Abstract

The selective-logging regime is one of the most important instruments to support the sustainable development of forests, and reinforce the multiple services that they provide, such as the maintenance of biodiversity. The consideration of some stand features like having a certain volume of dead wood and/or large trees may help to preserve and enhance biodiversity in forest ecosystems, but as a consequence they may decrease the benefits obtained from timber sale.

This paper presents a bioeconomic model that allows to determine the optimal forest management when biodiversity conservation is taken into account. It aims to analyze to which extent the requirement to fulfill some structural targets related to biodiversity preservation affects the optimal forest management regime, and the profitability of forests. With this purpose, an empirical analysis is conducted to determine the optimal selective-logging management of two diameter distributed stands of Pinus sylvestris in Catalonia, and the results are then compared with the outcome generated when some stand structural targets are imposed. The results show that the costs of biodiversity conservation in terms of reduced profitability can be significant.

Key words: biodiversity conservation, forest management, selective logging, dynamic optimization, distributed optimal control.
1. Introduction

Interest in forests has grown significantly in many countries over the last few years. The United Nations Economic and Social Council, in its decision 1995/226, endorsed the recommendation of the third session of the Commission on Sustainable Development to establish the Intergovernmental Panel on Forests (IPF). Its objective was to pursue consensus and coordinate proposals for action to support conservation, management and sustainable development of forests. The implementation of the proposals for action has been approved by the United Nations and the European Union. The sixth Environment Action Programme of the EU explicitly considers extending national and regional programs to further promote sustainable forest management as one of their primary objectives. Therefore, it has become necessary to design forest policies that support the sustainable management of forests, and that contemplate the multiple services that forests provide.

The maximization of timber benefits has resulted in clear-cutting being the prevailing logging technique in many regions. Traditionally, forest economists determined the optimal rotation period of the trees for timber production by the Faustmann formula (Conrad and Clark, 1987). However, apart from timber, forests are also a source of important by-products such as mushrooms, or cork (Croitoru, 2007) and they present scenic and recreational values (Scarpa et al., 2000). Moreover, forests maintain biological diversity by providing habitat for a wide range of species (Doyon et al., 2005; Sawadogo et al., 2005), for instance in form of a complex structure of tree and canopy (Mendenhall et al., 1990; Whittam et al., 2002). Finally, forests also grant important environmental services, such as protection of floods, avalanches and landslides, the enhancement of the water buffering capacity and the sequestration of carbon (van Kooten and Sohngen, 2007). If the multiple services that forests provide are taken into account, clear-felling would most likely cease to be the optimal logging technique.

Recently, the literature has begun to study the adoption of more respectful regimes such as Green-tree retention (GTR) (Koskela et al., 2007; St-Laurent et al. 2008). This type of logging technique implies a modification of the traditional clear-cutting regime by leaving some standing trees at the end of the rotation period until the new trees reach a precise height. Compared to the clear-cutting regime, it is believed that GTR enhances biodiversity in the forest ecosystems. Rosenvald and Lõhmus (2007) reviewed a large number of studies to determine when this regime meets its objectives. They found that although GTR improves the habitat for various taxa, silvicultural practices other than
Green-tree retention (for instance, the selective-logging regime) should be considered for the species with high biodiversity value and poor survival under this type of regime.

In this paper we present a theoretical model to determine the optimal selective-logging regime of a size-distributed forest when biodiversity conservation is taken into account. The law of motion of the economic model is governed by a partial integrodifferential equation that describes the evolution of the forest stock over time. Given the complexity of the problem it is not possible to obtain an analytical solution. To solve the problem numerically we employ a technique known as the “Escalator Boxcar Train”, previously tested by us (Goetz et al., forthcoming). The empirical part of the paper determines the selective-logging regime that maximizes the discounted net profits from timber production of a stand of *Pinus sylvestris*, and compares it with the optimal selective-logging regime when biodiversity is accounted for. The results show that the costs of biodiversity conservation in terms of reduced profitability can be significant. Therefore, assessing the link between the indirect indicators and biodiversity is crucial for the correct determination of the optimal forest management.

