The Economics of Tiered Pricing and Cost Functions: Are Equity, Cost Recovery, and Economic Efficiency Compatible Goals?

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Abstract

The paper develops a framework to analyze equity and economic efficiency measures of various pricing mechanisms for regulated products such as electricity or water. In particular, average cost, marginal cost, and increasing block rate pricing are compared. The analytical model recognizes that consumers are heterogeneous in their demand characteristics, and this heterogeneity will have to be incorporated in the design and assessment of alternative pricing mechanisms. Under certain circumstances, economic efficiency and cost recovery can be achieved in a manner that also reduces inequality, which is measured through changes in the Gini coefficient of consumer surplus. Under marginal cost or average cost pricing, the Gini coefficient is primarily

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affected by parameters of the demand function, but with increasing block rate pricing, the demand and cost parameters impact this measure. Under increasing block rates, a company with a heterogeneous mix of inputs has a greater ability to improve equity while still remaining revenue neutral and maintaining economic efficiency. The results are illustrated through the use of a numerical example.
1 Introduction

Regulated utilities such as electricity providers and municipal water suppliers have a unique presence in economic theory. Unlike most firms, the standard economic goal of profit maximization does not hold for these firms. Regulators have typically imposed a revenue neutral constraint; often resulting in a rate structure based on average cost pricing (Bonbright, Danielsen & Kamerschen 1988). It is well known in economic theory that average cost pricing leads to economic inefficiencies and deviations from socially optimal consumption. However, regulated utilities are frequently providers of products derived from limited natural resources (e.g., electricity derived from coal or natural gas, water provided by finite surface and groundwater sources). Therefore, regulators need to consider both the social goals of revenue neutrality and the impacts of price induced demand shifts in their choice of pricing mechanism. With increasing marginal costs, average cost pricing generally results in aggregate demand greater than the socially optimal level. This is a particularly relevant concern with regulated products such as electricity and water, due to concerns about the sustainability of current consumption levels.

One alternative to average cost pricing is increasing block rates (hereafter, IBR), or tiered pricing, where individuals pay a low rate for an initial consumption block and a higher rate as they increase use beyond that block. Increasing block rates are frequently used by regulated utilities in the United
States and worldwide. For example, Borenstein (2008) describes the adoption of IBR pricing by California electrical utilities during the 1980s. An OECD study of water rates in developed countries shows frequent use of increasing block rates (OECD 1999). Concerns about conservation have led to a widespread shift in pricing patterns; while only 4 percent of public water suppliers in the United States used IBR in 1982, over 30 percent did by 1997 (OECD 1999). Over the same period, the use of decreasing block rates fell from 60 to 34 percent of public water suppliers. Advocates of IBR argue that it can improve equity by offering the poor a subsidized rate on consumption. Others argue that tiered pricing will encourage overconsumption if the subsidized block is too large.

In this paper, we answer the question “when can tiered pricing be used to simultaneously improve equity, achieve economic efficiency, and retain revenue neutrality?”. We show that under certain conditions, a regulated utility can achieve all of these goals. However, the feasibility of this depends critically on the underlying cost structure of the utility. Specifically, utilities with a variety of input sources and without extremely poor customers are best able to achieve these joint goals.

To answer this question, we develop an analytical model of a regulated utility with heterogeneous customers. We demonstrate how shifts in parameters of the cost function or population distribution affect a utility’s design of a tiered pricing rate structure. We include a numerical example to illustrate how differences in the demand or supply function affect the equilibrium
under average cost, marginal cost, and tiered pricing. We calculate the Gini coefficient of consumer surplus for each type of pricing, and compare the equity gain associated with average cost or tiered pricing over marginal cost pricing.

Much of the previous literature on the economics of tiered pricing has examined the consumer response to a tiered pricing rate structure. Most of the work in this field has been empirical (Hewitt & Hanemann 1995, Castro-Rodríguez, Da-Rocha & Delicado 2002, Rietveld, Rouwendal & Zwart 2000, Bar-Shira, Finkelstain & Simhon 2006, Reiss & White 2005, Borenstein 2008). There has been a paucity of theoretical work examining the feasibility and implications of tiered pricing. Wilson (1993) briefly discusses the issue, but only in the context of decreasing block rates and a profit-maximizing monopolist. Certain limitations of tiered pricing in developing countries include the feasibility of every family unit having its own meter (Whittington 1992), although suggestions exist to remedy this problem such as customer specific block prices and quantities (Pashardes & Hajispyrou 2002). A related area of literature exists on two-part pricing with a profit-maximizing monopolist (Oi 1971, Spence 1977, Cassou & Hause 1999). There are three major differences between this literature and our research. Most importantly, we consider a regulated company or a state-owned utility, and therefore the objective of the rate structure choice will be different than an unregulated monopoly. Second, we examine and discuss the parameter values that allow tiered pricing to improve equity, as well as the limitations of this goal. Third, we do
not consider a fixed fee in the rate structure in the modeling, but different levels of a variable fee.

