Optimal environmental policy, vertical structure and imperfect competition

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Preliminary version. Comments welcome.

Abstract

Environmental policies should take into account imperfect competition along the vertical structure of polluting activities. This paper specifies what an optimal environmental tax would be when the polluting industry is an oligopoly competing *à la Cournot* and when it reduces its pollution by purchasing environmental goods to an eco-industry sector, also assumed to be an oligopoly competing in quantities. Therefore, the tax is a single instrument to regulate three kind of externalities, pollution and two restrictions in production. Consequently, the optimal tax is the result of a trade-off, mainly depending on price-demand elasticities. We put the emphasis on price-demand elasticity on the eco-industry sector, showing two aspects: a strategic one, consequence of the imperfect competition, but also a technical one, linked with the shape of the depollution function. We also show that a no-emission strategy from polluting firms implies an upper limit for the environmental tax, always lower than the threshold leading the firms to leave the market. This allows us to complete our comparison between marginal damage and environmental tax. It is notably shown that there exists a nonmonotonicity in that comparison.

Keywords: eco-industry, imperfect competition, optimal environmental taxation, end-of-pipe pollution abatement, vertical structure

JEL classification: D42, H23, Q58

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1 Introduction

One of the "by-products" of the increasing importance of environmental regulations since the 1980s has been, from polluting firms, the outsourcing of their cleaning-up activities. This fact has allowed the emergence of the eco-industry sector in the economy. A definition has been given by the European Commission in 1994: "Eco-industries may be described as including firms producing goods and services capable of measuring, preventing, limiting or correcting environmental damage such as the pollution of water, air, soil, as well as waste and noise-related problems. They include clean-technologies where pollution and raw-material used is being minimized[1]." According to recent studies[2], the sector represents 2% of GDP in the European Union and at least 2 million jobs. Furthermore, the eco-industry sector is highly concentrated and the main firms are supposed to hold a market power (Barton 1997).

This article presents the environmental taxation policy chosen by a regulator dealing with pollution and imperfect competition. We model two oligopolistic markets, with firms competing *à la Cournot*. Downstream, firms compete for the supply of a final good, purchased by consumers. Upstream, eco-industry firms compete to sell environmental goods and services to polluting firms. For instance, the model illustrates what an environmental policy should be when dealing with water pollution created by a highly concentrated industry. Technologies cleaning up water are supplied by only a few firms (Ondéo and Veolia Environnement in France), which means that the regulator has to face two distortions along the vertical structure of the industry.

We find that an optimal environmental taxation should be the result of a trade-off between two opposite incentives. On the one hand, the inefficient level of production in the final good market tends to induce a lower tax than the pigouvian one. On the other hand, the imperfect competition between eco-industries urges the regulator to increase the tax above the marginal damage of emissions. The overall effect is ambiguous, but two things can be added: first, the relative efficiency of the depollution function plays a key role to solve the trade-off and second, *ceteris paribus*, low (resp. high) values of environmental damage tend to induce a lower (resp. higher) tax than marginal damage. To complete the last point, we have added to the analysis another questioning: by cleaning up its overall pollution rather than paying the resultant tax levied, a firm can modify the regulator's behavior. This "no-emission" strategy of some polluting firms imply an upper

[1] Barton (1997)

[2] Ecotech Research and Consulting Limited (2002)

limit for the environmental tax[3]. It usually happens for high marginal damage of pollution. Therefore, it is shown that there exists a nonmonotonicity in the comparison between the tax chosen and the pigouvian reference, according to the value of marginal social damage of pollution.

If the importance of the eco-industry sector has been recognised by numerous reports from the OECD and the EU and by a few empirical studies by economists (Barton (1997) or Baumol (1995)), the theoretical approach of environmental policy has so far mainly ignored this stylised fact. However, the potential market power of these firms increase the costs of buying environmental goods above the marginal cost of production. This, in turn, can substantially modify the main results of the literature on the optimal environmental taxation.

From a theoretical point of view, there are two ways to deal with environmental policy in the presence of imperfect competition. First, a regulator can exhibit strategic behavior, especially in a context of international trade when commercial policy has been strictly controlled. In that case, environmental policy can be determined as a way to shift rents from foreign countries to national industries. This approach has been studied extensively. For instance, Barrett (1994) has shown that competition *à la Cournot* among polluting firms tends to induce a lower level of tax than marginal social damage. Conversely, competition *à la Bertrand* tends to induce a higher tax. This framework can be complicated (asymmetric information, decreasing returns to scale, R&D) but the idea remains the same: it is possible to adjust the level of tax according to the nature of competition in the market. This in turn should shift rents to national companies. Taking into account the eco-industry sector, Fees & Muehlheusser (2002) explore whether a national leadership in environmental policy can pay off when profits of the environmental industry are specifically introduced. Their results show that the Porter hypothesis can hold as soon as a learning curve is assumed to exist in the environmental sector[4]. In another context of international trade, Greaker (2004) model the optimal simultaneous choice of an emission standard by two countries when polluting firms compete *a la Cournot* and buy abatement services to an upstream monopolistic competitive sector. The author notably shows that a tougher standard can in some cases increase the competitiveness of the downstream sector, because it increases competition among eco-industry firms.

Our paper considers the second way of dealing with environmental policy in a context

[3]A corollary is that a polluting firm always prefers giving up pollution rather than leaving the market.

[4]These results are close to those presented by Greaker (2003), even though the author does not explicitly introduce the eco-industry sector in his analysis.

of imperfect competition: we assume that only firms behave strategically and that the regulator is only present to adjust taxation in order to maximize social welfare. One of the first papers looking for optimal environmental taxation with imperfect competition was Barnett (1980). The author put the emphasis on the importance of price-demand elasticity in the output market. This elasticity determines the level of distortion (compared to marginal damage) necessary to reach the optimal taxation. In a sense, our thesis follows the same methodology as Barnett. We use environmental taxation as an instrument that deals with two sources of misallocation: pollution *and* underproduction. However, the author was not interested in the way polluters were able to clean up. As in most papers, it was assumed that firms were able to produce and invest in environmental goods and services. Nevertheless, we argue that tougher environmental policies have contributed to the emergence of an eco-market. It is the role of the eco-industry sector to respond to these kinds of environmental demands. David & Sinclair-Desgagné (2005) were the first to address the consequences of the market power of the eco-industry sector in terms of optimal environmental policy. Due to the fact that the cost of reducing pollution is higher than usually assumed, the authors prove that there exists an incentive for the regulator to set up a tax higher than marginal damage. We would like to add to their study the conflict between imperfect competition on downstream and upstream industries. A first attempt to deal with both aspects has been made by Okuguchi (2004). The author demonstrates that in a context of Cournot oligopsonistic oligopoly, there exists a unique equilibrium under general assumptions and presents the implications of this optimal tax in three delineated cases. However, Okuguchi introduces imperfect competition in all factor markets and for the purposes of our article, competition in the environmental sector is paramount. We focus on that market. Nimubona & Sinclair-Desgagné (2005) present the trade-off that a regulator faces in the presence of imperfect competition between polluting firms and between environmental firms. They are in a sense very close to the purpose of this document and present similar results. However, the environmental demand is treated as exogenous in their model. By endogenising that demand, we manage to put the emphasis on a technical characteristic of the price-demand elasticity in environmental goods that does not appear in their working paper.

