industry,

\(^9\)Luan and Sudhir (2007) find that, on average, a consumer’s utility from a DVD would be reduced after having viewed the movie in a theater.
especially when products are sold sequentially and separately by exhibitors and distributors (analyzed in Section 5). In other markets, the seller can consider the possibility of bundling the two versions, but in the movie industry this would be difficult to enforce. However, in the proof of Proposition 1 we also consider briefly the case where the firm bundles the two versions.

We can now illustrate how the market is split between the two versions. Define $\theta_{ij}$ as the consumer that is indifferent between buying good $i$ and $j$, where $i = \{H, L, B, 0\}$ and $j \neq i$, at a price $p_i = \{p_H, p_L, p_H + p_L, 0\}$ respectively. We thus obtain that $\theta_{LH} = (p_H - p_L)/(u_H - u_L)$ is the consumer that is indifferent between buying the low and the high quality version separately, $\theta_{HB} = p_L/(u_B - u_H) = p_L/(u_L(1 - s))$ is the consumer that is indifferent between buying the high quality version and both versions, and $\theta_{io} = p_i/u_i$ is the consumer that is indifferent between buying product $i$ alone, and not buying anything. Finally, we define as $\theta_{BO} = (p_L + p_H)/u_B$ the consumer that is indifferent between buying both versions and not buying anything. Different market shares for the two products can arise according to the relative magnitude of the various indifferent consumers $\theta_{ij}$.

The quality of the two versions is given. The firm has a constant marginal cost of supplying each variety of the product, denoted as $c \geq 0$. The case $c = 0$, as in the rest of the literature, corresponds to pure information goods that can come in different versions.

4 Simultaneous and sequential introduction of the two versions

We start with a single-period analysis. The monopolist sells both versions simultaneously in one period. In section 4.2 we consider the possibility of the sequential introduction of the two versions.
4.1 Simultaneous product introduction

In order to show the main mechanism at work, we simplify the analysis here by considering first the simple case where $0 \leq s \leq 1$ and $c = 0$. That is, we consider the case of pure information goods that can be (partial) substitutes. The quality of versions $H$ and $L$ is exogenously given and the firm cannot modify it. The firm then sets the prices to maximize its profits, anticipating the way consumers decide to purchase one particular version, both, or none.

The following proposition describes the optimal strategy.

**Proposition 1** Imagine $c = 0$. When $0 < s \leq 2/3$, the firm offers the pattern $L/H/B$ at prices $p_L = \frac{(1-s)u_L}{2-s} + s(\frac{u_L}{2})$ and $p_H = \frac{u_H}{2(2-s)}$. When $2/3 < s \leq 1$, it offers the pattern $H$ at a price $p_H = \frac{u_H}{2}$.

**Proof.** See the Appendix.

As already anticipated, for $s = 1$, our model corresponds to the standard case of information goods analyzed, e.g., by Bhargava and Choudhary (2001). In line with the received literature, the cannibalization effect of introducing the lower quality variant always prevails over the market expansion effect, and the monopolist supplier is better off by supplying only the high-quality version. Indeed, we find that this result extends to allowing the possibility of buying both versions, as long as the degree of substitutability is sufficiently high ($s > 2/3$).

When the level of substitutability is low, however, the firm finds it profitable to offer both variants, and consumers self select the variant(s) that maximize individual utility. The pattern that emerges is $L/H/B$ (see Figure 1), according to types’ preferences: Consumers with a very low $\theta$ buy nothing, those with a low $\theta$ buy only $L$, those with an intermediate $\theta$ buy $H$, and those with a high $\theta$ buy both versions, $H$ and $L$.

This market segmentation is novel to the literature and is sustainable for a wide range of values of the degree of substitutability, as long as this is not too high. As said above, when $s$ is high the model boils down to a standard model with zero production costs and uniform
distribution of types, making it optimal to sell only $H$. The segmentation also disappears when products are completely independent ($s = 0$): In this case it is optimal to sell both versions at a price $(u_L + u_H)/2$ to every buyer.

The fact that, at $s = 0$, there is the pattern $B$ depends on having assumed zero marginal costs. To see why, imagine products are independent ($s = 0$), but the marginal cost is now $c$ for both versions. Since products are independent, they are both charged the monopoly price $(c + u_i)/2, i = H, L$. The consumer indifferent between buying product $i$ and not buying at all is $	heta_{i0} = p_i/u_i = 1/2 + c/(2u_i)$. Since $u_H > u_L$ it follows that $\theta_{H0} < \theta_{L0}$ (these indifferent types coincide only for $c = 0$). Thus, at $s = 0$, there must be now the following pattern: $H/B$. Consumers with a very low type buy nothing, those with $\theta_{H0} < \theta < \theta_{L0} \equiv \theta_{HB}$ buy only $H$, and those with $\theta \geq \theta_{L0} \equiv \theta_{HB}$ buy both versions.