The paper is organized as follows. The following section describes the features of the bioeconomic model and explains the different components of the growth process. Section 3 presents the empirical study which determines the optimal selective cutting regime of two particular stands, and conducts a sensitivity analysis of the previous results with respect to different parameters. Section 4 closes out the paper by presenting the conclusions.

2. Bioeconomic model

In order to specify the economic model, one needs to characterize the underlying biological model that describes forest dynamics.

2.1 Forests dynamics

In the previous economic literature the age of the tree is regarded as the structuring variable of the biological population (Wan, 1994; Salo and Tahvonen, 2002). In biological science, however, it is usually not the age but physiological or behavioral characteristics that determine the biological life cycle of the individual, for instance the size, life cycle stages, gender or genetic differences, or behavioral activities (Cushing,
1988). Likewise, from an economic point of view it is not the age but the size of the tree that is important. The price of lumber changes with the size of the tree but does not with age. As established by forest scientists the age of a tree is not very closely related to its size (Björklund, 1999). A large genetic variety between the trees and of the different local conditions of each tree makes it difficult to establish a functional relationship between size and age. Consequently, the age-size relationship resulting from an econometric estimation is very poor (Seymour and Kenefic, 1998).

Normally, the size of a tree, and consequently the size of a forest, is measured by the diameter at breast height, that is, the diameter of the trunk at height of 1.30 meters above the ground. Apart from defining size and time as the relevant exogenous determinants for the growth process, it is also taken into account that timber price per $m^3$ increase with the diameter of the tree. More specifically, time, denoted by $t$, and diameter, denoted by $l$, are incorporated as the domain of the control and state variables. The upper boundary of the diameter domain, $l_m$, can be interpreted as the maximum diameter that a tree can reach under perfect environmental conditions. It is assumed that a diameter-distributed forest can be fully characterized by its number of trees and by the distribution of the diameter of the trees. In other words, the spatial distribution or particular location of the trees is not accounted for. It is assumed that all individuals have the same environmental conditions and the same amount of space. Moreover, given that the diameter of a tree lies in the interval $[l_u, l_m)$, and that the number of trees is sufficiently large, the forest can be represented by a density function. This function is denoted by $x(t, l)$ and indicates the population distribution with respect to the structuring variable, $l$, at time $t$. Given this definition, the number of trees in the forest at time $t$ is given by

$$X(t) = \int_{l_u}^{l_m} x(t, l) dl.$$  (1)

In order to model the dynamics of the forest, the processes: growth, reproduction and mortality are determined in the following subsections, where the influence of the individual tree on the vital functions of other individuals (intra-specific competition) is taken into account.

- **Growth and competition between individuals:**
In order to express biotic or abiotic factors that influence the life cycle of the individual, biologists use the term environment. The height of a tree is determined only by its genetic information. However, its diameter is function of the time and the environment.

Let define \( g(E(t, l), l) \) as the change in the diameter of tree over time as a function of its current diameter, and of a collection of environmental characteristics, \( E(t, l) \), that affect the individual life cycle. In a context where the atmospheric and soil conditions are given, and in the absence of predators, these characteristics are given by the local environmental conditions of the tree and by the competition between the trees of the same stand for space, light and nutrients. Since our model does not consider the exact location of each tree, \( E(t, l) \) measures exclusively the intra-specific competition. The consideration of this form of competition is an innovative feature in forest management that helps to describe forest dynamics in a more realistic way.

The pressure of intra-specific competition on a tree with diameter \( \hat{l} \) is measured by the basal area of all trees. This feature of the model is supported by Álvarez et al. (2003), who analyzed various indices to evaluate the effect of intra-specific competition on the individual growth rate of the trees. They found that the basal area is the statistic that best explains the differences in diameter growth. Thus, the change in the diameter over time of a single tree is given by

\[
\frac{dl}{dt} = g(E(t, l), l),
\]

where the functional relationship between diameter and basal area is employed to determine \( E(t, l) \) which is given by

\[
E(t, l) = \int_{l}^{l_{n}} \frac{\pi}{4} n^2 x(t, n) dn.
\]

- **Mortality:**

The instantaneous death rate is denoted by \( \mu(E(t, l), l) \). It describes the rate at which the probability of survival of a tree with diameter \( l \), given the environmental characteristics \( E(t, l) \), decreases with time.