We also introduce the fact that the regulator's environmental policy must take into account corner solutions. They have been introduced when dealing with a stock of pollution[5]. In fact, assuming that a "no-pollution" strategy is chosen implies that the stock

[5]For instance, the impact of corner solutions on the shape of the environmental Kuznets curve.

of pollution does not increase. Therefore, the damage of pollution is not postponed to younger generations. It is also a key element when spatial heterogeneity is introduced among agents. Wu & Babcock (2001) show on an example that corner solutions can reverse a conventional finding of tax or standard superiority based on the relative-slopes rule. Empirical agricultural studies have also shown that a regulator's incentive could cause some cropland to be taken out of production. Even if we remain with a static and symmetric framework here, we manage to show how dealing with corner solutions can complete the analysis of the comparison between marginal damage and environmental tax.

Among the key elements of the model, both price-demand elasticities play a significant role. In fact, they strongly influence the choice of the tax because they indicate to what extent a polluting firm adjusts its level of production and pollution to a certain level of tax. In the model, the price-demand elasticity of the final good is exogenous. However, the price-demand elasticity of the environmental goods is endogenous. As usual, it is a function of the level of competition between firms. More surprisingly, we also show the link between that elasticity and the relative efficiency of the depollution function. As long as interior solutions are chosen, polluting firms reduce production and pollution when facing an increase in the environmental tax. This is not true anymore as soon as firms choose a no-emission strategy, both production and pollution being therefore independent of the tax level. With some specifications, the model is also a way to make a comparison between tax and marginal damage, according to the value of marginal damage of pollution. It is notably shown that there exists an upper limit for the tax, which implies a polluting firm always prefers not polluting rather than leaving the market.

We have structured the paper as follows: the next section presents and solves the model in the general case, where only interior solutions are studied. Section 3 solves the model with specified functions, which allows to introduce corner solutions. Then, section 4 makes the comparison between tax and marginal damage. At last, section 5 concludes and suggests different ways to develop this work further.

2 The model

There are n downstream firms (DFs) on the market and each firm produces a quantity x_i, bought by consumers. By producing this good, each firm also creates a negative externality, making a damage D equals to a constant marginal damage ν times the amount of pollution. Without any incentive to reduce pollution, polluting firms do not take into account this pollution in their behavior, which means that they do not need to buy inputs to the upstream oligopoly.

As a way to reduce pollution induced by DFs' activity, the regulator chooses to introduce an environmental tax, paid on the amount of waste created by each polluting firm[6]. In that case, it becomes important for DFs to consider buying goods (or services) coming from the eco-industry sector. These environmental goods and services help DFs to reduce the level of pollution created and consequently to reduce the amount of tax paid to the regulator. We assume that polluting firms are price-takers on the eco-industry market. It can be justified by the fact that the eco-industry sector supplies many industrial sectors and upstream firms (UFs) are able to discriminate between these markets. Nevertheless, it could have been interesting to look at other ways to analyze vertical relationships dealing with pollution (Hamilton & Requate 2004).

Given the sphere we have chosen to work in, we can now formulate the three-stage game we want to solve[7]:

1. The regulator chooses the optimal tax to control pollution;

2. the firms of the eco-industry sector, anticipating the demand, maximize their profits and fix the price of environmental goods, consequence of their competition in quantity;

3. polluting firms, given the tax and the price of environmental inputs, choose their optimal level of production and pollution.

We can notice that this modeling introduces firms as Stackelberg followers because they consider as given the tax level fixed by the regulator to maximize the social welfare (Petrakis & Xepapadeas 2003). As usual, this game is solved by backward induction.

2.1 Third stage

Each one of the n firms takes as given the level of tax and the price of environmental inputs. They are symmetric and they face a global demand $f(X)$. The cost of production

[6]It is generally assumed that there are three ways to reduce pollution : the first one is to reduce production, the second one is to invest in cleaner technologies and the third one consists in reducing *ex post* an end-of-pipe pollution. Here, the model catches the first and third ways.

[7]The last two stages could have been seen as simultaneous

of each firm is summarized in the function $c_d(x_i)$. So, downstream firms maximize the following profit function:

$$\Pi_i = f(X)x_i - c_d(x_i) - pa_i - ts(x_i, a_i) \; \forall \; i \; \in [1, \dots, n] \tag{1}$$

where p is the price of the environmental input a, t is the environmental tax and $s(x_i, a_i)$ is the net amount of pollution left by each polluting firm. We consider here an end-of-pipe pollution, meaning that DFs produce, and then consider the optimal level of polluted waste they should leave, considering that it is on that amount that they pay the tax[8]. So, the last function can be specified as follows:

$$s(x_i, a_i) = \epsilon(x_i) - w(a_i) \; \forall \; i \; \in [1, \dots, n] \tag{2}$$

where $\epsilon(x_i)$ measures the link between production and polluting waste and $w(a_i)$ expresses the amount of pollution cleaned by the purchase of a_i environmental goods. This function tends to reflect the growing difficulty in cleaning up polluted waste. A few assumptions need to be made about these functions[9]:

$$f'(X) < 0, \; c_d'(x_i) > 0, \; w'(a_i) > 0, \; w''(a_i) < 0, \; \epsilon'(x_i) > 0 \tag{3}$$

These assumptions are consistent with standard economics and should note create any problem of interpretation. Before presenting the first order conditions of this stage, it should be noted that we only solve the problem in the general case, i.e. the interior solutions, knowing that corner solutions could appear. They are introduced in the next section. So, each polluting firm maximizes its profits considering two variables, x_i and a_i.

$$\frac{\partial \Pi_i}{\partial x_i} = 0 \Rightarrow f(X) + f'(X)x_i - c_d'(x_i) - t\epsilon'(x_i) = 0 \tag{4}$$

$$\frac{\partial \Pi_i}{\partial a_i} = 0 \Rightarrow -p + tw'(a_i) = 0 \tag{5}$$

As usual, the existence and unicity of a solution is ensured by assuming the demand function is not overly convex compared with the convexity of the cost function. Due to the construction of the s function, the marginal efficiency of environmental goods does

[8]Even though it is quite a strong assumption, it is often made in that kind of modeling, for instance by Ulph (1996). Indeed, it does not signify that the output is not influenced by the level of tax.