This paper expands on the existing literature by asking a more fundamental question. Specifically, we determine when tiered pricing for water can achieve three common goals: economic efficiency, cost recovery, and equity improvement. We examine the parameters of the demand function, the underlying customer distribution, and the cost function that impact whether these two goals are feasible. We find that when cost recovery is feasible, the lowest level of consumer demand determines if an outcome is economically efficient.

One of the broader implications of our results is what we refer to as the “inequality in leads to equity out” result. This result shows that certain utilities have a greater capacity to offer tiered pricing, and hence, improve equity, while still achieving economic efficiency. Under plausible assumptions about the cost function, we determine that those agencies with diverse sources of inputs (i.e., electricity providers that utilize coal, natural gas, and hydropower; or water providers that have sources from multiple rivers and groundwater aquifers) are better able to use tiered pricing to improve equity than those that rely on a single input source.
2 General Model of Tiered Pricing

We first develop a general model of tiered pricing, and use it to derive conditions under which tiered pricing can achieve economic efficiency. For simplicity we assume that tiered pricing is characterized by two parameters. The subsidized price or ‘lifeline price’ that individuals pay is denoted by $w_L$, and is below marginal cost. The size of the block or ‘lifeline quantity’ (the maximum quantity individuals can purchase at the subsidized price) is denoted by $q_L$. Figure 1 shows an example of this, where $w^M$ denotes the long run marginal cost. The figure also includes two theoretical demand curves for types $\theta_L$ and $\theta_H$. This diagram shows how consumption under tiered pricing may not be economically efficient; and that this depends on how the parameters are chosen. Type $\theta_H$ consumes at the economically efficient level, since at the margin he/she faces a price equal to the long run marginal cost. However, type $\theta_L$ overconsumes with consumption equal to $q^{TP}_{\theta_L}$ instead of $q^*_{\theta_L}$.

3 Model

3.1 Demand Model

We model a utility maximizing individual, where consumers are heterogeneous, and utility is a function of the amount of the good consumed and heterogeneity parameter $\theta$. Heterogeneity could be due to differences in
family size or underlying preferences. With electricity this could be due to differences in underlying preferences for conservation or the number and type of appliances owned; with water this could be due to different landscaping choices. In the following framework, we use $\theta$ to represent income heterogeneity. We assume that $\theta$ is distributed over a finite interval $[\theta_L, \theta_H]$ with pdf $f(\theta)$. Denoting $q$ as the quantity of water consumed, $U(\theta, q)$ as utility, $I(\theta)$ as income for type $\theta$, and $c(q)$ as the cost of $q$ units of water, let the benefit function be additively separable as follows. The utility function is not assumed to be linear, therefore this generalization only requires that utility

![Figure 1: Basic Tiered Pricing Rate Structure](image)
can be measured with a money metric.

\[ B(\theta, q) = U(\theta, q) + I(\theta) - c(q) \]  

(1)

For simplicity, we assume that there are two segments in the cost function, with the higher price equal to the long-term marginal cost. In practice, tiered pricing is sometimes designed with many blocks. For example, a redesign of California electricity rates included up to five different price levels (Reiss & White 2005). We denote the lower price and block size (i.e., the lifeline price and lifeline quantity) by \( w_L \) and \( q_L \). Initially, the only restriction we impose is that \( q_L \) is non-negative. In theory, \( w_L \) could be negative, implying that consumers receive a per-unit payment for consumption below some minimum quantity.

As shown in Olmstead, Hanemann, and Stavins (2007), an increasing block rate mechanism creates a point of non-differentiability in the budget constraint (i.e., a kinked budget constraint). In addition, there is a non-zero probability that an individual will consume exactly at the tier \( q_L \). We define \( w_M \) as the long run marginal cost. This is the price for all consumption above the initial block, resulting in an individual cost function \( c(q) \) equal to
the following:

\[
c(q) = \begin{cases} 
(q - q_L)w^M + w_Lq_L & \text{if } q > q_L \\
q_Lw_L & \text{if } q = q_L \\
qw_L & \text{if } q < q_L 
\end{cases} 
\] (2)

An individual will choose to maximize their benefits:

\[
\max_q B(\theta, q) = U(\theta, q) + I(\theta) - c(q) 
\] (3)

We assume that \( \frac{\partial U}{\partial \theta} > 0, \frac{\partial U}{\partial q} > 0, \frac{\partial^2 U}{\partial q^2} < 0, \) and \( \frac{\partial^2 U}{\partial \theta \partial q} > 0. \) When \( q \neq q_L, \) this is piecewise differentiable, and we can determine the corner solution outcomes when \( q = q_L: \)

For \( \theta \) s.t.

\[
\begin{cases} 
U'(\theta, q_L) > w^M & \Rightarrow U'(\theta, q) = w^M \\
w_L < U'(\theta, q_L) < w^M & \Rightarrow q = q_L \\
U'(\theta, q_L) < w_L & \Rightarrow U'(\theta, q) = w_L 
\end{cases} 
\] (4)