The following proposition makes this reasoning more precise as it generalizes the previous model to the general case where $c \geq 0$ and $s \leq 1$.

**Proposition 2** Imagine $c > 0$. The optimal segmentation strategy and the corresponding profit maximizing prices take different values according to the degree of substitution $s$ between versions:
• When \( s^1 < s \leq 1 \), the firm supplies only the \( H \) version at a price \( p_H = (c + u_H)/2 \);

• When \( s^2 \leq s \leq s^3 \), the firm supplies the pattern \( L/H/B \) at prices
  \[
  p_H = \frac{c + u_H}{2} - \frac{su_L}{2(2-s)},
  \]
  and \( p_L = \frac{c}{2} + \frac{u_L(1-s)}{(2-s)} \);

• When \( s \leq s^1 \leq s^2 \), the firm supplies the pattern \( H/B \) at prices
  \[
  p_H = u_H - \frac{u_H(c(2-s) + su_L)}{2[u_L + u_H(1-s)]},
  \]
  and \( p_L = p_H u_L / u_H \);

• When \( s^3 \leq s \leq s^4 \), the firm supplies the pattern \( H/B \) at prices
  \[
  p_H = \frac{c + u_H}{2},
  \]
  and \( p_L = \frac{c + u_L(1-s)}{2} \);

• When \( s \leq s^3 \), the firm supplies the pattern \( B \) at prices
  \[
  p_H + p_L = (c + u_B)/2;
  \]

where the cut-off points are

\[
\begin{align*}
  s^1 &= \frac{u_L(3c^2 + 6cu_H - 5u_Lu_H + (c^2 + 4cu_H + (u_L - 6u_H)u_H + (4c + u_L^2)u_H^2)^{1/2}) - 4u_Hc^2}{u_L(2c^2 + u_H(4c - 6u_L)) - u_H2c^2}, \\
  s^2 &= \frac{2c}{c + u_L}, \\
  s^3 &= 1 - \frac{u_H}{u_L} < 0,
\end{align*}
\]

with \( s^1 > s^2 > s^3 \) when \( c \) is not too large.

**Proof.** See the Appendix. ■

The previous proposition shows that, for \( s^1 < s \leq 1 \), there is a region \( A \) where only the version \( H \) is provided. For \( s^2 < s \leq s^1 \) there is a region \( B \) where the pattern \( L/H/B \) is offered. The mechanisms at work in these two regions are the same ones as those described in Proposition 1. For \( s^3 \leq s < s^2 \) there is a region \( C \) where the pattern \( H/B \) is offered. Finally, for \( s \leq s^3 \) there is a region \( D \) where only \( B \) is supplied. (See Figure 2.)

The existence and the size of these regions depend on the value of \( c \). When \( c = 0 \), \( s^2 = 0 \) and region \( C \) disappears. As a result, for \( 0 \leq s < s^1 = 2/3 \) there is the pattern \( L/H/B \), as already found in Proposition 1. While Proposition 2 concentrates on the case when \( c \) is not too high, when \( c \) is high, it would be that \( s^2 < s^1 \). As a result, region \( B \) would disappear and for \( s^2 \leq s < 1 \) only service \( H \) would be provided.

(We need to discuss the role of \( c \) and the probability of introducing more versions.)
4.2 Sequential product introduction

We now consider the possibility of introducing two versions sequentially. With the sequential introduction, consumers might still choose among $i = L, H, B$. Following Moorthy and Png (1992), we consider that $H$ is offered in the first period and $L$ is available for purchase only in the second period. We assume that the firm and the consumers have the same discount factor $\delta \in (0, 1)$. Hence, $\theta_{ LH} = (p_H - \delta p_L)/(u_H - \delta u_L)$ is the consumer that is indifferent between buying separately either the high quality version in the first period or the low quality version in the second period. As before, $\theta_{ HB} = p_L/(u_L(1 - s))$ is the consumer indifferent between buying only $H$ or both version sequentially, $\theta_{ HO} = p_H/u_H$ is the consumer indifferent between buying $H$ and not buying anything, and $\theta_{ LO} = p_L/u_L$ is the consumer that is indifferent between buying $L$ in the second period and not buying anything.

To facilitate the presentation of the results we assume again that $0 \leq s \leq 1$ and $c = 0$, and that the qualities of the two products are exogenously given. The following proposition describes the optimal strategy of the firm, when the monopolist commits to its subsequent
Proposition 3 Assume that the firm commits in advance to its pricing strategy and that it has the same discount factor as consumers. Sequential versioning never arises. In particular, when \( \frac{2}{3} \leq s \leq 1 \) the firm only provides \( H \) in the first period. When \( 0 \leq s \leq \frac{2}{3} \), it provides the pattern \( L/H/B \) simultaneously in the first period.

The main finding is that the firm never introduces the products sequentially because the loss generated by the postponement of profits cannot be compensated by an improvement in extracting information rents.