[9]We also assume that conditions on the limit of the functions are satisfied.

not depend on the amount of pollution. From this program, it is possible to find x_i^* as a function of t and a_i^* as a function of t and p. We can notice that by construction, x_i does not depend on p, but it is also true that DFs question the opportunity of producing as much as before the introduction of the tax.

2.2 Second stage

Once the third stage is solved, we introduce the inverse demand function of environmental goods into UFs' programs. The overall demand is the sum of demands coming from the different polluting firms. This demand can be summarized as follows:

$$p(A) = tw'(a_i) \ \forall \ i \in \ [1, \ldots, n] \tag{6}$$

$$A = \sum_{i=1}^{n} a_i = \sum_{i=1}^{n} w'^{-1}(\frac{p}{t}) \tag{7}$$

Then, each one of the m firms of the eco-industry sector wants to maximize the following program:

$$\Pi_j = p(A)a_j - c_u(a_j) \ \forall \ j \ \in [1, \ldots, m] \tag{8}$$

where $c_u(a_j)$ is the cost function of UFs, with the usual properties. The first and second order conditions are:

$$\frac{\partial \Pi_j}{\partial a_j} = 0 \Rightarrow p'_{a_j}(A)a_j + p(A) - c'_u(a_j) = 0 \tag{9}$$

$$\frac{\partial^2 \Pi_j}{\partial a_j^2} < 0 \Rightarrow p''_{a_j}(A)a_j + 2p'_{a_j}(A) - c''_u(A) < 0 \tag{10}$$

We can check the existence and unicity of the solution. First, let us rewrite the FOCs, in order to introduce the elasticity of demand on that market. We have:

$$\frac{\partial \Pi_j}{\partial a_j} = 0 \Rightarrow p(A)[1 + a_j \frac{p'_{a_j}(A)}{p(A)}] - c'_u(a_j) \tag{11}$$

We denote $F(a) = p(A)[1 + a_j \frac{p'_{a_j}(A)}{p(A)}]$ and $G(a) = c'_u(a_j)$. A condition on the existence of a solution is $F(a) > 0$ as we have made the assumption that $G(a) > 0$. As we also know that $p(A) > 0$ and $p'_{a_j}(A) < 0$, it provides a condition on the inverse function of the price-demand elasticity:

$$-1 < a_j \frac{p'_{a_j}(A)}{p(A)} = e_{p(A)/a_j} < 0 \tag{12}$$

which means:

$$-\infty < e_{a_j/p(A)} < -1 \tag{13}$$

This condition can be interpreted easily: the price-demand elasticity that each firm faces has to be negative and elastic enough, at least at the equilibrium, otherwise it is always better for the firm, given the expected reactions, to increase its quantities, as the price will drop in a lower proportion, leaving room for growing profits. Let us now introduce the demand coming from the downstream market. We know that each polluting firm seeks to equalize the price of environmental goods with the marginal benefit of reducing the pollution of one unit:

$$p = tw'(a_i) \; \forall \; i \; \in [1, \ldots, n] \tag{14}$$

It is therefore possible to rewrite the inverse demand function in the first order condition of each eco-industry firm:

$$tw'(a_i)[1 + a_j \frac{\frac{1}{n} w''_{a_j}(a_i)}{w'(a_i)}] - c'_u(a_j) = 0 \tag{15}$$

We can immediately notice that:

$$e_{p(A)/a_j} = a_j \frac{p'_{a_j}(A)}{p(A)} = e_{w'(a_i)/a_j} \tag{16}$$

which gives a condition on $w'(a_i)$: $-1 < e_{w'(a_i)/a_j} < 0$[10]. The elasticity can then be interpreted through two aspects. First, there is a strategic effect coming from the competition in the eco-industry sector. Each firm takes into account the externality it creates on the market price by increasing its quantities. Second, this result introduces the link between technical characteristics (environmental goods' efficiency to clean up a certain amount of pollution) and the price-demand elasticity. In other words, the shape of the function w plays a significant role in the price-demand elasticity. For instance, if we assume that

[10] It is also possible to check that the conditions to ensure a solution to our program are also sufficient to be sure that the solution is unique (considering that the cost of production is increasing or constant).

$w(a)$ is a power function, the power μ of the function has to be contained between 0 and 1[11]. This signifies that $w'''(a)$ is positive. Considering we have already assumed $w'(a) > 0$ and $w''(a) < 0$, it means that pollution always decreases with an increase in the number of environmental goods purchased, the last unit purchased being less efficient than the previous one, this effect taking place at an increasing rate.

Proposition 1 *(i) The demand elasticity on the environmental market is determined by both a strategic effect induced by imperfect competition among eco-industry firms and by technical characteristics linked with the depollution functions of downstream firms; (ii) having an equilibrium on the environmental market imposes a restriction on the scope of $w(a)$.*

To conclude, we have shown the conditions under which the last two stages can have a solution. However, the firms react taking as given the value of the environmental tax. The next step consists for the regulator in choosing the environmental tax.

2.3 First stage

The regulator wishes to maximize the following welfare function:

$$W = \int_0^X f(u)du - nc_d(x_i) - mc_u(a_j) - n\nu s(x_i, a_i) \tag{17}$$

The first part of this function considers the consumers' surplus. Then, we take into account the cost functions of DFs and UFs, the supply of environmental goods being only a transfer between firms. The last part of the surplus measures the damage induced by pollution[12]. The optimal pollution tax requires to satisfy the following condition:

$$0 = nf(X)\frac{dx_i}{dt} - nc_d'(x_i)\frac{dx_i}{dt} - mc_u'(a_j)\frac{da_j}{dt} - n\nu[\frac{\partial s}{\partial x_i}\frac{dx_i}{dt} + \frac{\partial s}{\partial a_j}\frac{da_j}{dt}] \tag{18}$$

In order to make a comparison of our work with that of Barnett (1980), the results are presented as done in the article cited. The details leading to the following expression can be found in Appendix 1[13].