For any pair \( \{q_L, w_L\}, \) there are three groups of individuals that are formed, based on the first order conditions in Equation 4. We define \( \theta_1(q_L, w_L) \)
and \(\theta_2(q_L, w)\) as the two values of \(\theta\) that separate the types of individuals.

\[
\begin{align*}
\text{For} \quad & \begin{cases} 
\theta < \theta_1(q_L, w_L) & \Rightarrow \quad \frac{\partial U}{\partial q_L} < w_L \text{ and } q < q_L \\
\theta \in \left[\theta_1(q_L, w_L), \theta_2(q_L, w^M)\right] & \Rightarrow \quad w_L < \frac{\partial U}{\partial q_L} < w^M \text{ and } q = q_L \\
\theta > \theta_2(q_L, w^M) & \Rightarrow \quad \frac{\partial U}{\partial q_L} > w^M \text{ and } q > q_L
\end{cases}
\end{align*}
\]

A similar model for the producer is used in Bar-Shira, Finkelshtain, and Simhon (2006). However, a key distinction between their paper and ours is that their primary goal is the empirical estimation of producer response to tiered pricing. Their paper does not develop analytical results about the feasibility of tiered pricing.

**Lemma 1** Users may be divided into three groups - those that consume below the tier, those that consume at the tier, and those that consume above the tier. These three groups are defined by the conditions in Equation 5. For those individuals who do not consume at the tier, a higher level of \(\theta\) results in a greater quantity of water demanded.

**Proof** We can show that the total quantity demanded will increase with higher levels of \(\theta\). To show this, we totally differentiate the first order condition with respect to \(q\) and \(\theta\). For consumption that is not at the tier \(q_L\), the
following condition holds for any price $w$:

$$\frac{\partial U(\theta, q)}{\partial q} - w = 0 \quad (6)$$

Totally differentiating this condition with respect to $q$ and $\theta$, we have the following:

$$\frac{\partial^2 U}{\partial q^2} dq + \frac{\partial^2 U}{\partial \theta \partial q} d\theta = 0 \quad (7)$$

Rearranging this equation, we have the following:

$$\frac{dq}{d\theta} = -\frac{\partial^2 U}{\partial \theta \partial q} > 0 \quad (8)$$

### 3.1.1 Conditions for Equity Improvement

One of the primary reasons cited for using tiered pricing is to try to improve equity in access to services (Agthe & Billings 1987, OECD 1999). Tiered pricing is designed to assure that all consumers get a minimum benefit from water or electricity. We model this as a minimum level of utility, or well-being that is socially desirable, and we denote this level by $u$. This could be based on some standard such as a minimum amount necessary to meet basic living standards. While not defined by consumption, the minimum utility level is likely to be highly correlated with the size of the subsidized block. However, this general standard for $u$ recognizes that some substitution is possible between the regulated good and all other goods (i.e., income).

**Proposition 1** *For any social goal of minimum utility $u$, there exists at least*
one set \{q_L, w_L\} that can achieve this goal.

**Proof** We will show that for any choice of \(u\), there exists at least one set \{q_L, w_L\} that can achieve this goal. Consider the lowest type \(\theta_L\). Since \(\frac{\partial U}{\partial q} > 0 \forall q\), there exists \(\tilde{q}\) s.t. \(U(\theta_L, \tilde{q}) = u\). If the quantity and price pair are set by \(\tilde{q} = q_L\) and \(w_L = 0\), then the utility level of the lowest type is \(U(\theta_L, q_L) = u\). Since the price \(w_L\) is zero, the lowest type can afford this quantity. And, since \(\frac{\partial U}{\partial \theta} > 0\), if the social goal \(u\) is achieved for type \(\theta_L\), then it is achieved for all types.

### 3.1.2 Characteristics of Tiered Pricing

In evaluating tiered pricing rates, there are two characteristics we are particularly interested in: revenue neutrality and economic efficiency. The *distributional* outcome of a tiered pricing scheme is based on whether it achieves revenue neutrality, and the *efficiency* outcome is based on whether it achieves economic efficiency. We can measure the distributional cost by the total subsidy level, and the efficiency cost as the deadweight loss associated with inefficient pricing.

**Proposition 2** There is a maximum level of \(u\), denoted by \(\hat{u}\) such that any social goal where \(u > \hat{u}\) results in an economically inefficient outcome.