In Moorthy and Png (1992) sequential introduction can arise due to the difference of discount factors between the seller an the buyer, which we assume to be the same here. On the other hand, in Padmanabhan et al. (1997) sequential introduction is offered to create a signal on the uninformed customers that the product had some demand externalities. Afterward, the seller offers an upgraded version.\(^{10}\) While these are interesting reasons for justifying sequential offers, they do not appear to fit particularly well the movie industry. Instead, we propose next a new and simple explanation which relies on its vertical structure.

5 Sequencing and the vertical structure of the movie industry

This section applies our basic model of versioning to the vertical structure of the movie industry. To this end, imagine there is a distributor of a movie (which could be also the producer) that holds all the rights over the movie, and also decides whether to release the movie in theaters and/or through DVD stores. Following the notation of Section 3, we call \( H \) the movie exhibited in theaters (high quality version) and \( L \) the movie viewed with a

\(^{10}\)Levinthal and Purohit (1989) analyze a situation where the current version of the product loses its value due to obsolescence. In this case, the firm faces a tradeoff between the cost of waiting for new products sales and the cost of cannibalizing these sales.
DVD (low quality version). For simplicity, we assume that the distributor directly sells $L$ to consumers.\footnote{Equivalently, the distributor is able to write efficient complete contracts with DVD video stores. Mortimer (2007) and Ho et al. (2008) discuss at some length how some majors have established sophisticated agreements with stores.} By contrast, $H$ is exhibited in theaters. Reflecting the situation of the US movie industry after the Paramount decision, we assume that theaters and distributors are vertically separated, and the distributor sets an access charge to theaters.

We are therefore now studying a situation where versioning interacts with the vertical structure. Theaters have some market power that can be used when they negotiate the contract of each movie with distributors. The distributor cannot fully appropriate the revenues associated with the theatrical version, for which an independent movie theatre is needed. The lack of complete internalization of the effect of $H$ introduces important changes, both to the optimal versioning strategy and to sequencing, compared to the basic full monopoly case.

The game played has the following timing. First, the distributor decides whether or not to allow the exhibitor to show the movie. If it does not, $L$ only is sold by the distributor in the first period, $t_0$, and the game ends. If instead it does, then the theatre releases $H$ in $t_0$ and the distributor either simultaneously offers $L$, or waits until a later period, $t_1$, to sell it. We denote as $d$ the discount factor for $t_1$. If $t_0 = t_1$, then $d = 1$ and $H$ and $L$ are supplied simultaneously. If $d < 1$, this means that $L$ is introduced some time after $H$. That is, the DVD is released some months after the exhibition of the movie in theaters. We call “video window” the period of time that separates $t_0$ and $t_1$, and this period can be measured by $d = t_1 - t_0$. The sequencing decision and the length of the video window is decided jointly by the distributor and the exhibitor (more on this below). The smaller is $d$ the longer is the lapse of time taken to introduce $L$.\footnote{Using the notation from Section 4.2, if $\delta$ is the per-period discount factor, then $d = \delta^{t_1-t_0}$. In Section 4.2, we allowed only for a one-period sequencing, i.e., we simply set $t_1 = t_0 + 1$.}

Consumer preferences are the same as in Section 4. Thus the surplus of a consumer that buys $H$ only is $\theta u_H - p_H$, and if she buys $L$ only obtains a surplus $d(\theta u_L - p_L)$. If a consumer
buys both versions, her surplus is \( \theta u_B - p_H - dp_L \), where \( u_B = u_H + du_L(1 - s) \) and \( s \) is the level of substitutability between \( H \) and \( L \).

When the distributor decides to release the two versions, either simultaneously or sequentially, then it sets a linear rental price \( a \) to the exhibitor and, after that, the exhibitor and the distributor independently set the retail prices \( p_H \) and \( p_L \) respectively.

Next, we present the optimal strategies for the firms. In order to keep the model as simple as possible, in what follows we assume that the marginal production costs of either version are zero for the distributor, \( c = 0 \). Notice that we consider that the distributor sets a linear access charge for each ticket sold by the theater exhibitor. Linear access charges are the simplest way of accounting for the problem of imperfect vertical control and less-than-full extraction of rents that distributors face in the industry. At the end of this section we also briefly discuss the case where the distributor charges a rental price that is a percentage of the theatre box-office receipts.