[11] We need to respect Equation 12. The elasticity of $w'(a_i)$ can be simplified as follows: $a_j\mu\frac{(a_i)^{\mu-1}}{(a_i)^{\mu}} = a_j\mu(\frac{A}{n})^{-1} = \frac{na_j}{A}\mu = \mu$.

[12] Two comments can be made about the amount of taxes collected by the regulator: first we do not take into account any opportunity cost of gathering this money. Indeed, we assume taxes are reallocated with neutrality in the economy.

[13] This expression only gives an implicit function of t, t is on both sides of the equation.

$$t^* = \nu \frac{\left[\epsilon'(x_i)\frac{dx_i}{dt} - w'(a_i)\frac{da_i}{dt}\right]}{\left[\epsilon'(x_i)\frac{dx_i}{dt} - (1 - |e_{w'(a_i)/a_j}|)w'(a_i)\frac{da_i}{dt}\right]} + \frac{\frac{f(X)}{e_{x_i/f(X)}}\frac{dx_i}{dt}}{\left[\epsilon'(x_i)\frac{dx_i}{dt} - (1 - |e_{w'(a_i)/a_j}|)w'(a_i)\frac{da_i}{dt}\right]} \tag{19}$$

Before analyzing this expression, we consider the expected variations of a_i and x_i according to t. The following expressions are explained in Appendix 2:

$$\frac{dx_i}{dt} = \frac{\epsilon'(x_i)}{\frac{\partial^2 \Pi_i}{\partial x_i^2}} \tag{20}$$

$$\frac{da_i}{dt} = \frac{-w'(a_i)}{\frac{\partial^2 \Pi_i}{\partial a_i^2}} - m\frac{\frac{\partial^2 \Pi^j}{\partial a_j \partial t}}{\frac{\partial^2 \Pi^j}{\partial a_j^2}} \tag{21}$$

Lemme .1 *An increase in t always induces a reduction in the output produced in the downstream market and higher levels of environmental goods purchased*[14]*; the higher the number of eco-industry firms, the more important the efforts in terms of cleaning-up activities from polluting firms.*

About variations in x_i, the numerator is by definition positive (because of the positive link assumed between production and pollution) and the denominator is necessarily negative at the equilibrium. As far as the variations of a_i are concerned, they are the consequence of two effects: the first part of the RHS of Equation 21 is positive. It is also the case for the second part, which measures UFs' reactions to the tax. $\frac{\partial^2 \Pi_j}{\partial a_j \partial t}$ has to be positive, otherwise there is no solution for the profit maximization of an eco-industry firm. Indeed, the more important the number m of environmental firms, the higher the competition between them. Consequently, it becomes cheaper to buy environmental goods and DFs are more receptive to an increase in t.

2.4 Analysis of the results

Even though it is not possible to give an unambiguous result, it is still worth presenting three delineated cases. First, let us assume the elasticity of demand in the downstream market is infinite. This means that polluting firms have no power market, and that they

[14]This holds true when t is strictly positive and until a state of no-emission is reached.

take the price as given. DFs would then be a sample of n identical firms in a market of perfect competition. Consequently, the second part of Equation 19 will tend to 0. Once again, because of the condition on the price-demand elasticity in the upstream market, we can be sure that imperfect competition in the upstream market only induces a value of t^* higher than marginal damage, as soon as there is a market for environmental goods. As already explained in David & Sinclair-Desgagné (2005), "the price of abatement goods and services will normally be greater than their marginal cost. In this context, if the tax t was to be set equal to the marginal damage ν, the polluter would settle for an abatement level that is too small relative to the first-best".

On the other hand, another interesting case appears when the depollution function is linear in a ($w(a) = ka$). Then, the elasticity of $w'(a_i)$ is zero and the demand elasticity of the upstream market tends to $-\infty$. In other words, UFs must take the price of A as given, they cannot manipulate it and will seek to equalize their marginal cost to $\frac{t}{k}$. Each polluting firm becomes indifferent to buying the environmental goods or paying the resultant tax. However, it does not mean that the influence of imperfect competition in the upstream market disappears. Indeed, the final level of t^* remains uncertain. It is only when $k = 1$ that UFs must fix their price at their marginal cost. We therefore return to the solution already explained by Barnett (1980), among others. The optimal tax will be lower than marginal damage as soon as polluting firms hold a market power. This delineated case also shows that if the marginal efficiency of environmental goods is constant, the role played by the eco-industry is neutral in the process of the choice of optimal pollution.

Lemme .2 *The eco-industry sector holds a market power provided the marginal efficiency of environmental goods is decreasing.*

The last delineated case allows us to check that our model is consistent with the pigouvian approach, because when there is not any market power, the tax will be set equal to the marginal social damage. In a more general case, the expected consequences of imperfect competition on the environmental tax can be summarized in the next proposition:

Proposition 2 *(i) In a context of double oligopoly, the optimal tax is always the result of a trade-off between two antagonistic effects: the inefficient level of production in the final good market tends to induce a lower tax than the pigouvian one; however, the imperfect competition in the upstream market urges the regulator to increase the tax above the marginal damage of emissions; (ii) the overall effect is ambiguous.*

If we make a comparison of our results with those of Nimubona & Sinclair-Desgagné (2005) and Greaker (2004), the former put the emphasis on strategic effects on both markets whereas the latter add a price effect introduced by increased R&D. What we show here is the key role played by a technical effect, i.e. the importance of the relative efficiency of environmental goods and services.

3 The possibility of a no-emission strategy

Until now, we have only seen emissions on a period without considering their impact on the stock of pollution. Here, we study under which conditions, in a static model, it is possible to stop the increase in the stock of pollution, i.e. to annul emissions. Even though environmental goods get less and less efficient, we are going to show that cleaning up all polluted waste is always better than leaving the market for downstream firms.