**Proof** An economically efficient outcome is defined as an outcome where the value of the marginal unit equals the long run marginal cost for all individuals. We denote \(w\) as the long run marginal cost, and define \(\tilde{q}\) s.t. \(\frac{\partial U(\theta_L, q)}{\partial q} = w\).
at \( q = \tilde{q} \). We also define \( U(\theta_L, \tilde{q}) = \tilde{U} \). For any \( \epsilon > 0 \), \( \frac{\partial U(\theta_L, q)}{\partial q} < w \) at \( q = \tilde{q} + \epsilon \). At this point the marginal utility of consumption is less than the long run marginal cost, resulting in economic inefficiency. Therefore, setting \( q_L > \tilde{q} \) results in an economically inefficient outcome and \( q_{LMax} = \tilde{q} \) is the largest lifeline quantity that can be offered while still maintaining economic efficiency. Any social goal \( u > \tilde{U} \) cannot be achieved without some inefficiency in consumption. Therefore, \( \tilde{U} = \hat{u} \) and any \( u < \hat{u} \) can be achieved efficiently.

We define all \( u < \hat{u} \) as potentially efficient, meaning that there exists a set \( \{q_L, w_L\} \) that achieves the social goal of \( u \) and results in economically efficient consumption by all individuals.

### 3.2 Cost Structure and Tiered Pricing

Until this point, we have focused on the analysis of the demand model. However, in many cases a revenue neutrality condition is required. Therefore, the supply/cost function is critical in determining if a particular rate structure allows a utility to cover total costs without cross-subsidization from another revenue source. Revenue neutral pricing is one reason that average cost pricing is frequently used by regulated utilities (Bonbright et al. 1988). In this section we consider the feasibility of cost recovery under a tiered pricing rate structure.

We define the cost function \( TC(Q) \), where \( Q \) is the total quantity de-
manded by all individuals. The parameters on the total cost function will determine the choices for \( u \) that are potentially revenue neutral. A potentially revenue neutral choice is one that can be achieved without subsidization from the government or other sectors of the economy.

The total quantity demanded depends on the distribution of individuals, the long run marginal cost, and the choice of lifeline price and quantity. We assume that the long run marginal cost is constant over the range of interest. For each distribution \( f(\theta) \), we can determine the total quantity demanded by the following:

\[
Q = \int_{\theta_L}^{\theta_H} q(\theta, w_L, q_L, w^M) f(\theta) d\theta
\]

Equation 9 includes the integration of demand across three segments of the distribution. The first component is the total quantity demanded by all individuals who consume below the tier. For these individuals, their demand level is determined by the lifeline price, \( w_L \). The second component is the total demand by all individuals who consume exactly at the tier, \( q_L \). The third segment includes all individuals who consume above the tier. In this segment of the distribution, the quantity demanded is determined by the marginal cost.

\[
Q = \int_{\theta_L}^{\theta_1(q_L, w_L)} q(\theta, w_L) f(\theta) d\theta + \int_{\theta_1(q_L, w_L)}^{\theta_2(q_L, w^M)} q_L f(\theta) d\theta + \int_{\theta_2(q_L, w^M)}^{\theta_H} q(\theta, w^M) f(\theta) d\theta
\]

\[
Q(w_L, q_L, w^M, f(\theta))
\]
Total revenue is determined by the following:

\[ TR(Q) = \int_{\theta_L}^{\theta_H} c(q(\theta, w_L, q_L, w_M)) f(\theta) \, d\theta \]

(10)

\[ = \int_{\theta_L}^{\theta_1(q_L, w_L)} w_L q(\theta, w_L) f(\theta) \, d\theta + \int_{\theta_1(q_L, w_L)}^{\theta_2(q_L, w_M)} w_L q_L f(\theta) \, d\theta \\
+ \int_{\theta_2(q_L, w_M)}^{\theta_H} (w_L q_L + w_M (q(\theta, w) - q_L)) f(\theta) \, d\theta \\
= TR(Q(w_L, q_L, w_M, f(\theta))) \]

**Definition** For two utilities that produce at \( Q = Q^* \), we define the one with lower input costs as the utility with the lower value of \( \int_0^{Q^*} MC(q) \, dq \).

**Proposition 3** For two marginal cost functions that result in the same equilibrium, the one with lower input costs can support a higher level of \( \tilde{u} \) while still maintaining revenue neutrality.

**Proof** Consider two marginal cost function \( MC_1(Q) \) and \( MC_2(Q) \). Suppose that \( MC_1(Q^*) = MC_2(Q^*) \), and that \( MC_1(Q^* - \epsilon) < MC_2(Q^* - \epsilon) \) \( \forall \, \epsilon > 0 \).

The supply function is determined by the marginal cost curve, and since \( \frac{\partial MC_2(Q)}{\partial Q} > \frac{\partial MC_1(Q)}{\partial Q} \), \( MC_2 \) is a more price elastic supply function. The total revenue that can be distributed for subsidies via tiered pricing is determined by the total producer surplus. A supply function with a higher level of producer surplus will increase the feasible subsidy.
The difference in producer surplus is given by the following:

\[ PS_1 - PS_2 = (MC_1(Q^*)Q^* - \int_0^{Q^*} MC_1(q) dq) - (MC_2(Q^*)Q^* - \int_0^{Q^*} MC_2(q) dq) \]

\[ = \int_0^{Q^*} MC_2(q) dq - \int_0^{Q^*} MC_1(q) dq \]

\[ = \int_0^{Q^*} (MC_2(q) - MC_1(q)) dq > 0 \] (11)

Supply function \( MC_1 \) is more price inelastic, but has a greater level of producer surplus. Therefore, a higher level of \( \bar{u} \) that can be supported while maintaining revenue neutrality. This result is particularly important for comparing the feasibility of using tiered pricing in different locations. Those producers or locations with a range of low-cost inputs have a greater capacity to subsidize low-income consumers than those relying on a single input source or a range of high cost sources.