- **Reproduction:**
As far as reproduction is concerned, we assume that the totally of young trees with
diameter $l_0$ is planted, and no biological reproduction takes place. The control variables
of the model $u(t, l)$ and $p(t, l_0)$ denote the density of cutting in time $t$ with diameter $l$, 
and the flow of trees planted in time $t$ with diameter $l_0$ respectively.

These four processes allow us to model the forest dynamics based on the equations
described by de Roos (1997), and Metz and Diekmann (1986). It can be modeled as:

$$
\frac{\partial x(t, l)}{\partial t} = - \frac{\partial \left( g(E(t, l), l) x(t, l) \right)}{\partial l} - \mu(E(t, l), l) x(t, l) - u(t, l).
$$

### 2.2 Biodiversity considerations

There exist basically three types of biodiversity: the genetic, the species, and the
ecosystem biodiversity. We have focused on the second type. In the literature one can
find many indices for measuring biodiversity of the forest ecosystems, either directly,
by counting the number of species of a territory or indirectly, by assessing the
abundance of different factors that favor biodiversity.

Within the indirect factors, the amount of dead wood present in the forest ecosystem
the amount of large-diameter trees are accepted by the scientific community as essential
for determining the development of biodiversity (Harmon et al., 1986; McComb et al.,
1999). More specifically, the dead wood, including fallen trees and standing dead trees
(snags), is an indicator proposed by the European Environmental Agency in the
“Streamlining European 2010 Biodiversity Indicators” in order to asses and inform
about progress towards the European 2010 targets. Steele (1972) stresses that twenty
percent of animal species associated with wood are associated with dead wood, for
instance, the wood-consuming insects which feed insectivorous birds. It also allows the
development of flora such as lichens and mosses, among others.

Besides dead wood, large and old trees also play a very important ecological
function, since they offer a varied and abundant supply of natural cavities in its trunk
and branches for the nesting and refuge of birds (Camprodon, 2001). Old trees also
possess greater diversity of mycorrhizae, absent or very scarce in younger trees. In the
case of Scots pine, for instance, the cavities are found in 150-year old trees and above,
and with a diameter greater than 40 cm, while the trees are often cut within the range 80-120 year-old, which leads to a lack of these habitats in most managed forests.

Those two indicators are included in the methodology used in the Third National Forest Inventory to determine the status of the biodiversity of Spanish forest ecosystems. Thus, we will focus particularly on those structural elements that serve as indirect indicators of biodiversity in the forest ecosystems, such as the presence of dead wood and trees with a diameter greater than 40 cm, which are already out of turn and present some cavities.

A first approach to introduce biodiversity in the economic model would be to include the social benefits of biodiversity in the objective function, giving an economic value to biodiversity. As a result of the optimization one could identify the optimum percentage of dead wood and the optimal number of trees with a diameter greater than 40 cm.

Given the complexity and wideness of the concept of biodiversity, and the diversity of forests and conditions of the territory, the former approach can usually not be employed since requires an equation that links dead wood and large-diameter trees with biodiversity, which does not exist at present.

Thus, if the approach to include the biodiversity in the objective function is not feasible, an alternative option is to include biodiversity as a restriction in the economic problem. That is, to establish minimum levels of a comprehensive set of indirect indicators of biodiversity that the forest manager has to fulfill. In this way it is not necessary to have an exact relationship between these indirect aspects and biodiversity.

Moreover, following the latter approach one doesn’t need to give an economic value to biodiversity, one can identify instead the difference between the benefits of the private manager obtained with the optimal selective-logging regime without restrictions and the optimal logging with restrictions that allow maintain a suitable environment to preserve the biodiversity of a forest. These restrictions in the model will be defined by the existence of a minimum percentage of dead wood and of a minimum amount of large-diameter trees.
2.3 The decision problem of the forest manager