**Proposition 4** Imagine that the distributor charges a linear rental price \( a \) to the theater exhibitor. The profit maximizing prices and the distributor’s optimal segmentation strategy are the following:

- When \( s = 0 \), the pattern offered is \( L/B \) at prices \( p_L = \frac{u_L}{2} \), \( p_H = \frac{3u_H}{4} \) and \( a = \frac{u_H}{2} \);
- When \( s = 1 \) and \( u_L < u_L^1 \), the pattern offered is \( L/H \) at prices \( p_L = \frac{u_L(10u_H - du_L)}{2(du_L + 8u_H)} \), \( p_H = \frac{12u_H^2 - 2du_Lu_H - d^2u_L^2}{2(du_L + 8u_H)} \), and \( a = \frac{d^2u_L^2 + 8u_H^2}{2(du_L + 8u_H)} \);
- When \( s = 1 \) and \( u_L \geq u_L^1 \), the pattern offered is \( L \) and the retail price is \( p_L = \frac{u_L}{2} \);

where the cut-off point is

\[
    u_L^1 = \frac{2[d - 2 + (4 - 3d)^{1/2}]}{(1 - d)d} u_H.
\]

**Proof.** See the Appendix.

When \( s = 0 \), versions are independent and there is no cannibalization. The distributor thus commercializes the movie through both theaters and DVD stores. It charges theater ex-
hibitors and DVD’s consumers the monopoly price, and some consumers buy the two versions. Because of the double mark up imposed by movie theaters, the price $p_H$ is now particularly high, which explains why the pattern $L/B$ emerges now, instead of the pattern $B$ that we found in Proposition 1 for $s = 0$, or the pattern $H/B$ that we found in Proposition 2 for $s = 0$ and $c > 0$ (for both versions).

When $s = 1$, if the utility generated by a DVD is close enough to the utility of the theatrical version, the distributor does not segment the marked and just offers the video version. This case reflects the situation where the rental price obtained from the theater does not compensate the cannibalization from $H$ and the distributor prefers to sell alone the lower quality version.

On the other hand, when $s = 1$ the distributor still finds it profitable to segment the market if $u_L$ is low, i.e., the utility generated by the DVD is sufficiently low. Contrary to our result in Proposition 1, the vertical separation structure of the industry does not allow the distributor to fully internalize the profits by selling just the $H$ version. By contrast, it now offers both versions and uses the rental price charged to the theatre to extract part of its revenues. This case is also interesting as the preferences of the two agents over sequencing diverge.

**Proposition 5** The level of substitutability between the versions determines the preference of firms over the video window:

- When $s = 0$, the distributor and the theater agree on introducing the two versions simultaneously;
- When $s = 1$ and $u_L < u_L^1$, the exhibitor is interested in delaying the introduction of $L$ and the distributor prefers releasing the two versions simultaneously.

**Proof.** See the Appendix. ■

When the products are independent ($s = 0$), the two firms are interested in introducing the two products as soon as possible. Clearly, the introduction of $L$ at $t_0$ does not cannibalize
By contrast, when the products are perfect substitutes \((s = 1)\), the theater exhibitor obtains more profits with a long video window that reduces the cannibalization of \(L\) over \(H\). However, the distributor wants just the opposite because the increase of rental revenues from the theatre by a wide video window does not compensate the revenues loss from delaying the introduction of \(L\). In this situation, only if the bargaining power of the theater is high enough windowing may arise.

The previous Proposition points to the tension arising over sequencing. Some factors that affect bargaining power are the presence of other theaters in the region, the affiliation to an association of exhibitors or the repeated interactions between the parties. We stress that, in order to obtain the sequential introduction of the two versions in our setting, we need quite crucially both the vertical structure and incomplete contracts. In fact, the possibility of versioning arises only when \(s\) is high enough. From Section 4 we know that when \(s\) is high, if versions were controlled by a single monopolist (or if the distributor and the exhibitor could write perfectly efficient contracts to replicate the monopolist’s preferred solution) only the high-quality version would be introduced, and thus there would be no room for versioning at all. Instead, as shown by Proposition 4, when there is a vertical structure, when \(s\) is high the two versions can be introduced and firms can agree on establishing a video window. These results appear in our setting without externalities and with identical discount factors between the firms and customers.

For simplicity we have derived our results only in the extreme cases of perfectly separate or perfectly substitutable versions. More in general, it is possible to identify a threshold value of \(s\) such that, for values of \(s\) above this threshold, the exhibitor prefers sequencing, in contrast with the distributor.\(^{13}\)

As a last set of results, we consider in some further details the video window decision and the impact its length has on the rental price when sequencing occur. In particular, we now assume that, once the distributor has decided to allow the exhibitor to show the movie,
the decision on sequencing is taken jointly. We use the Nash axiomatic bargaining approach, which allows us to obtain the following

**Proposition** When the distributor decides to introduce both versions, imagine the distributor and the exhibitor bargain over the length of the video window. Then, in order for a window to exist, the degree of bargaining power of the exhibitor must be high enough. The length of the window also depends on the ratio $u_H/u_L$ in a non-linear way. The linear rental price set by the distributor increases with the length of the video window if $u_L$ is sufficiently low.

**Proof.** See the Appendix. ■

With a large video window ($d$ small), version $L$ becomes less profitable, especially when $u_L$ is low. In this context, as the window becomes even larger, the distributor increases the rental price to increase the profits obtained through $H$. However, the access charge is in more general not monotonic in $d$. When $u_L$ is high and the video window is short, the reverse will arise.