We focus on a representative firm on each market and deal with the successive relationships between firms and regulator, without excluding a priori the possibility that a polluting firm cleans-up the whole amount of pollution. It is shown that this possibility substantially modifies the results at different stages of the model. We specify some of the functions we use. Let us denote:

$$w(a_i) = a_i^{\mu} \ ; \ \epsilon(x_i) = \epsilon x_i \ ; \ f(X) = \alpha - \beta X \ ; \ c(x_i) = c_d x_i \ ; \ c_u(a_j) = c_u a_j$$

3.1 DFs' inverse demand

As we analyze the reduction of pollution as an end-of-pipe decision, it is possible to split DFs' program into two components. First, each polluting firm tries to minimize the cost of depollution:

$$\min_{a_i \in R_+} pa_i + t(\max\{\epsilon(x_i) - w(a_i);\ 0\}) \tag{22}$$

As we know that $\epsilon(x_i) - w(a_i)$ will never be negative because depollution is costly, we can rewrite the program:

$$\min_{w(a_i) \leq \epsilon(x_i)} pa_i + t(\epsilon(x_i) - w(a_i)) \tag{23}$$

The first order condition, in a Kuhn-Tucker program, can then be written as follows:

$$p - t\mu a_i^{\mu-1} + \lambda\mu a_i^{\mu-1} = 0 \tag{24}$$

with $\lambda \geq 0$. The interior solution induces that:

$$a_i^* = (\frac{p}{t\mu})^{\frac{1}{\mu-1}} = w'^{-1}(\frac{p}{t}) \tag{25}$$

If the corner solution is chosen, we have:

$$w(a_i) = a_i^\mu = \epsilon x_i \Rightarrow a_i^* = (\epsilon x_i)^{\frac{1}{\mu}} \tag{26}$$

Consequently, λ can be specified as follows:

$$\lambda = t - \frac{p}{\mu(\epsilon x_i)^{\frac{\mu-1}{\mu}}} \geq 0 \tag{27}$$

We have now determined an optimal level of a_i according to the values of the parameters. As we assume symmetric firms, the choice is the same for each polluting firm. Therefore, it is possible to reveal the cost of depollution as a two-part function (total and partial depollution) that only depends on t, p and x_i.

$$\begin{cases} C_t(p,t,x_i) &= p(\epsilon x_i)^{\frac{1}{\mu}} \\ C_p(p,t,x_i) &= p(\frac{p}{t\mu})^{\frac{1}{\mu-1}} + t(\epsilon x_i - (\frac{p}{t\mu})^{\frac{\mu}{\mu-1}}) \end{cases}$$

We can work out the marginal cost of depollution according to x_i. We have:

$$\begin{cases} \partial_{x_i} C_t(p,t,x_i) &= \frac{p\epsilon}{\mu}(\epsilon x_i)^{\frac{1}{\mu}-1} \\ \partial_{x_i} C_p(p,t,x_i) &= t\epsilon \end{cases}$$

It then becomes possible to find the value x_0 that equalizes both marginal costs[15].

$$x_0 = \frac{1}{\epsilon}\frac{p}{\mu t}^{\frac{\mu}{\mu-1}} \tag{28}$$

The solution of DFs' program is given by the comparison between marginal cost of depollution and marginal profit of production. The marginal profit of production depends on the number of firms, as each firm competing in quantity anticipates the expected reactions from the others:

$$\begin{cases} \alpha - (n+1)\beta x_i - c_d &= \frac{p\epsilon}{\mu}(\epsilon x_i)^{\frac{1}{\mu}-1} \\ \alpha - (n+1)\beta x_i - c_d &= t\epsilon \end{cases}$$

[15]We can check that the marginal cost function is continuous in x_0.

When a polluting firm wants to avoid paying the tax, we know that: $x_i = \frac{a_i^\mu}{\epsilon}$. As we also know, from the FOC of DFs' program for the interior solution that $p - t\mu a_i^{\mu-1} = 0$, we can now give the two inverse demand functions of a polluting firm for environmental goods a:

$$\begin{cases} p &= \frac{\mu}{\epsilon}\left(\frac{\alpha - c_d - (n+1)\beta(\frac{a_i^\mu}{\epsilon})}{a_i^{1-\mu}}\right) \\ p &= t\mu a_i^{\mu-1} \end{cases}$$

These expressions are equal in a_0, which gives us the transition between both demands. We have:

$$a_0 = [\frac{\epsilon(\alpha - c_d - t\epsilon)}{(n+1)\beta}]^{\frac{1}{\mu}} \tag{29}$$

Here again, we can check the continuity of the demand functions in a_0.

3.2 UFs' program

From these demand functions, it then becomes possible to solve UFs' programs in the general case where both interior and corner solutions are taken into account. The first rows of the following expressions refer to total depollution and the second rows to partial depollution decisions. We recall that: $a_i = A/n = \frac{ma_j}{n}$

$$\begin{cases} \Pi_{j_t} &= [\frac{\mu}{\epsilon}(\frac{\alpha - c_d - \frac{(n+1)\beta}{\epsilon}(\frac{ma_j}{n})^\mu}{(\frac{ma_j}{n})^{1-\mu}}) - c_u]a_j \\ \Pi_{j_p} &= (t\mu(\frac{ma_j}{n})^{\mu-1} - c_u)a_j \end{cases}$$

The first and second order conditions are therefore:

$$\begin{cases} \frac{\alpha - c_d}{\epsilon}\mu^2(\frac{ma_j}{n})^{\mu-1} - \frac{2(n+1)\beta}{\epsilon^2}\mu^2(\frac{ma_j}{n})^{2\mu-1} - c_u &= 0 \\ t\mu^2(\frac{ma_j}{n})^{\mu-1} - c_u &= 0 \end{cases}$$

$$\begin{cases} \frac{\alpha - c_d}{\epsilon}\mu^2(\mu-1)(\frac{ma_j}{n})^{\mu-2} - \frac{2(n+1)\beta}{\epsilon^2}\mu^2(2\mu-1)(\frac{ma_j}{n})^{2\mu-2} &< 0 \\ t\mu^2(\mu-1)(\frac{ma_j}{n})^{\mu-2} &< 0 \end{cases}$$

You can find in Appendix 3 the line of argument leading to the characterization of the three possible solutions for UFs' programs. First, both FOCs are positive in a_0: it means that the solutions of UFs' programs lead to total depollution. Second, the first FOC is positive and the second one is negative: the solution of the programs leads to a consumption of environmental goods equals to a_0 for each polluting firm. Then, the overall demand is shared between environmental firms. Third, both FOCs are negative: this is the general case, where DFs partly cleans up and pays the tax. Then, we just have to characterize the range of parameters that leads to these different situations. The first FOC is positive in a_0 as soon as:

$$\frac{\mu^2 a_0^{\mu-1}}{\epsilon}[\alpha - c_d - \frac{2(n+1)\beta}{\epsilon} a_0^{\mu}] - c_u > 0 \tag{30}$$

which after substitution leads to the following condition:

$$2t\epsilon - \frac{c_u \epsilon}{\mu^2}(\frac{\epsilon(\alpha - c_d - t\epsilon)}{(n+1)\beta})^{\frac{1-\mu}{\mu}} > \alpha - c_d \tag{31}$$