We assume that the forest is privately owned and managed over a planning horizon of $T$ years. Using the previous defined components of the model, the decision problem of the forest owner to maximize private net benefits from timber sale can be stated as:

\[
\begin{align*}
\int_0^T \left[ B\left(x(t, l), u(t, l)\right) \right] e^{-rt} dl 
&= \int_0^T C\left(p(t, l_0)\right) e^{-rt} dt \\
&+ \int_{l_0}^{l_*} S_T\left(x(T, l)\right) e^{-rt} dl,
\end{align*}
\]

subject to the constraint (4) and

\[
\begin{align*}
g\left(E(t, l_0), l_0\right) x(t, l_0) &= p(t, l_0), \\
x(t_0, l) &= x_0(l), \\
p(t, l_0) &\geq 0, \\
u(t, l) &\geq 0,
\end{align*}
\]

where $E(t, l)$ is given by equation (3), and $r$ denotes the discount rate. The function $B(x, u)e^{-rt}$ presents the discounted net benefits of timber production. It depends not only on the amount of logged trees but also on the amount of standing trees in order to take into account the maintenance cost of the forest. The strictly convex function $C\left(p\right)e^{-rt}$ expresses the discounted cost of planting trees with diameter $l_0$, the function $S_T\left(x\right)$ the value of the standing trees at the final point in time of the planning horizon. The restriction $g\left(E(t, l_0), l_0\right) x(t, l_0) = p(t, l_0)$ requires that the flux of the change in diameter at diameter $l_0$ coincides with the total flux of the diameter of trees planted with diameter $l_0$. The term $x_0(l)$ denotes the initial diameter distribution of the trees. Finally, the control variables, $u(t, l)$ and $p(t, l_0)$, must be nonnegative.

In order to account for biodiversity we need to incorporate three additional constraints in the decision problem:
Equation (9) requires that the amount of trees with a diameter larger than 40 cm is greater than a minimum ecological value, $b_{\text{min}}$. Equation (10) is an identity that specifies the volume of trees that die at time $\tau$, where $\alpha$ and $\beta$ are parameters that convert the amount of trees into volume. Finally, equation (11) establishes a minimum stock of dead wood in the forest, $S_{\text{min}}$. Many authors assume that all constituents of dead biomass are equally decomposable, and the single-exponential model is the most frequent model used to determine decomposition constants (Zhou et al., 2007). Therefore, based on the literature, we assume a constant decomposition rate of the dead wood in the forest ecosystem, denoted by $\delta$.

In practice, the necessary conditions of the optimization problem (D), two equations and a system of partial integrodifferential equations, can only be solved analytically under severe restrictions with respect to the specification of the mathematical problem (Muzicant, 1980). Thus, one has to resort to numerical techniques in order to solve the distributed control problem. Available techniques such as the method of finite differences, or finite elements may be appropriate choices (Calvo and Goetz, 2001). However, all of these methods require the programming of algorithms that are mostly unknown to economists or foresters.

To take the analysis further we propose to employ a numerical solution technique known as "Escalator Boxcar Train" used by de Roos (1988) to describe the evolution of physiologically-structured populations. He has shown that this technique is an efficient integration technique for structured population models. In contrast to the other available methods, the EBT can be implemented with standard computer software such as GAMS (General Algebraic Modeling System), used for solving mathematical programming problems.
Applying the EBT allows the partial integrodifferential equations of problem (D) to be transformed into a set of ordinary differential equations which are subsequently approximated by difference equations. Besides a brief presentation of the EBT method, Goetz et al. (forthcoming) show how this approach can be extended to account for optimization problems by incorporating decision variables. To transform the decision problem (D), we first divide the range of diameter into n equal parts, and define \( X_i(t) \) as the number of trees in the cohort \( i \), being \( i = 0, 2, 3, \ldots, n \), that is, the trees whose diameter falls within the limits \( l_i \) and \( l_{i+1} \) are grouped in the cohort \( i \). Likewise, we define \( L_i(t) \) as the average diameter, \( U_i(t) \) as the number of cut trees within cohort \( i \), and \( P(t) \) as the number of planted trees in cohort 0.

3. Empirical study

The purpose of the empirical analysis is to determine the optimal selective-logging regime of a diameter-distributed forest taking into account biodiversity conservation goals, that is, the selective logging regime that maximizes the discounted private net benefits from timber production of a stand of \( \textit{Pinus sylvestris} \) over a time horizon of 200 years while satisfying some requirements on the level of dead wood and large-diameter trees in the forest ecosystem. The election of \( \textit{Pinus sylvestris} \) was motivated by the fact that it is the most important commercial specie for timber production in Catalonia. The object of the study is located in Alta Garrotxa (municipality of Girona, Spain), since it is a region with a large extension of forest stands of \( \textit{Pinus sylvestris} \) (Ibàñez, 2004).