Linear rental prices are arguably a very simple type of wholesale contract, though this assumption captures the imperfect rent extraction that the distributor faces with respect to theatres. We also considered alternative contracts, in particular we studied the case where the distributor receives a fixed percentage for each ticket receipt. Intuitively, we found that when the two versions are perfect substitutes, if the percentage that accrues to the distributor is high and the video window is short, the closer is the versioning strategy of the distributor to that characterized by Proposition 1. However, if the video window is long enough, the distributor prefers to introduce the two versions. If instead the percentage is very low, only version $L$ is sold.

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14 Results are available from the authors.
6 Conclusions

The main result of this paper is that versioning can happen even for information goods with zero marginal costs and simple uniform distribution of types when consumers are able to buy the two versions of the product. The key parameter for this finding is the degree of substitutability between versions.

This result brings to an empirical question: To what extent are theatrical and non-theatrical consumer segments overlapping. If DVDs deter people from going to the theater, then versioning is less likely. However, consumers can enjoy consuming the same information goods or cultural products several times and using different versions, because the utility they derive is not lost with repetition, or because consumption of different versions allow them to appreciate different aspects of the same product. If this is the case, theatrical movies and DVDs can be partial substitutes, or even complements, and versioning should occur more often than otherwise found in the literature.

More analysis is needed for understanding how spillovers between versions (e.g., marketing campaigns) and word-of-mouth communications affect the commercialization strategy of movie distributors, in particular with respect to the decision to introduce variants sequentially or simultaneously. The possibility of delaying the introduction of a version of the product still many raises questions both at a theoretical and at an empirical level. Waterman et al. (2007) find how the “video window” has been quite stable (around 180 days) between 1988 and 1997, but it has been falling steadily since then. The authors also report that in more recent years, some producers and distributors announced a series of experiments with simultaneous theaters, video and pay-per-view television release. Our model predicts this trend, either when markets are not subject to cannibalization, or when distributors have stronger bargaining power than exhibitors when deciding on the length of the video window.

Piracy could also be analyzed in our simple framework. Pirate copies are themselves a different variant (Sundararajan, 2004b). They should be a very close substitute for DVDs, but a very poor substitute for the theatrical experience.
Finally, in this paper we mainly considered a monopoly setting. Introducing competition among content producers is also an essential next step we envisage in our future research.
7 Annex

Proof of Proposition 1. The firm’s problem when it only offers H is to set the price $p_H$ that maximizes $\pi_H = p_H(1 - \theta_{H0})$. The price that solves this problem is $p_H = u_H/2$ and the firm obtains $\pi_H = u_H/4$.

Next consider the case where the firm offers L to the low segment of consumers, H to the intermediate segment, and L and H to the high segment. It then sets $p_H$ and $p_L$ to maximize:

$$\pi_{LBH} = (p_H + p_L)(1 - \theta_{BH}) + p_H(\theta_{HB} - \theta_{LH}) + p_L(\theta_{LH} - \theta_{L0}).$$

(1)

Solving this problem, we obtain the following prices:

$$p_L = \frac{(1-s)u_L}{2-s} \quad p_H = \frac{2u_H - s(u_L + u_H)}{2(2-s)},$$

and the firm’s profits are:

$$\pi_{LBH} = \frac{u_H}{4} + \frac{(2-3s)u_L}{4(2-s)}.$$ 

(2)

It is simple to verify that $\pi_H < \pi_{LBH}$ for $s < 2/3$. In this range, it also immediate to confirm that at the equilibrium prices, $\theta_{BH} = \frac{1}{2-s} > \theta_{LH} = \frac{1}{2} > \theta_{L0} = \frac{1-s}{2-s}$.

Imagine now that the firm is able to bundle together the two products. In this case, the firm’s problem is to set the price $p_B$ that maximizes $\pi_B = p_B(1 - \theta_{B0})$. Solving this problem we obtain that the optimal price is $p_B = (u_H + u_L(1-s))/2$ and the firm’s profits are $\pi_B = (u_H + u_L(1-s))/4$. Finally, observe that $\pi_B > \pi_{LBH}$ and $\pi_B > \pi_H$ for any value of $s$. Thus, if feasible, the firm would bundle the two versions together. Q.E.D.

Proof of Proposition 2. Following the same steps as in Proposition 1, the firm’s expression for the profit when it only offers H and $c > 0$ is $\pi_H = (p_H - c)(1 - \theta_{H0})$. Maximizing this with respect to the price we obtain $p_H = (c + u_H)/2$ and the corresponding profit $\pi_H = (u_H - c)^2/(4u_H)$. Consumers between $\theta_{H0} = 1/2 + c/(2u_H)$ and 1 buy only H, and the others buy nothing. We call this region A.