In other words, if t is fixed above the previous threshold, polluting firms favor a strategy leading to total depollution. We are in the case where the stock of pollution will not increase. Second, we can characterize the sign of the second FOC in a_0 as follows:

$$t\mu^2(a_0^{\mu-1}) - c_u < 0 \tag{32}$$

when

$$\alpha - c_d > \frac{(n+1)\beta}{\epsilon}(\frac{c_u}{t\mu^2})^{\frac{\mu}{\mu-1}} + t\epsilon \tag{33}$$

If t is fixed below this threshold, an interior solution is chosen. Apart from these thresholds, two other possibilities have to be presented. First, the tax can be negative. It appears when the marginal damage of pollution is lower than the distortion caused by underproduction in the downstream market. Second, the level of tax can make firms leave the market. It appears when the optimal level of production is negative, i.e. $t > \frac{\alpha - c_d}{\epsilon}$. The next proposition summarizes the different cases:

Proposition 3 *Five kind of reactions are expected according to the environmental tax value*[16]*:*

- *If the tax is negative, there is no market for depollution as the subsidy is positively linked to the amount of pollution.*
- *If the final good market is large enough to support distortions, which means the following condition respected ($\alpha - c_d > \frac{(n+1)\beta}{\epsilon}(\frac{c_u}{t\mu^2})^{\frac{\mu}{\mu-1}} + t\epsilon$) then DFs buy environmental goods and pay the tax.*

[16] Given what we have proved earlier about the sense of variation in the difference between FOCs, we are sure that the tax t threshold of our second bullet point takes lower values that the threshold of the third bullet point.

- *If the previous condition is not respected and* $2t\epsilon - \frac{c_u\epsilon}{\mu^2}(\frac{\epsilon(\alpha - c_d - t\epsilon)}{(n+1)\beta})^{\frac{1-\mu}{\mu}} < \alpha - c_d$*, DFs purchase* a_0*, cleaning up all the pollution coming from the process of production.*

- *If* $2t\epsilon - \frac{c_u\epsilon}{\mu^2}(\frac{\epsilon(\alpha - c_d - t\epsilon)}{(n+1)\beta})^{\frac{1-\mu}{\mu}} > \alpha - c_d$*, DFs also refuse to pay the tax, the environmental goods purchased following the second demand function.*

- *If* t *is fixed above* $\frac{\alpha - c_d}{\epsilon}$*, polluting firms leave the market.*

3.3 The regulator's behavior

If the previous proposition characterizes the expected reactions from firms, we have not yet analyzed the regulator's behavior. In other words, we need to introduce the last stage of the resolution, the choice of t^*.

The regulator knows that there exists a tax limit above which DFs clean their whole polluted waste. First, the welfare function is continuous in the point where DFs give up pollution. Second, once this threshold is overtaken, x_i and a_i do not depend on t[17]. This can be shown by looking at the solution of DFs' program in the case of total depollution. The optimal level of a_i purchased is then fully determined by the level of production, so firms' both decisions are taken without any consideration for the level of tax.

It means that, in the range of parameters where it is optimal not to pay the tax, the overall welfare remains constant. There is no marginal income for the regulator if she chooses to increase the tax, simply because there is no more pollution and then no more income raised. Total depollution will always lead to productions a_0 and x_0, the optimal level of tax t_0 being the solution of Equation 33.

If we make the assumption that a tax is received as a bad signal by the agents, it is always better for the regulator to choose the lowest tax giving her the same welfare.

Proposition 4 *(i) As soon as total depollution is chosen by polluting firms, the overall surplus does not depend on variations of* t*; (ii) Consequently, an upper limit exists in the optimal taxation scheme of the regulator.*

[17] Indeed, we can remark that an exogenous shock on the level of final good production will also increase the level of environmental goods purchased, which would not necessarily have been the case when polluting firms polluted and paid the tax.

4 A comparison between tax and marginal damage

The aim of this section is to give an overview of the comparison between marginal damage and environmental tax, taking into account imperfect competition along the vertical structure of polluting activities and the possibility of corner solutions. All results are presented according to the value of marginal damage and allow to make comparative statics, but to give explicit thresholds, we need to fix $\mu = 1/2$. First, we can explicit the upper limit of taxation.

$$t^*_{max} = \hat{t} = \frac{4\epsilon c_u(\alpha - c_d)}{(n+1)\beta + 4\epsilon^2 c_u} \tag{34}$$

When marginal damage of pollution is higher than $\hat{t}$, the optimal tax will always be lower than marginal damage. First, it is interesting to see that this threshold decreases with n, the number of firms on the final good market. This can signify that as competition increases on the final good market, the regulator can concentrate her efforts to regulate underproduction on the environmental market and reduce pollution.

We can also notice that when $\alpha - c_d$ increases, t^*_{max} increases. It becomes less likely that the tax will be set below marginal damage. This is coherent, because an increase in α also increases the price-elasticity in the downstream market, which reduces DFs' market power. Then, the regulator is more able to focus on regulating the environmental externality, rather than underproduction. Furthermore, an increase in β will decrease t^*_{max}, leading to the opposite interpretation. Indeed, if c_u increases, t^*_{max} is higher: environmental goods become more expensive, cleaning-up the whole pollution is not so much interesting. At last, positive variations for low values of ϵ increase the threshold but for high values, it is the opposite.

A remark can be made about this threshold: whatever the value of the parameters, $\hat{t}$ is always lower than the threshold that decides a firm to leave the market ($t \geq \frac{\alpha - c_d}{\epsilon}$).

Proposition 5 *In the model, the polluting firm always prefers having a no-emission strategy rather than leaving the market.*

We can also present what the optimal tax will be for interior solutions and the specified functions chosen. After a few calculations, we find:

$$t^* = \nu\Big(\frac{2(n+1)^3\beta + 8c_u\epsilon^2(n+1)}{(n+1)^3\beta + 8c_u\epsilon^2 n}\Big) - \frac{8c_u\epsilon(\alpha - c_d)}{(n+1)^3\beta + 8c_u\epsilon^2 n} \tag{35}$$

First, if you look at the limit of the tax when $n \rightarrow +\infty$, you find that $t^* = 2\nu = \frac{1}{\mu}\nu$. Moreover, the tax is positively linked with marginal damage and negatively with $\alpha - c_d$. The wider the market is, the more important the distortions caused by underproduction are, so the regulator is prone to favor the downstream market via a low environmental tax. The opposite argument can be used for variations in β. Moreover, it is surprising to find that an increase in c_u decreases the level of tax. At last, variations for low values of ϵ decrease the threshold but high values increase it.