### 3.3 Determining Feasible Lifeline Price and Quantity Combinations

In this section we show how various lifeline price and quantity combinations can be chosen to achieve revenue neutrality, economic efficiency or both.
3.3.1 Finding Revenue Neutral Combinations

To find the pairs of \( \{q_L, w_L\} \) that result in revenue neutrality, we consider the isocost curves and isorevenue curves for a utility. Since \( \frac{\partial Q(q_L, w_L, w_M)}{\partial q_L} \geq 0 \) and \( \frac{\partial Q(q_L, w_L, w_M)}{\partial w_L} \leq 0 \), for the total quantity demanded to stay constant after a shift in these values, both the isocost and isorevenue curves are monotonically increasing in \( \{q_L, w_L\} \) space. Each isocost and isorevenue curve correspond to a single value of \( Q(q_L, w_L, w_M) \) for distribution \( f(\theta) \). The direction of the change in the value for the isocost curve is unambiguously increasing as the curves move away from the \( w_L \) axis. However, the direction of the change in revenue depends on whether demand is price elastic or price inelastic. When demand is price elastic, an increase in the price leads to a decrease in total revenue, while the opposite effect holds when demand is price inelastic. In the following diagrams, we assume that demand is price inelastic. Previous empirical research has shown this to be the case for regulated products like electricity and water. For example, Reiss and White (2005) finds price elasticity estimates of -0.39 for residential electricity demand while a meta-analysis of the price elasticity of urban water demand studies finds a mean of -0.41 and a median of -0.35 (Dalhuisen, Florax, de Groot & Nijkamp 2003).

Figure 2 shows one possible depiction of these curves. For simplicity, we draw both the isocost and isorevenue curves as straight lines, however this is not necessary as long as they are monotonic. While theory cannot predict which set of curves is steeper, we can predict that there will be a locus of intersection points where the total cost equals the total revenue. These points
Figure 2: Isorevenue, Isocost, and Revenue Neutral Combinations

are labeled as the set of revenue neutral combinations.

**Proposition 4** There is a maximum level of $u$, denoted by $u_{\text{Max}}$, such that any social goal where $u < u_{\text{Max}}$ results in a potentially revenue neutral outcome. A social goal of $u > u_{\text{Max}}$ requires subsidization from other sectors.

**Proof** The intuition behind this result is that any subsidy needs to be funded using producer surplus. Producer surplus is maximized when the total quantity consumed corresponds to marginal cost pricing. Figure 3 shows this result graphically. It compares the isoprofit curves with isouility curves for type $\theta_L$. An individual has a higher utility level when receiving a larger life-line quantity and a lower lifeline price. There is a maximum level of utility
Figure 3: Maximum Utility under Revenue Neutrality

that intersects the \( \pi = 0 \) curve. This is the maximum social goal \( u \) that can be achieved while still maintaining revenue neutrality.

**Proposition 5**  
*There is a maximum price for \( w_L \) (denoted by \( \hat{w}_L \)), and any \( w_L > \hat{w}_L \) leads to positive profits for the regulated utility.*

**Proof**  
Let \( q_L = \tilde{q}_L \) s.t. \( MB(\theta_L, \tilde{q}_L) = w^M \). First, we consider \( w_L = w^M \) (the long run marginal cost). Since the long run marginal cost is greater than the average cost, the total profit is positive (\( \pi(w^M, \tilde{q}_L) > 0 \)). Now, let \( q_L = \hat{q}_L \), but set \( w_L = 0 \). Setting a price of \( w^M \) for consumption over \( \hat{q}_L \), with the lifeline quantity available for free leads to negative profits (\( \pi(0, \hat{q}_L) < 0 \)). By the Intermediate Value Theorem, if \( \pi(w^M, \tilde{q}_L) > 0 \) and \( \pi(0, \hat{q}_L) < 0 \), \( \exists \hat{w}_L \in [0, w^M] \) s.t. \( \pi(\hat{w}_L, \hat{q}_L) = 0 \).
Let $q_L = \tilde{q}_L - \epsilon$ for any $\epsilon > 0$. Since the total subsidy is distributed over a smaller quantity, the maximum level of $w_L < \tilde{w}_L$. Now let $q_L = \tilde{q}_L + \epsilon$ for any $\epsilon > 0$ (this is an economically inefficient outcome). The same proof applies as with $\tilde{q}_L$. The total profit for $w_L = w^M$ is positive ($\pi(w^M, \tilde{q}_L + \epsilon) > 0$). The total profit for $w_L = 0$ is negative ($\pi(0, \tilde{q}_L + \epsilon) < 0$). Again, by the Intermediate Value Theorem, $\exists \bar{w}_L \in [0, w^M]$ s.t. $\pi(\bar{w}_L, \tilde{q}_L + \epsilon) = 0$.