When the firm offers the pattern L/H/B, the prices and the associate profits are:
\[ p_H = \frac{c + u_H}{2} - \frac{su_L}{2(2-s)}; \quad p_L = \frac{c}{2} + \frac{u_L(1-s)}{(2-s)}, \]  
\[ \pi_{LHB} = \frac{u_H}{4} + \frac{(2-3s)u_L}{4(2-s)} + \frac{c[c(2-s) - 4(1-s)u_L]}{4(1-s)u_L}. \]  

It can be verified that the value of \( s \) that equals \( \pi_H \) and \( \pi_{LHB} \) is
\[ s^1 = \frac{u_H(3c^2 + 6cu_H - 5u_Lu_H + (c^2 + 4cu_H + (u_L - 6u_H)u_H + (4cu_L + u_H^2)u_H^{1/2}) - 4u_HC^2}{u_L(2c^2 + u_H(4c - 6u_L))} - u_H2c^2 \]

Thus, as long as \( \theta_{HB} = \frac{1}{2} \theta_{LH} = \frac{1}{2} > \theta_{LO} = \frac{1}{2} + \frac{c}{u_L}, \) then (3) is a candidate solution. We call this region B. This solution is valid until \( s \) is not too low.

When \( s = s^2 = \frac{2c}{c+u_L} \), then at the prices given by (3) it is the case that \( \theta_{LO} = \theta_{H0} = \theta_{LH} \). This implies that for \( s < s^2 \), if the prices are as in (3), the first marginal buyers will choose \( H \) instead of \( L \), since now \( \theta_{H0} < \theta_{LO} \), and no one customer buys \( L \) alone. When this occurs, the firm will choose \( p_H \) and \( p_L \) to maximize:
\[ \pi_{HB} = (p_H + p_L - 2c)(1 - \theta_{BH}) + (p_H - c)(\theta_{HB} - \theta_{H0}). \]  

In this case, the prices and the associated profit would be:
\[ p_H = \frac{c + u_H}{2}; \quad p_L = \frac{c + u_L(1-s)}{2}, \]
\[ \pi_{HB} = \frac{u_H + u_L(1-s)}{4} - c + \frac{c^2[(1-s)u_L + u_H]}{4(1-s)u_Hu_L}. \]

In this region, that we call \( C \), the pattern is \( H/B \) and the indifferent types are \( \theta_{HB} = \frac{1}{2} + \frac{c}{2u_L(1-s)}; \theta_{H0} = \frac{1}{2} + \frac{c}{2u_L} \). This solution holds as long as \( \theta_{HB} \geq \theta_{H0} \), which results in \( s \geq s^3 = 1 - \frac{u_H}{u_L} < 0 \). Notice, however, that with these prices some consumers may want to buy \( L \) instead of \( H \). Thus we also have to check that \( \theta_{LO}u_L - p_L \leq 0 \) at the prices given by (7). This is satisfied when \( s \leq \hat{s} = c(1/u_L - 1/u_H) < s^2 \), and (7) is the solution for \( s^3 \leq s \leq \hat{s} \).

When \( \hat{s} \leq s \leq s^2 \), we are still in region \( C \) as the segmentation pattern is \( H/B \), but the prices take a different expression. In particular, the firm sets \( p_L = p_H(u_L/u_H) \) in order to
make sure that that \( \theta_{L0} = \theta_{LH} \). The couple of prices that satisfy this condition and maximize (6) are

\[
\begin{align*}
p_H &= \frac{u_H}{2} - \frac{u_H[c(s - 2) + s u_L]}{2[u_L + u_H(1 - s)]}, \quad p_L = p_H(u_L/u_H). \\
\end{align*}
\]

(8)

In this part of region C the indifferent types are \( \theta_{HB} = \frac{1}{2} + \frac{c(2 - s) - su_L}{u_L + u_H(1 - s)} \), \( \theta_{H0} = \frac{1}{2} + \frac{c(2 - s)}{u_L + u_H(1 - s)} \).

Finally, when \( s \) is very negative (strong complementarities), all consumers who buy prefer to buy \( B \). For \( s < s^3 \) the firm maximizes the following profit \( \pi_B = (p_H + p_L - 2c)(1 - \theta_{B0}) \). The optimal price is \( p_H + p_L = c + u_B/2 \) and the firm’s associate profit is \( \pi_B = (u_B - 2c)/(4u_B) \), where \( u_B = u_H + u_L(1 - s) \). The indifferent type is \( \theta_{B0} = \frac{1}{2} + \frac{c}{u_B} \). This is region \( D \).