Between both values already presented, there is one main difference: $\hat{t}$ does not depend on marginal damage of pollution whereas t^* increases in ν. So, for a certain value of ν, t^* is higher than $\hat{t}$ and the regulator chooses a second-best policy. The switch to the second-best policy happens when:

$$\hat{\nu} \geq \hat{t}\frac{(n^2 + 2n + 3)\frac{\beta}{2} + 4c_u\epsilon^2}{(n+1)^2\beta + 4c_u\epsilon^2} \tag{36}$$

$\hat{\nu}$ is always lower than $\hat{t}$. As soon as marginal damage is higher than $\hat{\nu}$, the tax chosen by the regulator is $\hat{t}$, consequence of the no-emission choice of polluting firms. For environmental damages included between $\hat{\nu}$ and $\hat{t}$, the tax is above marginal damage. Afterwards, the tax remains below ν.

In the range of parameters leading to an interior solution, we have shown that there exists a trade-off between the upstream distortion, the downstream one, and pollution. In that case, it is also possible to make a comparison between tax and marginal social damage. The next threshold can also be expressed as a function of ν:

$$\tilde{\nu} = \frac{8(\alpha - c_d)c_u\epsilon}{(n+1)^3\beta + 8c_u\epsilon^2} \tag{37}$$

It can be checked that $\tilde{\nu}$ is always lower than $\hat{\nu}$. When marginal damage is lower (resp. higher) than $\tilde{\nu}$, the regulator chooses a tax below (resp. above) the pigouvian one. There is also the limit case where it is better to subsidy pollution. It appears when ν is lower than:

$$\underline{\nu} = \frac{4(\alpha - c_d)c_u\epsilon}{(n+1)^3\beta + 4c_u\epsilon^2)} \tag{38}$$

We can now summarize and formulate all the results in the following proposition.

Proposition 6 *There exists a nonmonotonicity in the comparison between tax and mar-*

ginal damage, mainly due to the fact that the optimal tax has an upper limit. The comparison can be expressed in the following table:

$0 < \nu \leq \underline{\nu}$	$\underline{\nu} \leq \nu \leq \tilde{\nu}$	$\tilde{\nu} \leq \nu \leq \hat{\nu}$	$\hat{\nu} \leq \nu \leq \hat{t}$	$\nu > \hat{t}$
$t < 0$	$t = t^* < \nu$	$t = t^* > \nu$	$t = \hat{t} > \nu$	$t = \hat{t} < \nu$

5 Conclusion

In conclusion, this work has demonstrated the overall impacts of imperfect competition on the vertical structure of polluting activities. In this context, the optimal tax to regulate pollution is set up as the result of a trade-off between two antagonistic elements. The first effect tends to reduce the tax as a way to regulate efficiently production in the downstream market. The second effect pushes up the tax in order to compensate the reduction in environmental goods supplied by the upstream industry.

A few key elements should decide which effect will get the better. Among them, the price-demand elasticities on both markets are fundamental parameters. The elasticity in the downstream market is exogenous in our model. Nevertheless, it is shown that the elasticity in the upstream market is determined by both the depollution function, i.e. the amount of environmental goods necessary to reduce marginal pollution and strategic effects as a result of the imperfect competition in the upstream market. Furthermore, it is also proved that a no-emission strategy from firms imply that the regulator has no interest in breaking an upper limit in the tax level. This, in turn, implies a nonmonotonicity in the comparison between tax and marginal damage.

However, this model is only a first step in this new strand of research, which attempts to take explicitly into consideration the eco-industry sector in environmental policies. First, following Hamilton & Requate (2004), it could be interesting to try to model differently the vertical relationships between eco-industries and polluting firms. Second, a change in the nature of competition would introduce new approaches in the analysis. An asymmetric oligopoly would generalize this work and could underline interesting intuitions coming from the appearance of no-emission strategies from some of the firms. Third, introducing dynamics in the pollution stock could modify substantially a multi-period environmental policy.

6 Appendix

Appendix 1: an expression of the optimal tax

We know, from stage 2 and stage 3 of our game, that:

$$c_d'(x_i) = f(X) + \frac{\partial f(X)}{\partial x_i} x_i - t\epsilon'(x_i) \tag{39}$$

$$p(A) = tw'(\frac{A}{n}) \tag{40}$$

$$c_u'(a_j) = p(A) + \frac{\partial p(A)}{\partial a_j} a_j \tag{41}$$

So, we can rewrite Equation 18 as follows:

$$0 = -n\frac{\partial f(X)}{\partial x_i}\frac{dx_i}{dt}x_i + n\epsilon'(x_i)\frac{dx_i}{dt}t - m\{p(A) + \frac{\partial p(A)}{\partial a_j}a_j\}\frac{da_j}{dt} - n\nu[\epsilon'(x_i)\frac{dx_i}{dt} - w'(a_i)\frac{da_i}{dt}] \tag{42}$$

As we also know that:

$$p(A) + \frac{\partial p(A)}{\partial a_j}a_j = tw'(\frac{A}{n})(1 - |e_{w'(\frac{A}{n})}|) \tag{43}$$

$$m\frac{da_j}{dt} = \frac{dA}{dt} = n\frac{da_i}{dt} \tag{44}$$

we have now:

$$t\Big[\epsilon'(x_i)\frac{dx_i}{dt} - (1 - |e_{w'(a_i)}|)w'(a_i)\frac{da_i}{dt}\Big] = \nu\Big[\epsilon'(x_i)\frac{dx_i}{dt} - w'(a_i)\frac{da_i}{dt}\Big] + \frac{\partial f(X)}{\partial x_i}\frac{dx_i}{dt}x_i \tag{45}$$

which, rearranging the expression, gives:

$$t^* = \nu\frac{\Big[\epsilon'(x_i)\frac{dx_i}{dt} - w'(a_i)\frac{da_i}{dt}\Big]}{\Big[\epsilon'(x_i)\frac{dx_i}{dt} - (1 - |e_{w'(a_i)}|)w'(a_i)\frac{da_i}{dt}\Big]} + \frac{\frac{\partial f(X)}{\partial x_i}\frac{dx_i}{dt}x_i}{\Big[\epsilon'(x_i)\frac{dx_i}{dt} - (1 - |e_{w'(a_i)}|)w'(a_i)\frac{da_i}{dt}\Big]} \tag{46}$$