From Proposition 5, we know that there is a maximum value of $w_L$, and that $\tilde{w}_L < w^M$. Figure 4 shows this result, and also illustrates the effect on total revenue of choosing a $q_L, w_L$ combination that is not in the revenue neutral locus of points.
3.3.2 Finding Economically Efficient and Revenue Neutral Combinations

The previous analysis considered how the choice of lifeline price and quantity affect revenue neutrality. However, we are also concerned with achieving a second goal, economic efficiency. As shown in the proof of Proposition 2, there is a maximum lifeline quantity that permits economic efficiency. Figure 5 shows two possible values for this quantity. When the maximum lifeline quantity is $q_{L_{\text{Max}}}$, the combinations that achieve both goals are labeled. However, there may be parameter values that result in an empty set of $\{q_L, w_L\}$ pairs that satisfy revenue neutrality and economic efficiency. For example, if the maximum lifeline quantity is $q_{L_2}$, any economically efficient outcome will result in some positive level of surplus earned by the utility.

4 Numerical Illustration

We consider the case where $f(\theta) \sim U[0, 1]$. We use a linear marginal utility function, as is frequently used in the literature (Mussa & Rosen 1978, Caswell & Zilberman 1986, Castro-Rodríguez et al. 2002). There are two primary reasons that we decide to use a linear function. First, it implies there is a maximum price (i.e., a choke price) that consumers are willing to pay for the good. The maximum price or choke price is denoted by $w^p$ and is the same for all individuals. This reflects the availability of an outside option or backstop technology such as relying exclusively on bottled water, or using solar panels.
Figure 5: Satisfying Revenue and Efficiency Goals with Tiered Pricing
to produce electricity. The second reason for using a linear function is that it implies there is a satiation level for the good. We assume the marginal utility function is denoted by the following:

\[ MU(\theta, q) = w^P - \frac{1}{\theta + a}q \text{ where } a > 0 \]  

(12)

While the choke price \( (w^P) \) does not depend on an individual's type; the level of consumption where demand is satiated does depend on \( \theta \). This assumption is also made in other literature on tiered pricing (Castro-Rodríguez et al. 2002). The parameter \( a \) indicates the quantity where demand is satiated. A higher value of \( a \) implies that increased consumption is necessary to
satiate demand. This can be used to calculate the aggregate demand at any price by the following:

\[
AD(p) = \int_0^1 (\theta + a)(w^P - p)f(\theta)d\theta \\
= \int_0^1 (\theta + a)(w^P - p)d\theta \\
= (a + \frac{1}{2})(w^P - p)
\]

The marginal cost function is denoted by \(V(Q)\), where \(V' \geq 0\) and \(V'' \geq 0\). We assume a linear MC curve as follows:

\[
V(Q) = b_1Q \text{ where } b_1 > 0 \quad (13)
\]

### 4.1 Average Cost Pricing

As mentioned earlier, many regulated utilities utilize average cost pricing to maintain revenue neutrality. Using this model, we calculate the equilibrium price and quantity under average cost pricing. These results will be compared to the market outcome under marginal cost (i.e., economically efficient) pricing. The two conditions that must be satisfied under average cost pricing are 1) revenue neutrality condition (total revenue equals total cost); and 2) market clearing condition (average cost equals aggregate marginal benefit). We calculate the total cost function from the marginal cost function in Equation
Total revenue is the product of quantity and aggregate demand:

\[ TR = Q(p)(w^P - \frac{Q(p)}{a + \frac{1}{2}}) \quad (15) \]

The average cost and the aggregate marginal benefit are determined from the total cost function and the aggregate demand function, respectively. Solving for the two necessary conditions, we find the following average cost equilibrium:

\[
P_{AC} = \frac{b_1 w^P (a + \frac{1}{2})}{2 + b_1 (a + \frac{1}{2})} \quad (16)
\]

\[
Q_{AC} = \frac{2 w^P (a + \frac{1}{2})}{2 + b_1 (a + \frac{1}{2})} \quad (17)
\]

Economic theory predicts that this outcome will not be economically efficient, and that with increasing marginal costs, will lead to excessive consumption and underpricing of resources.