3.1. Data and specification of the parameters and functions

The net benefit function of the economic model, \( B(x(t,l),u(t,l)) \), consists of the net revenue from the sale of timber at time \( t \), minus the costs of maintenance, which comprise clearing, pruning and grinding the residues. The net revenue is given by the sum of the revenue of the timber sale minus the cutting costs defined as:

\[
\left[ \sum_{i=0}^{n} \left( \rho(L_i(t)) - vc(L_i(t))mv(L_i(t))U_i(t) \right) - mc(X(t)) \right] \text{ where } X(t) = \sum_{i=0}^{n} X_i(t). 
\]

The terms in the first square brackets denote the sum of the revenue of the timber sale minus the cutting costs of each cohort \( i \), and the term in the second square bracket, \( mc(X(t)) \), accounts for the maintenance costs. The parameter \( \rho(L_i) \) denotes the timber price per cubic meter of
wood as a function of the diameter; \( v(L) \) is the total volume of a tree as a function of its diameter; \( m_v(L) \) is the part of the total volume of the tree that is marketable; \( vc \) is variable cutting cost, and \( fc \) is the fixed cutting cost. Timber price per cubic meter was taken from a study by Palahí and Pukkala (2003), who analyzed the optimal management of a *Pinus sylvestris* forest in a clear-cutting regime. They estimated a polynomial function given by
\[
\rho(L) = \text{Min}\left[{-23.24 + 13.63 \sqrt{L}, 86.65} \right],
\]
which is an increasing and strictly convex function, for a diameter lower than 65cm. At \( L = 65 \) the price reaches its maximum value, and for \( L > 65 \) it is considered constant. Data about costs were provided by the consulting firm Tecnosylva, which elaborates forest management plans throughout Spain. The logging cost comprises logging, pruning, cleaning the underbrush, and collecting and removing residues, and it is given by 15 € per cubic meter of logged timber. According to the data supplied by Tecnosylva, the maintenance cost function is approximated by \( mc(X(t)) \), and is given by \( mc(X(t)) = 44.33 + 0.0159 X + 0.0000186 X^2 \). The planting cost is linear in the amount of planted trees and is given by \( C(P) = 0.73 P \). The thinning and planting period, \( \Delta t \), is set at 10 years, which is a common practice for a *Pinus sylvestris* forest (Cañellas et al., 2000).

### 3.2. Simulations of forest growth

To proceed with the empirical study, different initial diameter distributions of a forest were chosen. These distributions were specified as a transformed beta density function \( \theta(l) \) since it allows a great variety of distinct shapes of the initial diameter distributions of the trees to be defined (Hunter, 1990). The shape parameters are denoted by \( \gamma \) and \( \phi \), and we allowed these parameters to take on either a value of 0.5, 1, or 2. The initial forest consists of a population of trees with diameters within the interval \( 0 \text{ cm} \leq l \leq 50 \text{ cm} \). We generated a wide variety of initial diameter distributions considering all possible combinations of the three possible values of the parameters \( \gamma \) and \( \phi \). The density function of the diameter of trees, \( \theta(l; \gamma, \phi) \), is defined over a closed interval, and thus the integral

\[
\int_{l_1}^{l_2} \theta(l; \gamma, \phi) dl
\]
gives the proportion of trees lying within the range \([l_i, l_{i+1})\). We defined \(l_0 = 0\) and \(l_m = 80\), as minimum and maximum commercial diameters of the tree, respectively. For the simulation of the forest dynamics, we concentrate on the diameter interval \([0, 50]\), because thereafter the growth rate of the trees is close to zero. This interval was divided into 10 subintervals of identical length. In this way, the diameter of the trees of each cohort differ at most by 5 cm, and thus, the size of the trees of each cohort can be considered as homogeneous. The initial number of trees in any cohort, \(x_i(0)\), is calculated for each combination of \(\gamma\) and \(\varphi\) with three different basal areas (15, 20 and 25 m²/ha), to isolate the effects of the initial distributions and the density of the stand on the optimization results.