It is easier to interpret this equation if we introduce the price-demand elasticities in the downstream and upstream markets. The equation can then be rewritten as follows:

$$t^* = \nu \frac{\left[\epsilon'(x_i)\frac{dx_i}{dt} - w'(a_i)\frac{da_i}{dt}\right]}{\left[\epsilon'(x_i)\frac{dx_i}{dt} - (1-|e_{w'(a_i)}|)w'(a_i)\frac{da_i}{dt}\right]} + \frac{\frac{f(X)}{e_{x_i/f(X)}}\frac{dx_i}{dt}}{\left[\epsilon'(x_i)\frac{dx_i}{dt} - (1-|e_{w'(a_i)}|)w'(a_i)\frac{da_i}{dt}\right]} \tag{47}$$

Appendix 2: variations of a_i and x_i according to t

The first step to reveal these variations is to make the total differentiation of the DFs' FOCs. We have:

$$\begin{pmatrix} \frac{\partial^2 \Pi_i}{\partial x_i^2} & \frac{\partial^2 \Pi_i}{\partial x_i \partial a_i} \\ \frac{\partial^2 \Pi_i}{\partial a_i \partial x_i} & \frac{\partial^2 \Pi_i}{\partial a_i^2} \end{pmatrix} \begin{pmatrix} dx_i \\ da_i \end{pmatrix} = \begin{pmatrix} \epsilon'(x_i) & 0 \\ -w'(a_i) & 1 \end{pmatrix} \begin{pmatrix} dt \\ dp \end{pmatrix}$$

The first matrix is the Hessian one already studied. We know that it is easy to invert it. Setting up $Y = \frac{\partial^2 \Pi_i}{\partial x_i^2}$ and $Z = \frac{\partial^2 \Pi_i}{\partial a_i^2}$, we are able to write:

$$\begin{pmatrix} dx_i \\ da_i \end{pmatrix} = \begin{pmatrix} \frac{\epsilon'(x_i)}{Y} & 0 \\ \frac{-w'(a_i)}{Z} & \frac{1}{Z} \end{pmatrix} \begin{pmatrix} dt \\ dp \end{pmatrix}$$

We then have the expected reactions of DFs according to variations in t and p. The problem is that p does depend on t (p is a function of A and A varies with t), which means that the impacts on x_i and a_i are not fully specified at this stage. We can rewrite the variations of p according to t as follows :

$$\frac{dp}{dt} = p'(A)(\frac{dA}{dt}) \tag{48}$$

As $\frac{dA}{dt}$ is also equal to $m\frac{da_j}{dt}$, the variations of p according to t can be calculated from the total differentiation of UFs' FOC. We find that:

$$mp'(A)(-\frac{\frac{\partial^2 \Pi_j}{\partial a_j \partial t}}{\frac{\partial^2 \Pi_j}{\partial a_j^2}}) = L < O \tag{49}$$

Finally, the variations of x_i and a_i can be written as follows:

$$\begin{pmatrix} dx_i \\ da_i \end{pmatrix} = \begin{pmatrix} \frac{\epsilon'(x_i)}{Y} & 0 \\ \frac{-w'(a_i)}{Z} & \frac{1}{Z} \end{pmatrix} \begin{pmatrix} 1 \\ L \end{pmatrix} dt$$

Consequently, we have:

$$\frac{dx_i}{dt} = \frac{\epsilon'(x_i)}{\frac{\partial^2 \Pi_i}{\partial x_i^2}} \tag{50}$$

$$\frac{da_i}{dt} = \frac{-w'(a_i)}{\frac{\partial^2 \Pi_i}{\partial a_i^2}} + \frac{1}{\frac{\partial^2 \Pi_i}{\partial a_i^2}} p'(A)(-m\frac{\frac{\partial^2 \Pi^j}{\partial a_j \partial t}}{\frac{\partial^2 \Pi^j}{\partial a_j^2}}) \tag{51}$$

Noting that $\frac{\partial^2 \Pi_i}{\partial a_i^2} = p'(A)$, we have:

$$\frac{da_i}{dt} = \frac{-w'(a_i)}{\frac{\partial^2 \Pi_i}{\partial a_i^2}} - m\frac{\frac{\partial^2 \Pi^j}{\partial a_j \partial t}}{\frac{\partial^2 \Pi^j}{\partial a_j^2}} \tag{52}$$

Appendix 3: Solutions for UMs' program

It is not immediate that in the case of total depollution, the second order condition will be satisfied. To show it, we need first to rewrite the expression:

$$SOC_t = \frac{\mu^2}{\epsilon}(\frac{ma_j}{n})^{\mu-2}[(\mu-1)(\alpha-c_d) - \frac{4\beta}{\epsilon}(2\mu-1)(\frac{ma_j}{n})^{\mu}] \tag{53}$$

The first term between brackets is necessarily negative (as μ is by construction contained between 0 and 1). The sign of the second term depends on μ. If $\mu > 1/2$, then the SOC is always satisfied. However, if $\mu < 1/2$, then there exists $\hat{a}$ for which this expression is equal to zero. Furthermore, for $\frac{ma_j}{n} > \hat{a}$ the second order condition will be positive, which means that the FOC's solution could be a minimum. However, when $\mu < 1/2$, $\lim_{a\to+\infty} FOC_t = -c_u$. This only means that the increasing part of the FOC function will be for negative values of the function. Then, we are sure that the solution of UFs' FOCs are a maximum.

Nevertheless, we have not yet precised on which part of the profit function the solution of the program will be found. First, we can notice that the profit function is continuous in a_0. However, the marginal profit is not continuous, which means that finding a solution on one part of the profit function could not necessarily lead to a global optimum. Fortunately, we can show that the jump of the function in a_0 will always be in the same direction. To prove it, we only have to make the difference between both FOCs in a_0.

$$\Delta = t\mu^2(\frac{ma_j}{n})^{\mu-1} - c_u - \frac{\alpha - c_d}{\epsilon}\mu^2(\frac{ma_j}{n})^{\mu-1} + \frac{4\beta}{\epsilon^2}\mu^2(\frac{ma_j}{n})^{2\mu-1} + c_u \tag{54}$$

$$\Delta = \mu^2 (\frac{ma_j}{n})^{\mu-1} [\frac{\alpha - c_d - t\epsilon}{\epsilon}] > 0 \quad (55)$$

As the term into brackets is also the survival condition for DFs, we are sure that the sign of this difference is constant and does not depend on the parameters. In other words, the first order condition in a_0 of the partial depollution is always above the total depollution one. Consequently, only three cases are possible for the solution of UM's program.

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