### 4.2 Economically Efficient Outcomes

A economically efficient outcome requires that every type of individual pays the marginal cost for the last unit of water that they consume, and results in the same price and quantity as under marginal cost pricing. Calculating
Table 1: Comparison of Equilibrium Price and Quantity under Marginal and Average Cost Pricing

<table>
<thead>
<tr>
<th>Marginal Cost Price</th>
<th>Average Cost Price</th>
<th>Marginal Cost Quantity</th>
<th>Average Cost Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{b_1 w^P (a + \frac{1}{2})}{1 + b_1(a + \frac{1}{2})}$</td>
<td>$\frac{b_1 w^P (a + \frac{1}{2})}{2 + b_1(a + \frac{1}{2})}$</td>
<td>$\frac{w^P (a + \frac{1}{2})}{1 + b_1(a + \frac{1}{2})}$</td>
<td>$\frac{2w^P (a + \frac{1}{2})}{2 + b_1(a + \frac{1}{2})}$</td>
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</table>

these gives the following:

$$P^* = \frac{b_1 w^P (a + \frac{1}{2})}{1 + b_1(a + \frac{1}{2})}$$

$$Q^* = \frac{w^P (a + \frac{1}{2})}{1 + b_1(a + \frac{1}{2})}$$

We calculate the level of producer surplus, which gives a measure of the total surplus that can be distributed using subsidized pricing.

$$PS = \frac{b_1}{2} Q^*^2$$

4.3 Comparison of Average and Marginal Cost Pricing

Average or marginal cost pricing are both frequently recommended or used for regulated utilities. While average cost pricing does lead to revenue neutrality, it unambiguously leads to economically inefficient consumption levels. The following comparison clearly shows that the equilibrium total quantity demanded is higher under average cost pricing; while the equilibrium price is lower.
4.4 Finding Tiered Pricing Parameters

With the lifeline price denoted by $w_L$ and the lifeline quantity denoted by $q_L$, an efficient outcome with tiered pricing requires that the total subsidy to all individuals equal the available producer surplus, or that the following hold. It also requires that the lifeline price and quantity be set so that type $\theta = 0$ uses water efficiently. These requirements are summarized in the following two conditions:

\begin{align*}
(P^* - w_L)q_L &= \frac{b_1}{2} Q^2 \quad \text{(21)} \\
w^P - \frac{1}{a} q_L \geq b_1 Q^* \quad \text{(22)}
\end{align*}

Equation 21 gives the revenue neutrality constraint, while Equation 22 gives the efficiency compatibility constraint. The set of $\{q_L, w_L\}$ that satisfies both of these constraints is the feasible set for policy makers implementing tiered pricing who wish to maintain economic efficiency.

Solving this set of equations we find that an economically efficient outcome is feasible when $a \geq \frac{1}{2}$, and is not feasible when $a < \frac{1}{2}$. This is the parameter value that determines if the maximum $q_L$ set by Equation 22 lies to the right or left of the minimum $q_L$ set by Equation 21. The value of $a$ represents minimum satiation levels. This result is important, as it shows that if there are customers with very low levels of demand, setting economically efficient tiered pricing rates may not be feasible.

These constraints can be analyzed graphically as shown in Figure 6, which
Lifeline Quantities ($q_L$)
Lifeline Prices ($w_L$)
Efficiency compatibility constraint
Revenue neutrality constraint

Figure 6: Feasible Set of Lifeline Prices and Quantities

illustrates the set of \( \{q_L, w_L\} \) that is feasible for a particular set of exogenous values of \( a \) and \( b_1 \). The feasible set contains the locus of points which satisfy the revenue neutral constraint and have a lifeline quantity below that defined by the efficiency compatibility constraint. For different parameter values of \( a, b_1 \) and \( w_P \), the feasible set of \( \{q_L, w_L\} \) will shift to reflect the different conditions.

Using the previous results we can show the effects of changes in the key parameters of the model both analytically and graphically.
4.4.1 Changes in the Marginal Utility Function

Figure 7 shows the results of a change in the demand function. The change examined is an expansion of demand, and the results show that as total demand expands, the feasible set of lifeline prices and quantities expands in size. However, the lifeline price increases, due to increased costs to the utility from a greater aggregate level of demand. The specific numerical values used are less important than the direction of the change based on underlying parameter values.
4.4.2 Changes in the Marginal Cost Function

We are also interested in how shifts in the marginal cost function affect the feasibility of tiered pricing. Figure 8 shows the impact of a change in the marginal cost curve on the feasible choice set of lifeline price and lifeline quantity. A steeper marginal cost curve limits the ability to subsidize consumption, due to higher costs. This result is in part due to the assumption of a linear marginal cost curve, which has a constant rate of change. With other functional forms, the total amount of producer surplus will be the primary indicator of a utility’s ability to subsidize consumption.
4.5 Equity Implications of Tiered Pricing

The previous results allow us to examine the equity implications of a shift from marginal cost pricing to tiered pricing. We use the same analytical model, and continue to assume that \( \theta \) is distributed uniformly between 0 and 1. We also continue to assume that a utility chooses \( \{q_L, w_L\} \) to satisfy both the revenue neutral and the economic efficiency constraints.

4.5.1 Measuring Equity

There are several methods that could be used to measure equity. One possibility would be to use a Gini coefficient of actual water consumption. However, due to differences in demand functions, it is not desirable to have an equal distribution of water resources, as it results in economic inefficiency. We choose to use the Gini coefficient of consumer surplus. As it is typically not economically efficient for consumer surplus to be equally distributed, we focus instead on changes in the Gini coefficient, and on the difference between pricing mechanisms as opposed to the measure itself.