To determine the dynamics of the forest, the growth of a diameter-distributed stand of *Pinus sylvestris* without thinning was simulated with the bio-physical simulation model GOTILWA (Growth Of Trees Is Limited by Water). The model simulates growth and mortality and allows exploring how the life cycle of an individual tree is influenced by the climate, characteristics of the tree itself and environmental conditions. The model is defined by 11 input files specifying more than 90 parameters related to the site, soil composition, tree species, photosynthesis, stomatal conductance, forest composition, canopy hydrology, and climate. Based on the previously specified initial diameter distributions, we simulated the growth of the forest over 150 years. Thereafter, the growth process practically comes to a halt and therefore, the time period of 150 years is sufficient to determine the factors of the growth process of the trees.

The generated data of the series of simulations allows to estimate the function \(g(E, L_i)\), which describes the change in diameter over time. The type of the function was specified as a von Bertalanffy growth curve (von Bertalanffy, 1957), generalized by Millar and Myers (1990) which allows the rate of growth of the diameter to vary with environmental conditions specified as the total basal area of all trees whose diameter is greater than that of the individual tree. The concrete specification of the function is given by \(g(E, L_i) = (l_m - L_i)(\beta_0 - \beta_1 BA)\). The exogenous variables of this function are Diameter at Breast Height (DBH) and Basal Area (BA) provided by GOTILWA. The parameter \(\beta_0\) and \(\beta_1\) are proportionality constants and were estimated by OLS. The estimation yielded the growth function: \(g(E, L_i) = (80 - L_i)(0.0070177 - 0.000043079 BA)\).
Other functional forms of $g(E, L)$ were evaluated as well, but they explained the observed variables to a lesser degree.

The value of the tree volume parameters, $tv(L)$, has also been estimated using the data generated with GOTILWA. The tree volume is based on the allometric relation $tv(L)=0.00157387L^{1.745087}$. A study by Cañellas et al. (2000) provides information that allows the marketable part of the tree volume, $mv(L,t)$, to be estimated as a function of the diameter. The marketable part of the timber volume of each tree is an increasing function of the diameter and is given by $mv(L)=0.699+0.0004311L$.

The mortality function was designed based on the survival function of González et al. (2005), and it is given by:

$$
\mu(E,L) = 1 - \frac{1}{\left(1 + \text{Exp}\left(-3.954 + 0.035BA - 2.297(L/A)\right)\right)^2}
$$

Mortality depends on the diameter of the individual, the basal area, and the average age of the stand, denoted by $A$. In accordance with the literature, we assumed that the amount of dead wood in the forest decreases at a constant rate, $\delta=0.2928$. This rate implies that 50% of biomass of trees that die at time $t$ has been decomposed after 20 years and only 10% of the dead-wood biomass remains after 80 years.

### 3.3. Mirabosc Database

The bioeconomic optimization problem was solved for the case of 2 real stands of $Pinus sylvestris$. For this purpose, the initial distributions were specified to reflect the diameter distributions of two stands of Alta Garrotxa. The first one consists eminently of trees with smaller diameters than the second one. Thus, we considered Stand 1 as a young population of trees and Stand 2 as a mature population of trees. In this way the effect of the different initial distributions on the optimization results can be observed.

These initial distributions were obtained from the database Mirabosc of the IEFC (Inventari Ecològic i Forestal de Catalunya). It is an inventory set up from 1988 to 1998 by the CREF (Centre de Recerca Ecològica i Aplicacions Forestals) for the Catalan forests. This database offers a large variety of data such as biomass, above ground production of wood, leaves, branches, and the diameter distribution of the inventoried stand. The initial diameter distributions used are shown in table 1, where Stand 1 represents the young forest distribution and Stand 2 is the mature forest distribution.
3.4. Optimization results

For the numerical solution of the problem considered the Conopt3 solver, available within GAMS, was employed. For a given initial distribution, the numerical solution of the problem determines for every 10 year period the optimal logging, $U_i$, and planting, $P$, the optimal values of the state variables, $X_i$ and $L_i$, and consequently economic variables, such as the revenue from timber sales, cutting costs, planting costs, and of maintenance costs. Optimizations with different random initializations of the control variables also were carried out to assure that the numerical method provides solutions that are independent from the initially chosen values for the numerical optimization technique. All optimizations were carried out on a per-hectare basis.