What remains to be shown is that the four regions are non-empty. We have already shown that \( s^2 > s > s^3 \). We must therefore only discuss when \( 1 > s^1 > s^2 \). The expression for \( s^1 \) given by (5) is a bit cumbersome, but it can be shown to decrease in \( c \) in the relevant range. Thus it takes a maximum when \( c = 0 \), in which case it simplifies to \( s^1 = 2/3 < 1 \). Secondly, still at \( c = 0 \), \( s^2 = \frac{2c}{c + u_L} \) simplifies to \( s^2 = 0 < s^3 \). By continuity, the four regions always exist for sufficiently low levels of \( c \). As \( c \) increases, region \( B \) shrinks, until it disappears when \( s^1 = s^2 \).\(^{15}\) Also notice that, in order for the problem to make economic sense, \( c < u_L \), thus \( s^2 \) is always bounded below 1, and regions \( A, C, D \) are always nonempty. Q.E.D.

**Proof of Proposition 3.** The expression for the firm’s profit when it offers the products sequentially is

\[
\pi_{SEQ} = (p_H + \delta p_L)(1 - \theta_{HB}) + p_H(\theta_{HB} - \theta_{LH}) + \delta p_L(\theta_{LH} - \theta_{L0}).
\]

(9)

Solving this, we obtain that the optimal prices are

\[
\begin{align*}
p_H &= \frac{u_H}{2} - \frac{\delta su_L}{2(2 - s)}, \quad p_L = \frac{(1 - s)u_L}{2 - s}.
\end{align*}
\]

and the firm’s profits are

\[
\pi_{SEQ} = \frac{u_H}{4} - \frac{\delta(3s - 2)u_L}{4(2 - s)}.
\]

\(^{15}\) As a numerical example, when \( u_H = 1 \) and \( u_L = 0.6 \), region \( B \) exists as long as \( c < 0.18 \).
From equation (2) recall that the firm’s profit with simultaneous introduction of the versions when $0 \leq s \leq 2/3$ is 

$$\pi_{LHB} = \frac{u_H}{4} + \frac{(2-3s)u_L}{4(2-s)}.$$ 

It is thus $\pi_{SEQ} < \pi_{LHB}$ for all $\delta < 1$.

On the other hand, recall that the profit of the firm when it only offers $H$ in the first period is $\pi_H = u_H/4$. Thus $\pi_H > \pi_{SEQ}$ for $s > 2/3$. Q.E.D.

**Proof of Proposition 4.** Consumers are potentially split between different combinations of products. $\theta_{LH} = (p_H - dp_L)/(u_H - du_L)$ is the consumer that is indifferent between watching the movie through a DVD and watching it in a theater, $\theta_{HB} = dp_L/(u_B - u_H)$ is the consumer that is indifferent between watching the movie in a theater and also buying a DVD on top, and $\theta_{LB} = p_H/(u_B - du_L)$ is the consumer that is indifferent between buying a DVD and also going to the theater in addition. Moreover, $\theta_{i0} = p_i/u_i$ is the consumer that is indifferent between buying the version $i = \{H, L\}$ at the price $p_i = \{p_H, p_L\}$ and not buying anything. Finally, $\theta_{B0} = (p_L + p_H)/u_B$ is the consumer that is indifferent between buying both versions and not buying anything.

When $s = 0$ and the pattern offered by firms is $L/B$, the problem of the exhibitor is max

$$\pi_{LB}^e = (p_H - a)(1 - \theta_{LB})$$

and the problem of the distributor is max $\pi_{LB}^d = dp_L(1 - \theta_{L0}) + a(1 - \theta_{LB})$. Computing the Nash equilibrium in prices and, after substituting them in the distributor’s profits, yields

$$\pi_{LB}^d = \frac{2au_H + du_Lu_H - 2a^2}{4u_H}.$$ 

The access charge that maximizes this function is $a = u_H/2$ and with this result we can obtain the retail prices of the proposition. Finally, the profits of the two firms are:

$$\pi_{LB}^e = \frac{u_H + 2du_L}{8}; \quad \pi_{LB}^d = \frac{u_H + 2du_L}{8}.$$  \hspace{1cm} (10)

It remains to be shown that $L/B$ is indeed the pattern that maximizes the distributor’s profits. As goods are independent ($s = 0$) it is clear that these are the maximum profits that the distributor can obtain with a linear access charge.

Next, consider that $s = 1$ and firms offer the pattern $L/H$. The exhibitor sets $p_H$...
to maximize $\pi_{LH} = (p_H - a)(1 - \theta_{LH})$ and the distributor sets $p_L$ to maximize $\pi_{dLH}^d = dp_L(\theta_{LH} - \theta_{L0}) + a(1 - \theta_{LH})$. Solving these problems and substituting the prices in the distributor’s profit we obtain:

$$\pi_{dLH}^d = \frac{(a - u_H)[du_L(du_L - u_H) - a(du_L + 8u_H)]}{(du_L + 4u_H)^2}.$$  

From this function we obtain the profit-maximizing access charge reported in the Proposition, and likewise we compute the retail prices after substitution. Finally, the profits of the firms are:

$$\pi_{LH}^e = \frac{(u_H - du_L)(2u_H + du_L)^2}{(du_L + 8u_H)^2}; \quad \pi_{LH}^d = \frac{(2u_H + du_L)^2}{4(du_L + 8u_H)}.$$  