As shown above, as long as both constraints are satisfied the choice of \( \{q_L, w_L\} \) under tiered pricing does not affect the equity measure. This result is because the total surplus available is fixed, and its distribution is determined by population, as each individual receives an equal share. With the model used, we find the following analytical functions for the Gini coefficient under marginal cost, average cost, and tiered pricing:

Figure 9 shows the impact of changes in either the demand or cost pa-
<table>
<thead>
<tr>
<th></th>
<th>Marginal Cost Pricing</th>
<th>Average Cost Pricing</th>
<th>Efficient Tiered Pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gini coefficient</td>
<td>$\frac{1}{6a+3}$</td>
<td>$\frac{1}{12a+6}$</td>
<td>$\frac{1}{(6b_1 w^P(a+0.5)^2)+6a+3}$</td>
</tr>
</tbody>
</table>

Table 2: Comparison of Gini Coefficient under Marginal, Average, and Tiered Pricing

Figure 9: Gini Coefficient Measures with Varying Parameters

Parameters on the Gini Coefficient. A range of parameters for $a$ and $b_1$ are considered, with the parameter $w^P$ held constant. The results show that reductions in either of these parameters reduce equity. However, the impacts from the two parameters are not symmetric.

One important result is that it is only the demand function that affects the equity measure under marginal cost or average cost pricing. The parameter $a$ measures the demand response to increased prices, and also corresponds
to various levels of satiation. A higher value of $a$ implies that demand is satiated at a greater consumption level. This decreases the Gini coefficient, corresponding to a more equitable outcome. This result is due to the fact that all individuals consume more as $a$ increases. A comparison of marginal and average cost pricing show that average cost pricing is unambiguously more equitable than marginal cost pricing. This is due to a lower price, which has a relatively larger impact on low-level consumers.

The results also show that moving from marginal cost pricing to efficient tiered pricing leads to a reduction in the Gini coefficient. The level of the reduction depends on $a$, $b_1$, and $w^P$. Since the total level of producer surplus is constant, and equal to $w_L \times q_L$, the choice of these parameters does not affect the Gini coefficient. This result is especially important, since the parameters of the supply function are only important in determining equity under tiered pricing, and not under marginal or average cost pricing.

5 Conclusion

The choice of rate structure for a regulated natural resource affects aggregate consumption, economic efficiency, and the distribution of the benefits from natural resource use. There are several social goals that can be targeted through the rate structure choice. For example, marginal cost pricing promotes economically efficient consumption levels, average cost pricing leads to revenue neutrality, and subsidized rates benefit the poor.
In some circumstance, tiered pricing can allow the full repayment of costs to meet financial obligations, economically efficient consumption levels, and the redistribution of resources to support equity goals. However, the capacity of tiered pricing to achieve these outcomes is limited and depends on exogenous underlying parameters. This result indicates that a tiered pricing rate structure needs to be designed carefully, with consideration given to a firm’s cost structure and customer distribution. Ad hoc choices for the lifeline price and lifeline quantity, or the simple duplication of a successful rate structure from another location are unlikely to be successful.

The feasibility of economic efficiency primarily depends on the marginal consumer. In cases where the marginal consumer has a very low level of demand, it limits the feasibility of compensating them while still having them consume an economically efficient quantity. As a result, distributions of consumers that include very poor customers are likely to result in economically inefficient consumption. However, the social cost of this economic inefficiency may be small enough that doing so is acceptable. Our analysis also shows that if the lifeline quantity is set too high, consumption levels will not be economically efficient. This will result in an excessive transfer from those consumers with high levels of demand to those with low levels of demand.

Achieving improved equity with tiered pricing is particularly effective when there are various sources of low-cost inputs for a utility. We refer to this as the “inequality in leads to equity out” result. Since tiered pricing is a mechanism to redistribute producer surplus to consumers it is most effective
with high levels of producer surplus, a direct result of low input costs. Results comparing the Gini coefficient under average cost, marginal cost, and tiered pricing show that there is an improvement in equity in a transition from marginal cost pricing to tiered pricing. However, there is a limit to the extent of redistribution that is possible under efficient consumption levels.

In addition to the issues discussed above, it is critical to recognize that feasible tiered pricing formulas cannot be set once and left unchanged. Changes in the demand function, availability of new technology, or in the underlying distribution of customers will may all result in differences in the optimality of tiered pricing. In addition, in this paper we have made the assumption that demand is constant, and have not adjusted for seasonality. Tiered pricing could be paired with another form of pricing such as peak load pricing in cases where regulators want to discourage consumption at certain times of the day or season. When regulators really learn how to use tiered pricing effectively, it can be used in combination with other pricing mechanisms, leading to multiple dimensions of efficiency and equity considerations.
References