The empirical analysis begins by calculating the optimal selective-logging regime for a stand of predominantly young trees (Stand 1), given a discount rate of 2%. Figure 1 depicts the evolution of the number of standing trees, the average forest age and the dead-wood volume over time when biodiversity is not considered and when it is considered. Specifically, in the latter case, the selective logging regime has been computed so that it assures a minimum amount of 10 trees with a diameter over 40cm ($h_{\text{min}} = 40$), and a minimum ecological of 10m$^3$/ha of dead wood in the forest ($S_{\text{min}} = 10$). Figure 1 shows that the introduction of biodiversity conservation goals does not have a very significant influence on the optimal selective-logging regime. When biodiversity is considered, it is optimal to slightly increase the investment in the forest in the first periods. However, the number of standing trees decreases in the long-run (see Figure 1a). The introduction of a minimum amount of large-diameter trees in the forest slightly increases its average age in the long-run (see Figure 1b). However, when biodiversity is considered, the dead-wood laying up in the forest ecosystem is more than doubled (see figure 1c).
Figure 1: Variation in the evolution of the main variables over time when accounting for biodiversity

1a) 

1b) 

1c)
Figure 2 depicts the number of trees in each cohort together with their corresponding average diameter, at different years of the planning horizon, to illustrate the optimal evolution of the forest over time and to compare the steady-state distribution of the forest whiteout biodiversity and with biodiversity conservation targets. It shows that in both cases it takes more than 100 years to reach a diameter distribution of the trees which is relatively stable over time. It can also be observed that the introduction of biodiversity in the economic model leads to an increase in the number of cohorts in the steady-state.

Figure 2: Evolution of the diameter distribution over time

Tables 2 and 3 present a more detailed analysis of the numerical results for the case where biodiversity is not considered and the case where it is considered. In particular, it is important to analyze the discounted sum of the net benefits obtained from forest management. In the remaining part of the paper we will refer to it as the NPV of the benefits for short. For a stand of predominantly young trees (Stand 1), Table 2 shows that when biodiversity does not form part of the management objective, the NPV of the benefits over 200 years is about 4514 €/ha. However, when biodiversity goals are incorporated in the formulation of the decision problem, the NPV of the benefits decreases to 4164 €/ha, since the forest owner needs to maintain an amount of big trees that are already out of turn. Thus, the incorporation of biodiversity in the economic problem leads to a decrease of 7.7% in the NPV of benefits obtained from the forest management. This is a very significant result, since it implies that forest owners are not likely to adapt their management regime to incorporate biodiversity targets unless they
are required to. Therefore, it is necessary to design policies that efficiently promote conservation and enhancement of forest biodiversity.

The results of the optimization for the mature stand (Stand 2), presented in Table 3 show a similar pattern in the logging regime. When biodiversity is incorporated in the model, the number of logged trees decreases initially from 389 to 320. Moreover, the number of planted trees at the beginning of the time horizon needs to increase from 108 to 287 in order to achieve the required structure to maintain a minimum volume of dead wood in the forest over time. However, in a mature stand, the logging begins in the initial period of the planning horizon. Consequently, the forest owner obtains his/her first benefits in the initial year and the losses in the NPV of benefits due to the incorporation of biodiversity are less pronounced. The losses of the NPV of benefits in this case amount to 3.9%.

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To analyze to what extent the losses from biodiversity conservation targets can be retained at a low level, we conducted a sensitivity analysis with respect to the required minimum volume of dead wood in the forest, ranging from 0 to 40m$^3$/ha, for the case of the young forest distribution (Stand 1). Figure 2 depicts the NPV of the different optimization scenarios. The figure shows that losses of the NPV of benefits can be substantial. In the extreme case where it is assumed that the forest owner is required to maintain a minimum stock of dead wood of 40m$^3$/ha the NPV of the benefits are only 2201,41€/ha and thus, the losses are above 50% of the benefits obtained from the forest management when biodiversity is not considered.
Figure 2: Net present value of the benefits of forest management over 200 years as a function of the minimum required amount of dead wood.