(11)

It remains to be demonstrated that this pattern $L/H$ offers the distributor more profits than the alternative patterns $L/H/B$, $L$, or $H$. To see this, observe that with the pattern $L$ the distributor sets the price $p_L = u_L/2$ and obtain the profit $\pi_{L}^d = u_L/4$. It can be shown that $\pi_{L}^d > \pi_{LH}^d$ when $u_L > u_L^1$, where

$$u_L^1 = \frac{2(4 - 3d)^{1/2}}{(1 - d)d}u_H < u_H.$$  

When only the pattern $H$ is offered, the distributor sets the access charge $a = u_H/2$, the theater sets the price $p_H = 3u_H/4$, and the distributor obtains profits $\pi_{H}^d = u_H/8$. These profits are lower than those obtained in (11) for any positive value of $d$.

Finally, observe that for $s = 1$ it is satisfied that $u_B = u_H$. Therefore, the pattern $L/H/B$ can not generate more profits than $L/H$. Q.E.D.

Proof of Proposition 5. Assume that $s = 1$. From $\pi_{LH}^e$ and $\pi_{LH}^d$ in (11) we obtain:

$$\frac{\partial \pi_{LH}^d}{\partial d} = \frac{u_L(du_L + 2u_H)(du_L + 14u_H)}{4(du_L + 8u_H)^2} > 0.$$  

\[16\text{In this case the distributor will always sell } L \text{ at the earlies possible date.}\]

\[17\text{The factor } \frac{2[4 - 2(d - 3d)^{1/2}]}{(1 - d)d} \text{ takes values in } (1/2, 1) \text{ for } 0 < d < 1.\]
\[
\frac{\partial \pi_L^e}{\partial d} = -\frac{u_L(du_L + 2u_H)(d^2u_L^2 + 22u_Lu_H + 4u_H^2)}{(du_L + 8u_H)^3} < 0.
\]

This establishes the result of the proposition. Q.E.D.

**Proof of Proposition 6.** From Proposition 4 we know that a video window can exist when
\( s = 1 \) and \( u_L < u_L^1 \), in which case the profits \( \pi^e_{LH} \) and \( \pi^d_{LH} \) are given by eq. (11). We also know that \( u_L^1 < u_H \). The length of the movie window is determined by in a Nash bargaining, where the outside option for the exhibitor is zero, while the outside option for the distributor is to sell \( L \) alone and derive \( u_L/4 \). Thus the length of the window is obtained from the following maximization problem:

\[
\max \Omega = \left( \frac{(2u_H + du_L)^2}{4(du_L + 8u_H)} - \frac{u_L}{4} \right)^\alpha \left( \frac{(u_H - du_L)(2u_H + du_L)^2}{(du_L + 8u_H)^2} \right)^{1-\alpha}
\]

where \( 0 < \alpha < 1 \) is the degree of bargaining power of the distributor and \( 1 - \alpha \) is that of the exhibitor.

Simple but tedious calculations show that, in the relevant range, \( \Omega \) is monotonic in \( \alpha \), with \( \frac{\partial \Omega}{\partial \alpha} \bigg|_{\alpha=1} > 0 \) and \( \frac{\partial \Omega}{\partial \alpha} \bigg|_{\alpha=1} < 0 \). By continuity, if the degree of bargaining power of the distributor is sufficiently high, there will not be a video window. Conversely, if the degree of bargaining power of the exhibitor is sufficiently high, there will be a video window.

When an interior solution exists, this is characterized by \( \partial \Omega/\partial d = 0 \) and we denote it by \( d^* \). Again, after some calculations, we can show that \( \partial d^*/\partial \alpha > 0 \) while the sign \( \partial d^*/\partial (u_H/u_L) \) is ambiguous, i.e., the window is not necessarily longer the higher the quality differential. The following diagram plots the optimal interior solution as a function of the quality differential, for three different values of the distributor’s bargaining power (recall that the higher is \( d \), the shorter the window).

For the last part of Proposition, recall from Proposition 4 that \( a = \frac{d^2u_L^2 + 8u_H^2}{2(du_L + 8u_H)} \). The derivative of \( a \) with respect to \( d \) yields:

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Figure 3: Length of the movie window (starting from the top: $\alpha = 1/3; 1/4; 1/5$)

\[
\frac{\partial a}{\partial d} = \frac{u_L(d^2u_L^2 + 16du_Lu_H - 8u_H^2)}{2(du_L + 8u_H)^2}.
\]

The sign depends on the bracket in the numerator. Clearly, if $u_L$ is low compared to $u_H$, then $\frac{\partial a}{\partial d} < 0$. This is also the case if $d$ is small. However, we cannot rule out that $\frac{\partial a}{\partial d} > 0$ when $d$ is high and $u_L$ close to $u_L^1$. Q.E.D.

References