4. Conclusions

This paper presents a theoretical model that allows us to determine the optimal management of a diameter distributed forest where biodiversity conservation is taken into account. The growth and mortality processes depend not only on individual characteristics but also on environmental characteristics by considering the distribution of the individual characteristics over the entire population. This modeling framework allows us to take into account the fact that the life cycle of each individual tree is affected by the other trees since they compete for light, nutrients and space. The density dependent formulation of the biological growth process leads to a partial integrodifferential equation. The corresponding economic decision problem for determining the optimal forest management can be formulated as a distributed optimal control problem where the control variables and the state variable depend on both time and diameter of the tree.

The resulting necessary conditions of this problem include a system of partial integrodifferential equations that usually cannot be solved analytically. For this reason, the method called Escalator Boxcar Train is used to solve the problem numerically.
In the empirical part, the paper determines the optimal selective logging regime of a diameter-distributed and privately owned forest without biodiversity considerations and with biodiversity considerations, that is, when the forest manager is required to fulfill some requirements over various indirect indicators of biodiversity in forest ecosystems. The chosen indicators are the volume of dead wood existing in the forest and the amount of large-diameter trees. With this purpose an empirical analysis over two real stands of *Pinus sylvestris* in Alta Garrotxa (Catalunya) is conducted, obtaining the optimal management plans. The results demonstrate that properly assessing the link between these indirect indicators and biodiversity is crucial for the correct determination of the optimal forest management. Moreover, the results of this study suggest that the costs of biodiversity conservation in terms of reduced profitability can be significant.
References


Steele, R.C., 1972. Wildlife conservation in woodlands. HMSO. London


Table 1. Initial diameter distributions of the analyzed stands

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<tr>
<th>Diameter Class</th>
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<th>Diameter Class</th>
<th>Nº of trees/ha</th>
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Stand 1
Basal Area: 23.42
UTM X Coordinate: 456600
UTM Y Coordinate: 4664800

Stand 2
Basal Area: 47.82
UTM X Coordinate: 448500
UTM Y Coordinate: 4674900
Table 2: Optimal Selective-Logging Regime of a Young Stand (Stand 1)

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<tr>
<th>Year</th>
<th>Number of trees</th>
<th>Planted trees</th>
<th>Logged trees</th>
<th>Logged volume (m³/ha)</th>
<th>Dead Wood Volume</th>
<th>Average Forest Age (years)</th>
<th>Net revenue from timber sale (€/ha)(b)</th>
<th>Maintenance cost (€/ha)</th>
<th>Planting cost (€/ha)</th>
<th>Net benefit (€/ha)</th>
<th>Discounted net benefit (€/ha)</th>
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Discounted sum over 200 years: 9226.19 - 4353.08 - 359.10 = 4514.01

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<th>Dead Wood Volume</th>
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<th>Net revenue from timber sale (€/ha)(b)</th>
<th>Maintenance cost (€/ha)</th>
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Discounted sum over 200 years: 9063.71 - 4496.74 - 402.76 = 4164.22

(a) The number of trees in the forest is calculated just after the trees are planted, and before the thinning takes place.
(b) All monetary values apart from the discounted net benefit in the last column of the table are expressed as current values.
Table 3: Optimal Selective-Logging Regime of a Mature Stand (Stand 2)

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<th>Dead Wood Volume</th>
<th>Average Forest Age (years)</th>
<th>Net revenue from timber sale (€/ha)(b)</th>
<th>Maintenance cost (€/ha)</th>
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<th>Net benefit (€/ha)</th>
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Discounted sum over 200 years

| Considering only timber                                                                 | 13582.74 | -3994.41 | -514.37 | 9073.96 |
| Considering timber and biodiversity                                                   | 13518.12 | -4217.94 | -583.99 | 8716.18 |

(a) The number of trees in the forest is calculated just after the trees are planted, and before the thinning takes place.

(b) All monetary values apart from the discounted net benefit in the last column of the table are expressed as current value.